A

COMPLETE SYSTEM

OF

ASTRONOMY;

BY THE

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IN THE

UNIVERSITY OF CAMBRIDGE.

VOL. II.

SECOND EDITION,

WITH ADDITIONS AND CORRECTIONS.

LONDON:

PRINTED BY G. WOODFALL, ANGEL COURT, SKINNER STREET,

AND SOLD BY W. H. LUNN, SOHO SQUARE; J. DEIGHTON, AND J. NICHOLSON, CAMBRIDGE.

1814.

THIS VOLUME contains the PHYSICAL PRINCIPLES of ASTRONOMY, with their application to all the phænomena which arise from the mutual attraction of the bodies in our system. The author has endeavoured to render this difficult subject easy to be understood, by fully explaining all the principles, and omitting no material steps in the investigations; he trusts therefore that what he has here done may prove a considerable help to students in physical Astronomy, and tend to diffuse a more general knowledge of that subject. The Tables and Catalogues of the fixed Stars will be found very useful to the practical Astronomer.

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CHAPTER XXXI.

ON THE GENERAL PRINCIPLES OF CENTRIPETAL FORCES.

Art. 805. IF a body revolve about an immoveable center of force, and be constantly attracted to it, it will always move in the same plane, and describe areas about that center proportional to the times. For let S be the center of force, and suppose a body to be projected at P in the direction PQc, and take $PQ=Qc$; then by the first law of motion, the body would move uniformly in the direction PQc, and describe PQ, Qc in the same time, if no other force acted upon it. But when the body comes to Q, let a *single* impulse act at S, sufficient to draw the body through QV in the time it *would* have described Qc, or *did* describe PQ, and complete the parellelogram $VQcC$, and the body, in the *same* time, will describe QC; therefore PQ, QC are described in the *same* time. Now by EUCLID, B. i. p. 37, the triangle $SCQ=ScQ$, and by B. i. p. 38, $ScQ=SPQ$; therefore $SCQ=SPQ$, or equal areas are described in equal times. For the same reason, if a *single* impulse act at C, D, E, *&c.* at equal intervals of time, then $SCQ=SCD=SED=$ *&c.* Now as this is true whatever be these equal intervals, let them be diminished *sine limite*, and the limit gives a force which acts constantly; and as the reasoning respecting the equal description of areas in equal times, holds true up to the limit (as no point can be assumed before it comes to the limit when it is not true), it must be true in the limit; hence, when the force acts constantly, equal areas will be described in equal times (219, 641,) and the body will describe a curve about S. And as no force acts *out* of the plane of SPQ, the whole curve must lie in that plane. FIG. 187.

806. Draw SY perpendicular to QP produced; then the area $SPQ = \frac{1}{2} PQ \times SY$, which varies as $PQ \times SY$; therefore PQ varies as $\frac{\text{area } SPQ}{SY}$; but PQ varies as the velocity V, when the time is given; hence, V varies as $\frac{\text{area } SPQ}{SY}$. Now in the curve which is the limit of $PQCDE$, &c. SY becomes a perpendicular to a tangent to the curve. Hence, V varies as $\frac{\text{area } SPQ \text{ des. in a given time}}{\text{perpendicular on the tan.}}$; but in the *same* curve, the area SPQ is given (805) when the time is given; therefore in the *same* curve, V varies as $\frac{1}{SY}$, considering SY as a perpendicular on a tangent, to that point of the curve where the body is.

807. If equal areas be described about S in equal times, the force must tend to S. For let $SPQ = SQC$; now $SPQ = SQc$, therefore $SQC = SQc$; hence, by EUCLID, B. i. p. 39, Cc is parallel to QS, therefore $QcCV$ is a parellelogram; now, by supposition, the body describes QC in consequence of the impulse at Q, and it would have described Qc if no impulse had acted; therefore QV must represent that motion impressed at Q, which, in conjunction with the motion Qc, can make the body describe QC, and QV is directed to S.

808. Draw Qv parallel to $zCxPw$ in its limiting state. Now (NEWTON's Prin. Book I. Sect. 1. Lem. 10. Cor.) QV being the space described in a given time by the impulse of the force acting at Q, the limiting ratio of QV in one point of the curve to QV in another, will express the ratio of the forces in these two points; but $QV = 2Qx = 2Cv$; hence, the limiting ratio of Cv to Cv in two points will express the ratio of the forces. Now by making P and C approach to Q as their limit, they will arrive at Q at the same time, because PQ, QC are described in the same time; but when P and C arrive at Q, the line zw ceases to *cut* the curve, and only *touches* it at Q, and therefore it becomes a tangent; consequently Qv (which is the ultimate direction of PC) is a tangent to the curve at Q. Hence, to find the proportion of the forces in any two points P, p,
FIG. 188. of a curve, draw the tangents PX, px, to those points, take two arcs PQ, pq described in the *same* time, and draw QR parallel to PS, and qr parallel to pS; then diminish this time, and consequently the arcs PQ, pq, and make them vanish, and that ratio to which RQ, pq approach as their limit, is the ratio of the forces at P and p.

FIG. 187. 809. If the areas be *accelerated*, the force tends in *consequentia*; for if SQC be greater than SQP or SQc, then C must fall *above* a line drawn from c parallel to QS; hence, the other side QV, of the parallelogram $QcCV$, must fall *above* QS, and therefore if QV be produced, it must fall *above* S; hence, S must have moved up into that line, or the same way the body has moved, because the motion QV arises from the force at S, and therefore S must have been in the line QV. For the same reason, if the areas be *retarded*, the force tends in

antecedentia; for then, as SQC is less than SPQ or SQc, C must fall *below* a line drawn from c parallel to QS; hence, V will fall *below* QS, and therefore QV produced will fall *below* S; consequently S must have moved in a direction contrary to the motion of the body.

810. Def. If the circle PAB touch the curve WPZ at P, and RQT be drawn, making any finite angle with the tangent PF; then if RQT move up to P, and the limiting ratio of $RQ : RT$ be a ratio of equality, that circle is called a *circle of curvature* to the curve at P. FIG. 189.

811. Let PV be a chord of that circle passing through the center of force S; draw SY perpendicular to the tangent, and RQT parallel to PV, and join TP, TV; then, by the Definition, the limit of $RQ : RT$ is a ratio of equality. Now the angle $RTP =$ the alternate angle TPV, and the angle TPR between the chord and tangent $=$ the angle PVT in the alternate segment, therefore the triangles $RPT = TVP$ are similar; hence, $PV : PT :: PT : TR = \frac{PT^2}{PV}$; now when the time is given, TR becomes ultimately proportional to the force (808); also (Newt. Prin. Lem. 7. Lib. 1. Sect. 1.) the limit of the chord PT to the arc PT is a ratio of equality; but as the time is given, the arc PT is ultimately proportional to the velocity V; hence, the force varies as $\frac{V^2}{PV}$. In the *same* curve (806) V varies as $\frac{1}{SY}$; hence, the force varies as $\frac{1}{SY^2 \times PV}$. Now this being proved for the circle, it must be true for the curve, as the limit of the sagittas RQ, RT is (810) a ratio of equality, and the limit of the two arcs is also a ratio of equality, as follows from Newton's Prin. Lib. 1. Lem. 7. Sect. 1. Hence, in different parts of the *same* curve, the force varies as $\frac{1}{SY^2 \times PV}$.

812. As $\frac{V^2}{PV}$ varies as the force F in general, therefore V varies as $\sqrt{F \times PV}$; that is, the velocity in the same or different curves varies in the subduplicate ratio of the force and chord of curvature conjointly.

813. To find the force tending to the focus S of an *ellipse*. Let P be the place of the body, H the other focus, AC, BC the semi-axis major and minor, SY, HZ, perpendiculars to a tangent at P, and DC parallel to the tangent. FIG. 190. Put $s =$ the sine of the angle SPY or HPZ, radius being unity, then $s = \frac{SY}{SP}$, and $s = \frac{HZ}{HP}$; hence, $s^2 = \frac{SY \times HZ}{SP \times HP} =$ (as, by Conics, $SY \times HZ = BC^2$, and $SP \times HP = CD^2$) $\frac{BC^2}{CD^2}$; but $s^2 = \frac{SY^2}{SP^2}$, therefore $\frac{SY^2}{SP^2} = \frac{BC^2}{CD^2}$; hence, $SY^2 = \frac{SP^2 \times BC^2}{CD^2}$; also, by Conics, the *chord of curvature* $= \frac{2CD^2}{AC}$; hence, (811)

the force tending to S varies as $\frac{CD^2}{SP^2 \times BC^2} \times \frac{AC}{2CD^2}$, which varies as $\frac{1}{SP^2}$, AC and BC being constant. The same proof holds for the *hyperbola*. If the curve be a *parabola*, SY^2 varies as SP; also, the *chord of curvature* $= 4SP$, and therefore it varies as SP; hence, the force tending to the focus varies as $\frac{1}{SP^2}$. The force therefore tending to the focus of every conic section varies *inversely as the square of the distance*.

814. Draw QT perpendicular to SP; then by similar triangles, $QT^2 : QP^2 :: SY^2 : SP^2 ::$ (from the equation in the last Article) $BC^2 : CD^2$; hence, $\frac{QT^2}{QR} : \frac{QP^2}{QR} :: \frac{2BC^2}{AC} : \frac{2CD^2}{AC}$; but (811) the chord of curvature $= \frac{QP^2}{QR}$, and it also $= \frac{2CD^2}{AC}$ by Conics; hence, $\frac{QT^2}{QR} = \frac{2BC^2}{AC} =$ the latus rectum L. This is for the *ellipse* and *hyperbola*. Now if the major axis of the ellipse be increased *sine limite*, it approaches the *parabola* as its *limit*, and as all the above reasoning holds up to the limit, it must be true for the parabola. Hence, in every conic section $L = \frac{QT^2}{QR}$, when the force tends to the focus.

815. If the force vary inversely as the square of the distance, the body must describe a conic section having the center of force in its focus. For let SP, the angle SPY, the velocity and force be given; then SY is given; also, as the force is given, QR is given (808) when the time is given; and the time being given, the space PQ described with the given velocity is given; hence, QT is given; therefore $\frac{QT^2}{QR}$ is given; make $\frac{QT^2}{QR} = L$ the latus rectum of a conic section, these quantities having been proved (814) to be equal in all the conic sections. But if we know SP, SY and L, we have data sufficient to describe a conic section (see my Conic Sect.); therefore the body *may* revolve in the conic section described with these data; and if it *may*, it *must*, as a body cannot, with all the same data, describe two different curves.

816. Take PQ indefinitely small; then the area SPQ varies as $QT \times SP$; hence, SPQ^2 varies as $QT^2 \times SP^2$, or (814) as $L \times QR \times SP^2$; but (the time being given) QR varies (808) as the force; and about the *same* common center the force varies as $\frac{1}{SP^2}$; hence, QR varies as $\frac{1}{SP^2}$, therefore $QR \times SP^2$ is constant; hence, when the time is given, SPQ^2 varies as L in *different* conic sections about the *same* common focus. About *different* foci, where the absolute forces are different, if $A =$ the absolute force, then the force varies as $\frac{A}{SP^2}$, therefore QR varies as $\frac{A}{SP^2}$; hence, SPQ^2 varies as $A \times L$.

817. As (812) V varies as $\sqrt{\text{force} \times \text{ch. curv.}}$ therefore at the same distance the force being the same, we have the velocity (V) in the conic section at P : velocity (v) in a circle at the same distance :: $\sqrt{\frac{2CD^2}{AC}}$: $\sqrt{2SP}$ :: (in the ellipse and hyperbola) $\sqrt{\frac{2SP \mp HP}{AC}}$: $\sqrt{2SP}$ :: $\sqrt{HP}$: $\sqrt{AC}$. In the *ellipse*, $HP = 2AC - SP$; hence, $V : v :: \sqrt{2AC - SP} : \sqrt{AC}$, which is always *less* than $\sqrt{2} : 1$. If the major axis of the ellipse be increased *sine limite*, the ellipse approaches to the parabola as its limit, and the above ratio approaches to $\sqrt{2} : 1$ as its limit; hence, in a *parabola*, $V : v :: \sqrt{2} : 1$ (645). In the *hyperbola*, $HP = 2AC + SP$; hence, $V : v :: \sqrt{2AC + SP} : \sqrt{AC}$, which is always *greater* than $\sqrt{2} : 1$. At the mean distance in the ellipse, $SP = AC$; hence, $V = v$ at that point.

818. Let M = the major axis of an ellipse, N = the minor, P = the periodic time about the focus, m = the whole area, n = the area SPQ described in a given time; then $m = n \times$ *the number* of these areas; but *the number* of these areas must vary as P, because equal areas are described in equal times, and therefore the greater P is, the greater will be *the number* of areas in proportion; hence, m varies as $n \times P$, therefore P varies as $\frac{m}{n}$. Now by the property of the ellipse, m varies as $M \times N$, and $L = \frac{N^2}{M}$; therefore (816) n varies as $\frac{N \times A^{\frac{1}{2}}}{M^{\frac{1}{2}}}$, consequently P varies as $\frac{M^{\frac{3}{2}}}{A^{\frac{1}{2}}}$. Hence, in different ellipses about the *same* common focus, P varies as $M^{\frac{3}{2}}$. This law was discovered by KEPLER (218). If the ellipse become a circle, M becomes the diameter, which varies as the radius R; therefore in *different* circles about *different* centers, P varies as $\frac{R^{\frac{3}{2}}}{A^{\frac{1}{2}}}$; and about the *same* center, P varies as $R^{\frac{3}{2}}$.

819. If, by the same law of force, bodies revolve in *different* circles about *different* centers, and A = the absolute force, R = the radius, then the force varies as $\frac{A}{R^2}$; therefore (812) V varies as $\sqrt{\frac{A}{R^2} \times 2R}$, or as $\frac{A^{\frac{1}{2}}}{R^{\frac{1}{2}}}$. Hence, about the *same* common center, V varies as $\frac{1}{R^{\frac{1}{2}}}$.

820. The angle QSP varies as $\frac{QT}{SP}$; but QT varies as $\frac{\text{area } SPQ}{SP}$; therefore the angle QSP varies as $\frac{\text{area } SPQ}{SP^2}$. Hence, in the *same* orbit, the angular ve-

locity varies inversely as the square of the distance, the area SPQ being constant (805) when the time is given, let the orbit be what it will.

821. By Art. 817. we have, in the ellipse, $V : v :: \sqrt{HP} : \sqrt{AC}$; hence, $V = v \times \frac{\sqrt{HP}}{\sqrt{AC}}$; now this is true whatever be the minor axis of the ellipse; if therefore we diminish the minor axis and make it vanish, the body will move in a right line, and as all the reasoning holds true up to that limit, it must be true in the limit.

822. Hence, draw Pp perpendicular to AM, and make the minor axis of the ellipse vanish, then S coincides with A, H with M, P with p, HP becomes Mp, and the body descends in a right line to p from rest at M; hence, (821) V (the velocity acquired from M to p) $= v \times \frac{\sqrt{pM}}{\sqrt{AC}}$; but (819) v varies as $\frac{1}{\sqrt{Ap}}$; let $v = \frac{a}{\sqrt{Ap}}$; and $V = \frac{a\sqrt{pM}}{\sqrt{Ap \times AC}}$; that is, the velocity acquired in falling from a state of rest, is equal to a certain constant quantity a multiplied into the square root of the space described divided by the square root of the space to be described multiplied into half the whole space.

823. Produce SP to L, and take $PL = PH$; then (822) the velocity (m) of a body falling from L to $P = \frac{a\sqrt{PL}}{\sqrt{SP \times \frac{1}{2}SL}} = \frac{a\sqrt{PH}}{\sqrt{SP \times AC}}$; also, let $n = \frac{a}{\sqrt{SP}}$ the velocity in the circle at P, and $p =$ the velocity in the ellipse; then

$$m : n :: \frac{\sqrt{PH}}{\sqrt{AC}} : 1 :: \sqrt{PH} : \sqrt{AC}; \text{ also}$$

$$n : p :: \sqrt{AC} : \sqrt{PH} \ (817)$$

$$\therefore m : p :: \sqrt{AC} \times \sqrt{PH} : \sqrt{PH} \times \sqrt{AC}; \text{ hence, } m = p,$$

That is, a body revolving in an ellipse, must fall externally through a space equal to the distance of the body from the other focus, to acquire the velocity in the ellipse.

FIG. 189. 824. Let $PL = \frac{1}{4}PV$, then a body, with the force at P continued *constant*, must fall down PL to acquire the velocity in the curve. For let $m =$ the velocity down RQ with the force at P, $p =$ the velocity down PL by the same force, $p =$ the velocity in the curve PQ; then in the time RQ is described by a falling

body; PQ is described in the curve, and the velocity through RQ is (by Mechanics) represented by $2RQ$; hence, $m : p :: 2RQ : PQ$, therefore

$$m^2 : p^2 :: 4RQ^2 : PQ^2,$$
$$\text{Also, } m^2 : n^2 :: RQ : PL, \text{ by Mechanics;}$$

but $n=p$, therefore $PL=\frac{PQ^2}{4QR}=(811)\ \frac{1}{4}\ PV.$

Cor. 1. Hence, by Mechanics, if $v=$velocity in the curve at P, $F=$force at that point, $v=\sqrt{2F\times\frac{1}{4}PV}=\sqrt{\frac{1}{2}F\times PV}$; therefore, $v \propto \sqrt{F\times PV}$.

Cor. 2. If a body revolve in a circle about the center, $\frac{1}{4}PV=\frac{1}{2}r$ ($r=$radius); hence, $v=\sqrt{F}$. Further, let $F\propto r^n$. Put the radius of the earth $=1$, gravity at the earth's surface $=1$, and the velocity of a body revolving about the earth at its surface $=1$; then, as the velocity varies as the square root of the force and space, we have $1 : v :: \sqrt{1\times\frac{1}{2}} : \sqrt{r^n\times\frac{1}{2}r}$; hence, $v=r^{\frac{n+1}{2}}$.

Cor. 3. As the periodic time (p) in a circle varies as r directly and v (velocity) inversely, $p\propto r^{\frac{1-n}{2}}$. If $n=-2$, $p\propto r^{\frac{3}{2}}$, which is the case in the planetary system.

825. If the curve be a circle, and the force be in the center, then $PL=$ half the radius. Hence, as (817) the velocity in an ellipse at the mean distance $=$ the velocity in a circle, a body at that point of an ellipse must fall down half the distance to acquire the velocity in the ellipse, the force remaining constant.

826. When a body revolves about a center of force, that part of its motion which is perpendicular to the radius vector gives the body a tendency to recede from the center; and the force with which the body thus recedes is called a *centrifugal* force. Let S be the center of força, PK the curve described, PT a tangent to it, SY perpendicular to PT, and PQ an indefinitely small arc; draw Qw perpendicular to SP, and with the center S describe the circular arc Qx, and let RQ be parallel to SP, and PV be the chord of the circle of curvature. Let PQ represent the motion of the body in the curve, in a given time, then Pw represents that part of the motion which is towards the center, and by which alone the body would be found, at the end of the given time, at the distance Sw; but on account of the motion wQ, it is found at the end of the same time at the distance SQ or Sx; the perpendicular motion wQ has therefore made the body recede from S through a space equal to wx, which therefore represents the *centrifugal* force. Also, the centripetal force is repre-

FIG. 191.

sented (808) by QR. Now $wx = \frac{xQ^2}{2xS}$ ultimately; but xQ^2 varies as $\frac{\overline{\text{area } SxQ^2}}{xS^2}$; therefore wx varies as $\frac{\overline{\text{area } SxQ^2}}{2xS^3}$ which varies as $\frac{\overline{\text{area } SPQ^2}}{SP^3}$ ultimately. Hence, in the *same* curve, the centrifugal force varies as $\frac{1}{SP^3}$, the area SPQ described in a given time being given (805). And in *different* curves, if the distance be the same, the centrifugal forces are as the squares of the areas described in a given time.

Cor. 1. Hence, the centripetal force in the curve : the centrifugal force :: $QR : xw$ :: $\left[\text{because (811) } QR = \frac{QP^2}{PV}, \text{ and } xw = \frac{xQ^2}{2QS} = \frac{xQ^2}{2PS}\right] \frac{QP^2}{PV} : \frac{xQ^2}{2PS}$ :: (as by sim. tria. $QP^2 : Qx^2 :: SP^2 : SY^2$) $\frac{SP^2}{PV} : \frac{SY^2}{2SP} :: 2SP^3 : SY^2 \times PV$.

Cor. 2. Let the curve be an ellipse whose major axis is $2a$, and the excentricity $= w$, and the body be at the greatest distance from the center of force, which is supposed to be in the focus; then the centripetal force : centrifugal :: $SP : \frac{1}{2} PV :: a + w : \frac{a^2 - w^2}{a} :: a : a - w$.

827. If the ratio of $a + mx : b + nx$ be constant, x only being variable, then $a : b :: m : n$. For if $x = 0$, the ratio becomes $a : b$; hence, $a + mx : b + nx :: a : b$, therefore (alternando and dividendo) $mx : a :: nx : b$, consequently $a : b :: m : n$.

FIG. 192. 828. Let VPA be an ellipse whose focus is S, and center C, VW a curve so constructed, that Sp may be always equal to SP, and the angle VSp to VSP in a given ratio $G : F$, then the areas VSp, VSP will be in the same given ratio. Now let a body revolve from V to P about the center of force S, in the same time in which another body revolves from V to p. Then as the area VSP varies as VSp, and (805) the area VSP varies as the time, the area VSp varies as the time, consequently the body describing Vp is (807) urged by a force tending to S. Let Pv be the chord of curvature. Now (827) the centrifugal forces of the two bodies are as $G^2 : F^2$, or as $\frac{G^2}{SP^3} : \frac{F^2}{SP^3}$, let them therefore be represented by these quantities; hence, the difference of the centrifugal forces is $\frac{G^2 - F^2}{SP^3}$. Now if p recede from the center by a centrifugal force which is greater by $\frac{G^2 - F^2}{SP^3}$ than that by which P recedes, it is manifest that p must be acted upon by a centripetal force which is greater by the same quantity, in order to destroy it, so that the bodies may continue at the same distance. Now (828)

$SY^2 \times Pv : 2SP^3 :: \frac{F^2}{SP^3}$ (the centrifugal force in the ellipse at P) $: \frac{2F^2}{SY^2 \times Pv}$ the centripetal force in the ellipse at P; hence, the force in the orbit VW at p $= \frac{2F^2}{SY^2 \times Pv} + \frac{G^2 - F^2}{SP^3}$. But $SY^2 = \frac{AC \times R \times SP^2}{CD^2}$, and $Pv = \frac{2CD^2}{AC}$, R being half the latus rectum, and CD the semi-conjugate diameter to PC; hence, the force at $P = \frac{F^2}{R \times SP^2}$, and at $p = \frac{F^2}{R \times SP^2} + \frac{G^2 - F^2}{SP^3}$; therefore the ratio of these forces is $\frac{F^2}{SP^2} : \frac{F^2}{SP^2} + \frac{RG^2 - RF^2}{SP^3}$, agreeable to what Sir I. NEWTON has proved in his PRINCIPIA, lib. i. sect. 9. If we apply this to the moon, the forces will be nearly as $F^2 : G^2$ or $60 : 61$. Hence, the periodic time which is diminished in the ratio of $G : F$, will be less in the ratio of $121 : 120$ nearly.

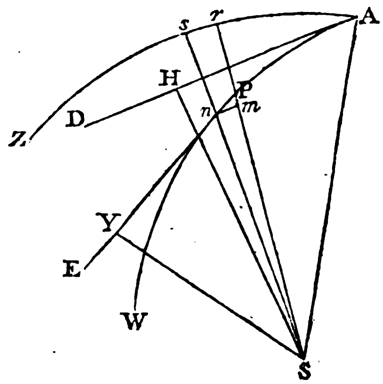
829. If the orbit VPA be very near to a circle, the force $\frac{F^2}{SP^2} + \frac{RG^2 - RF^2}{SP^3}$ may be made to vary very nearly as any power of SP, or as SP^{n-3}. For $\frac{F^2}{SP^2} + \frac{RG^2 - RF^2}{SP^3} : SP^{n-3} :: F^2 \times SP + RG^2 - RF^2 : SP^n ::$ (putting $T - x = SP$, where T is the greatest distance, and neglecting all the terms where the powers of x enter above the first, as being very small when compared with the rest) $F^2T - F^2x + RG^2 - RF^2 : T^n - nT^{n-1}x$; now (827) this ratio will be constant, if we assume the constant terms on each side in the same ratio to each other as the coefficients of the variable terms; but if the ratio of the two last quantities be constant, the ratio of the two first must be very nearly so, as we have only neglected terms which are very small in respect to those which are retained; assume therefore $F^2T + RG^2 - RF^2 : T^n$ as $-F^2x : -nT^{n-1}x$ or as $F^2 : nT^{n-1}$, and the ratio of the two first terms above becomes very nearly constant; but as R and T are very nearly equal, the proportion becomes $G^2 : F^2 :: 1 : n$, very nearly; hence, $G = \frac{F}{\sqrt{n}}$. Now the two bodies at P, p, being always at the same distance from S, must come to an apside at the same time; but the body in the ellipse comes to an apside when $F = 180°$; hence, $G = \frac{180°}{\sqrt{n}}$ the angle described between the apsides by the body in the curve VW, when the force varies as the $n-3$ power of the distance. If $n-3 = -2$, the distance of the apsides is $180°$; if $n-3$ be a negative number *greater* than 2, the distance of the apsides is *greater* than $180°$, but if it be *less* than 2, the distance is *less* than $180°$. That is, if the force vary inversely as the square of the distance, the apsides of the orbit described are at rest; if the force vary in a greater or less

ratio than the inverse square of the distance, the apsides are progressive or regressive.

830. Let the force vary as $\frac{b \times SP^m \pm c \times SP^n}{SP^3}$, and the greatest distance be assumed unity; then by exactly the same process, we find $G = 180° \times \sqrt{\frac{b \pm c}{bm \pm cn}}$ the angle between the apsides. In the *former* case, if the distance ($d°$) of the apsides be given, we can find the law of force; for if $d° = \frac{180°}{\sqrt{n}}$, then $n = \frac{180^2}{d^2}$; consequently the law of force is $SP^{\frac{180^2}{d^2} - 3}$.

The conclusions hitherto deduced when the force varies in the inverse square of the distance, have been upon supposition that the bodies were of indefinitely small magnitudes; we shall afterwards consider what will be the consequence if the bodies be spherical, and of finite magnitudes.

831. Suppose the law of force to vary as any power of the distance, to find the curve which the body describes. Let S be the center of force, and the



body be projected at A in the direction AD, and let APW be the curve described. With the center S describe the circular arc AZ; let P be any place of the body; draw the tangent PE, on which let fall the perpendicular SY, and SH on AD; also, draw Sn indefinitely near SP, and nm perpendicular to SP, and produce SP, Sn, to r, s. Put $SA = a$, $SH = p$, $Ar = z$, $SP = x$, $b =$ velocity at A, $v =$ velocity at P, $Pm = \dot{x}$, $rs = \dot{z}$. Now the velocity being inversely as the perpendicular on the tangent, $v : b :: p : SY = \frac{pb}{v}$; therefore $PY = \sqrt{x^2 - \frac{p^2b^2}{v^2}} = \frac{\sqrt{x^2v^2 - p^2b^2}}{v}$; and by sim. tri. $\frac{\sqrt{x^2v^2 - p^2b^2}}{v} : \frac{pb}{v} :: \dot{x} : mn = \frac{pb\dot{x}}{\sqrt{x^2v^2 - p^2b^2}}$; hence, $x : a :: \frac{pb\dot{x}}{\sqrt{x^2v^2 - p^2b^2}} : \dot{z} = \frac{pab\dot{x}}{x\sqrt{x^2v^2 - p^2b^2}}$, expressing the flux-

ional equation of the curve in terms of the angle described and distance. But (see my Fluxions, Art. 82.) $v^2 = b^2 + \frac{2}{n+1} \times \overline{a^{n+1} - x^{n+1}}$; or if b (vel. of proj.) : vel. $a^{\frac{n+1}{2}}$ in Cir. (Art. 824. Cor. 3.) :: $m : 1$, then $b^2 = m^2 a^{n+1}$; hence, $v^2 = \overline{m^2 + \frac{2}{n+1}} \times a^{n+1} - \frac{2}{n+1} \times x^{n+1}$; therefore by substitution, $\dot{z} =$

$$\frac{pab\dot{x}}{x\sqrt{m^2 + \frac{2}{n+1} \times a^{n+1} \times x^2 - \frac{2}{n+1} \times x^{n+3} - p^2 m^2 a^{n+1}}},$$ the fluent of which can only be found in particular cases.

Cor. At the apsides, $SP = SY$, or $x = \frac{pb}{v} = \frac{pb}{\sqrt{m^2 + \frac{2}{n+1} \times a^{n+1} - \frac{2}{n+1} \times x^{n+1}}}$, and $x\sqrt{m^2 + \frac{2}{n+1} \times a^{n+1} - \frac{2}{n+1} \times x^{n+1}} - pb = 0$, the equation to the apsides. Now to find the *number* of apsides, by squaring the first equation, we get $\overline{m^2 + \frac{2}{n+1}} \times a^{n+1} \times x^2 - \frac{2}{n+1} \times x^{n+3} - p^2 b^2 = 0$, which equation may have 4 possible roots when n is an even number, and 3 when n is an odd number; but this being the square of the original equation, some of the roots are introduced by that operation, and the equation to the apsides can never have more than 2 possible roots, so that no orbit of this kind can have more than 2 apsides, that is, there are only two different distances of the apsides; but there is no limit to the number of repetitions of those, without their falling upon the same points. If $n = -3$ or a greater negative number, the equation can have only 1 possible root, and therefore the orbit but one apside.

832. Given as in the last Article, to find at what point of the curve, the motion of the body towards the center is the greatest. We have found $mn = \frac{pb\dot{v}}{\sqrt{x^2 v^2 - p^2 b^2}}$; now *dato tempore*, the area SPn (A) is given, and $mn = \frac{2A}{x}$; hence, $\dot{v} = \frac{2A}{pb} \times \sqrt{v^2 - p^2 b^2 x^{-2}} = \frac{2A}{pb} \times \sqrt{m^2 + \frac{2}{n+1} \times a^{n+1} - \frac{2}{n+1} \times x^{n+1} - p^2 b^2 x^{-2}}$ $=$ a maximum; or $\frac{2}{n+1} \times x^{n+1} + p^2 b^2 x^{-2} =$ a minimum; put therefore the fluxion $= 0$, and we get $x = \overline{pb}^{\frac{2}{n+3}}$ the distance required. From this we shall get a very remarkable coincidence. We have $v : b :: \frac{1}{SY} : \frac{1}{p}$; hence, $pb = SY \times v$; also, $x = SP$; therefore $SP^{\frac{n+3}{2}} = SY \times v$; let $C =$ chord of curvature at P. Now (Art. 824. Cor. 1.) as the force in the curve and in the circle of curvature at P is the same, the velocities will be as the square roots of the chord of curva-

ture, and the velocity in the circle (Art. 824. Cor. 2.) $= SP^{\frac{n+1}{2}}$; hence, v : $SP^{\frac{n+1}{2}} :: \sqrt{C} : \sqrt{2SP}$, and $v = SP^{\frac{n+1}{2}} \times \sqrt{\frac{C}{2SP}}$; therefore $SP^{\frac{n+2}{3}} = SY \times SP^{\frac{n+1}{2}} \times \sqrt{\frac{C}{2SP}}$, and $2SP^3 = SY^2 \times C$. But (Art. 826. Cor. 1.) the former represents the centripetal force, and the latter the centrifugal force; hence, in general, *at the point of the orbit when the centripetal and centrifugal forces are equal, the velocity towards the center is a maximum.* If the curve be an ellipse with the force tending to the *focus*, this point is at the extremity of the ordinate to the major axis passing through the focus. If the force tend to the center, it is where the distance is a mean proportional between the semi-axes major and minor.

FIG. 193. 833. Let O be the center of the circle $ABCD$, draw OP perpendicular to the plane, and let P be a corpuscle attracted to the circle; describe about O the circle vw, and let the attraction of P to any particle $v = \frac{1}{Pv^2}$. Put $PO = a$, $Pv = x$, $p = 3{,}14159$; then $Ov^2 = x^2 - a^2$, and $p \times \overline{x^2 - a^2}$ = the area of the circle vw; hence, the fluxion of that area is $2px\dot{x}$; and by the resolution of forces, $x : a :: \frac{1}{x^2} : ax^{-3}$ the attraction of P to v in the direction PO; hence, the fluxion of the attraction of the corpuscle P to the circle vw will be $2p \times ax^{-2}\dot{x}$, whose fluent is $2p \times -\frac{a}{x}$; but when $x = a$, $vO = o$, and the attraction vanishes; hence, the fluent corrected, or the attraction of P to the circle vw is $2p \times \overline{1 - \frac{a}{x}}$; and when $x = PA$, the attraction to the whole circle becomes $2p \times \overline{1 - \frac{a}{PA}}$.

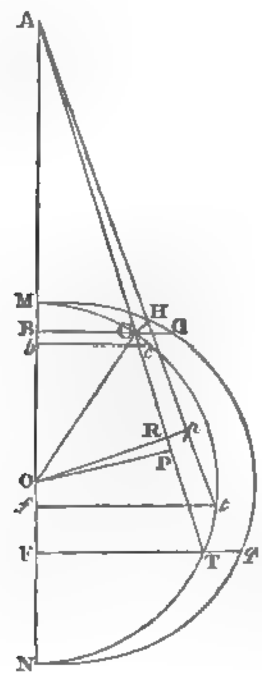
FIG. 194. 834. Let $ABCD$ be a sphere, P a corpuscle *without* the sphere; draw $PAOC$ through the center O, and let $BvDw$ be a section perpendicular to it. Put $AO = a$, $OP = b$, $AP = b - a = c$, $PK = y$, and let $PB = c + x$, then $AK = y - c$, and $CK = 2a - y + c$, therefore $\overline{y - c} \times \overline{2a - y + c} = BK^2 = BP^2 - PK^2 = \overline{c + x}^2 - y^2$; hence, (as $b = a + c$) $y = \frac{2bc + 2cx + x^2}{2b}$; therefore (833) the attraction of P to the circle $BvDw$ is $2p\left(1 - \frac{2bc + 2cx + x^2}{2b \times \overline{c + x}}\right)$ or $p \times \frac{2ax - x^2}{b \times \overline{c + x}}$; also, $\dot{y} = \frac{c\dot{x} + x\dot{x}}{b}$; hence, the fluxion of the attraction of P to the sphere is $p \times \frac{2ax\dot{x} - x^2\dot{x}}{b^2}$, whose fluent is $p \times \frac{ax^2 - \frac{1}{3}x^3}{b^2}$, the attraction to ABD; and when

$x = 2a$, the attraction to the whole sphere becomes $\frac{4pa^3}{3b^2}$, which varies as $\frac{a^3}{b^2}$. Now if the density d of the sphere should vary, the attraction must, cæteris paribus, vary as d; hence, for all spheres, the attraction varies as $\frac{da^3}{b^2}$. But the quantity of matter m varies as da^3; therefore the attraction varies as $\frac{m}{b^2}$. Hence, if the spheres were evanescent in magnitude, with the same quantity of matter, the attraction would be the same; consequently the attraction of a corpuscle to a sphere is just the same as if all the matter of the sphere were collected into its center. If the corpuscle be at the surface of the sphere, then $a = b$, and the attraction varies as ad. If P be *within* the sphere, in like manner find the attraction of P to the parts lying between P and A, P and C, and the difference of their attractions will be the whole attraction of P to O; and this comes out to vary as PO.

835. Hence, if the particles of two spheres A, B, attract each other by a force varying in the inverse square of the distance, the attraction is the same as if the whole quantity of matter in each sphere were collected into its respective center; because the attraction of *each* corpuscle of one sphere A to the other sphere B is the same as if the whole quantity of matter in B were concentrated into its center, and therefore the attraction of the *whole* sphere A to B must also be the same as if the whole quantity of matter in B were collected into its center. Hence, what has been proved for two corpuscles attracting each other when the force varies inversely as the square of the distance, holds true for two spheres, the particles of which attract each other according to the same law. If therefore (815) the particles of two spheres attract each other with a force varying in the inverse square of the distance, one sphere will describe a conic section about the other in its focus. Now all the planets are spherical, and (217) they revolve about the sun in ellipses having the sun in the focus of each. Hence, we conclude, that each planet is attracted to the sun by a force which varies inversely as the square of the distance of their centers, and that the constituent particles also attract each other by a force varying according to the same law.

836. To find the attraction of an oblate spheroid, which is very nearly a sphere to a point situated in the minor axis produced. Let MN be the minor axis of the oblate spheroid $MQqN$, $MCTN$ the inscribed sphere, O the center; draw ACT, Act indefinitely near each other, BCQ, bc, ft, FTq,* perpendicular

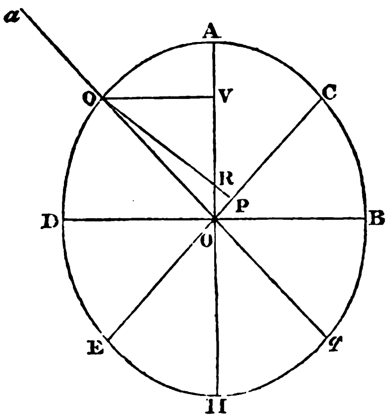
* c should have been drawn from the intersection of At, Mt.



to AMN, OP perpendicular to AT, ORp, CH, to At, and join OC. Put $AO = a$, $MO = c$, $c + b =$ semiaxis major, then $QC = \frac{b}{c} \times BC$, and if $CP = x$, $OP = \sqrt{c^2 - x^2}$; and let $p = 6,283$, &c. Then $\frac{pb}{c} \times BC^2 =$ the annulus generated by QC about B, and as the force is supposed to vary inversely as the square of the distance from each particle, $\frac{pb}{c} \times \frac{BC^2 \times Bb}{AC^2} =$ attraction to the annulus whose thickness is Bb in the direction CA, and $CA : AB :: \frac{pb}{c} \times \frac{BC^2 \times Bb}{AC^2} : \frac{pb}{c} \times BC^2 \times \frac{AB \times Bb}{AC^3}$ the attraction (A) in the direction AO. But by sim. tri. $\frac{BC^2}{AC^2} = \frac{PO^2}{AO^2}$; also, $Bb : Cc :: BC : CO$, and (sim. tri.) $Cc : Hc :: CO : PO :: CO : \frac{AO \times BC}{AC} :: CO \times AC : AO \times BC$, also $Hc : HC :: PO : PC$, and $HC : Rp :: AC : AR$ or AP, and by composition of these ratios $Bb : Rp :: BC \times CO \times AC^2 \times PO : CO \times AO \times BC \times PC \times AP$; hence, $Bb = \frac{AC^2 \times PO \times Rp}{AO \times PC \times AP}$; and the above attraction $(A) = \frac{pb}{c} \times \frac{PO^3 \times AC^2 \times Rp}{PC \times AO^4}$. In like manner the attraction (a) to

the annulus $Tq = \frac{pb}{c} \times \frac{PO^3 \times AT^2 \times Rp}{PC \times AO^4}$; and as $AC^2 + AT^2 = 2AP^2 + 2PC^2$, $A + a = \frac{pb}{ca^4} \times \frac{PO^3}{PC} \times \overline{2AP^2 + 2PC^2} \times Rp$; but $Rp = \overline{OP} = \frac{x\dot{x}}{\sqrt{c^2 - x^2}}$; hence, $A + a = \frac{2pb}{ca^4} \times \overline{c^2 - x^2} \times \overline{a^2 - c^2 + 2x^2} \times \dot{x}$, whose fluent, when $x = c$, is $\frac{2pb}{ca^4} \times \overline{\frac{2}{3}a^2c^3 - \frac{2}{5}c^5}$ the whole attraction of the matter in the spheroid above the sphere; to this add (Art. 834.) $\frac{2pc^3}{3a^2}$ the attraction to the sphere, and we get the attraction to the spheroid $= \frac{2pc^3}{3a^2} + \frac{4pbc^3}{3a^2} - \frac{4pbc^4}{5a^4}$.

837. Let $ABHD$ be a spheroid generated about the minor axis DB,



O the center, QOq any diameter, COE its conjugate, to which draw QRP perpendicular, and QV to AH. By Conics $VO^2 = \frac{AO^2}{DO^2} \times \overline{DO^2 - QV^2}$, and $RV = \frac{DO^2}{AO^2} \times VO$ (see my Conic Sections); hence, $RV^2 = \frac{DO^4}{AO^2} - \frac{DO^2 \times QV^2}{AO^2} + QV^2$. Let the ellipse be nearly a circle, and $c = DO$, $b =$ the difference of the semi-axes, $QV = x$, then $QR^2 = \frac{c^4 - c^2x^2}{c^2 + 2cb} - x^2$ very nearly, $= c^2 - 2cb + \frac{2bx^2}{c^2}$; and if $t = \sin.\ r = \cos.\ ARQ$, $x^2 = QR^2 \times t^2$, and $QR^2 - \frac{2b}{c} \times QR^2 \times t^2 = c^2 - 2cb$; therefore $QR^2 = \frac{c^2 - 2cb}{1 - \frac{2bt^2}{c}} = c^2 - 2cb + 2cbt^2$, and $QR = c - b + bt^2$ nearly. But $QV^2 = QR^2 \times t^2 = \overline{c - b + bt^2}^2 \times t^2$; hence, $QO^2\ (= QV^2 + VO^2) = AO^2 - 2bct^2$

and $QO = AO - bt^2 = c + b - bt^2 = c + br^2$. And as $QO \times CO = AO \times DO$, $CO = \frac{AO \times DO}{QO} = \frac{c^2 + cd}{c + br^2} = c + b - br^2$.

838. Produce OQ to a, to find the attraction of the spheroid to a. If about the same center, and in the same plane, be described a circle and an ellipse equal in area to it and very nearly a circle, a point in a perpendicular to that plane from the center of the circle, will be attracted to each equally very nearly, the peripheries differing very little from each other. And if the ellipsis be a very little inclined from that plane about its minor axis, neglecting the difference of the increments and decrements of the forces, the forces will remain nearly the same. Let therefore any number of planes be drawn perpendicular to $ADHB$, and these ellipses will be similar, and the attraction of each to a will be equal to the attraction of its respective circle as above described. Now of the ellipse passing through EOC, the semi-axes are OC, OA, and to this the other ellipses are similar; therefore the whole attraction to the given spheroid in the direction Oa to a will be equal to the attraction to another spheroid revolving about its axis Qq and whose equatorial radius is a mean between OC, OA, or (last Art.) between $c + b$ and $c + b - br^2$, or $c + b - \frac{1}{2} br^2$, with the minor axis $c + br^2$; and the difference of these two axes $= b - \frac{3}{2} br^2$. Substitute these for the axes in Art. 836. and by reduction the attraction $= \frac{2pc^3}{a^2} + \frac{4pc^2b}{3a^2} + \frac{2pc^4b}{5a^4} - \frac{6pc^4bt^2}{5a^4}$.

Hence, the attraction of a spheroid to a point without, varies partly as $\frac{1}{a^2}$ and partly as $\frac{1}{a^4}$; these, therefore, conjointly constituting a force not varying as $\frac{1}{a^2}$, will affect the motion of the moon's apogee, and introduce an equation of the moon's motion, as determined by de la PLACE.

FIG. 195. 839. A body P attracting another body Q, exerts its influence equally upon every particle of Q, and therefore the acceleration of Q to P is the same whatever be the quantity of matter in Q, and will be in proportion to the quantity of matter in P, the magnitudes of the bodies being supposed to be indefinitely small, so that every particle of matter in one body may be supposed to be equidistant from every particle in the other. In like manner it appears, that the acceleration of P towards Q, from the attraction of Q, is in proportion to the quantity of matter in Q. Let therefore P and Q attract each other, and G be their center of gravity; then the acceleration of Q towards P from the action of P : the acceleration of P towards Q from the action of Q :: $P : Q$ :: $GQ : GP$;

hence, the spaces Qa, Pb, moved over by Q and P from their mutual attractions in an indefinitely small time, will be as $GQ : GP$; consequently Ga must be to Gb in the same ratio, therefore G continues to be their center of gravity. Hence, the center of gravity is not affected by the mutual attractions of the bodies.

840. Now let Q and P be projected in the directions Qr, Ps, opposite and parallel to each other, with velocities as $GQ : GP$, or as $Ga : Gb$, and let Qr, Ps be the spaces that would have been described in the time in which the bodies would have moved over Qa, Pb by their mutual attractions, and complete the parallelograms $Qrqa$, $Pspb$; then the bodies at the end of that time will be found at q and p. Now the spaces described in the same time being as the velocities, Qr, or aq, : Ps, or bp, :: $Ga : Gb$; also, the angle $qaG = pbG$; hence, the triangles Gaq, Gbp are similar, consequently the angle $aGq = bGp$, and therefore pGq is a straight line; also, $Gq : Gp :: Ga : Gb :: GQ : GP :: P : Q$; hence, G is the center of gravity of P and Q when they come to p and q; consequently the center of gravity will still be at rest, and the bodies will describe similar figures about G. Now let us conceive each body to be acted upon, at the same time, by equal accelerative forces in the same direction, then the *relative* motions of the two bodies will not be altered, and they will still continue to describe similar figures about G which is now in motion. And by varying the motion of the system, we may vary the absolute initial velocities of P and Q as we please. Hence, if P and Q be projected with *any* velocities, they will continue to revolve about their center of gravity, and describe similar figures about it. And the center of gravity not being disturbed by their mutual attractions (839), will continue to move on uniformly in a straight line. Hence, the center of gravity of the solar system remains at rest, or moves uniformly in a straight line, the latter of which is probably the case (729). Sir I. NEWTON has therefore concluded, that the earth and moon revolve about their center of gravity; but FRISI has maintained, that this will not be the case, unless the earth and moon had been at first projected in opposite directions, with velocities inversely as their quantities of matter; it appears however, from what is proved above, that this is by no means necessary.

841. Let the bodies be spherical, and the particles attract each other by a force varying in the inverse square of the distance, then (834) the whole attraction will vary inversely as the square of the distance of their centers. Now as the attraction of Q to P varies as $\frac{1}{QP^2}$ it must vary as $\frac{1}{QG^2}$, because QP is to QG in a given ratio; and as G is in the line QP, the acceleration of Q towards G must be the same as towards P; hence, Q is attracted towards G with a force which varies as $\frac{1}{QG^2}$; therefore (815) Q describes about G a conic sec-

tion having G in the focus. For the same reason, P will describe a similar figure about G in its focus. Also, as $QG : QP$ in a given ratio, and G is always in the line QP, the angular velocity of Q about G must be equal to its angular velocity about P, because, in respect to any fixed line LM, QG and QP always make the same angle; therefore Q describes a figure about P similar to that which it describes about G, and in the same periodic time. Now all the planets are attracted to the sun by a force varying according to the above law; hence, each planet describes about the center of gravity of itself and the sun, an ellipse having that center in their focus, except so far as they disturb each other's motions by their mutual attractions.

842. Conceive a body Z to be placed at G, whose attraction upon Q shall be equal to that of P; then as the attraction varies as the quantity of matter directly and the square of the distance inversely, we have $\frac{Z}{QG^2} = \frac{P}{QP^2}$; hence, $Z = P \times \frac{QG^2}{QP^2}$. Now (818) the squares of the periodic times of bodies revolving about the focus of an ellipse vary as the cubes of the major axes directly and the absolute forces inversely; therefore, if the periodic time be given, the major axis must vary as the cube root of the absolute force. Hence, the major axis of the ellipse which Q describes about Z : the major axis of the ellipse which Q would describe about P at rest in the same periodic time $:: P^{\frac{1}{3}} \times \frac{QG^{\frac{2}{3}}}{QP^{\frac{2}{3}}} : P^{\frac{1}{3}} :: QG^{\frac{2}{3}} : QP^{\frac{2}{3}}$; also, from similar figures, the major axis of Q about P at rest : the major axis of Q about $Z :: QP : QG$; hence, by compounding these ratios, we get the major axis of the ellipse which Q describes about P when they revolve about the center of gravity : the major axis of the ellipse which Q would describe about P at rest in the same periodic time $:: QP^{\frac{1}{3}} : QG^{\frac{1}{3}} :: \overline{P+Q}^{\frac{1}{3}} : P^{\frac{1}{3}}$. This is the same conclusion as that deduced by Sir I. Newton in a very different manner, in his *Principia*, Lib. I. Sect. 2. Pr. 60. Hence, as the quantity of matter in the earth : that of the moon :: 78 : 1, the major axis of the ellipse which the moon describes about the earth (they revolving about their common center of gravity) : the major axis of the ellipse which the moon would describe about the earth at rest in the same time $:: 79^{\frac{1}{3}} : 78^{\frac{1}{3}} :: 429 : 427$.

FIG. 196. 843. Let a body E revolve about a body S in a circle, and at the same time let a body M revolve about E in a circle and be carried with E about S; to find the disturbing force of S upon M revolving about E, supposing ME to be very small when compared with SE, and the force to vary inversely as the

square of the distance. As the relative situation of M to E is just the same as if E was at rest, let us suppose E to be at rest. Produce SAE to D, draw CEB perpendicular to AD, and MK to CB. Now let ES represent the attractive force of E to S, then $\frac{1}{SE^2} : \frac{1}{SM^2} :: SE$: the force of M to $S = \frac{SE^3}{SM^2}$; hence, by the resolution of forces, $SM : SE :: \frac{SE^3}{SM^2}$: the force of S upon M in a direction parallel to ES, which therefore $= \frac{SE^4}{SM^3} = \frac{SE^4}{\overline{SE-MK}^3} =$ (by division) $SE + 3MK$, omitting the other terms of the series on account of their smallness. Hence, as the force of E to S is represented by SE we have $3MK$ for the difference of the forces with which E and M are drawn in directions perpendicular to BC, and therefore it represents the disturbing force of S upon M in that direction; produce therefore KM to r, and take $Mr = 3MK$, and Mr will represent this disturbing force. This force is called the *ablatitious* force, because it tends to draw M *from* E; and in the opposite semicircle BDC, the force is conceived to act in the contrary direction; because M being there less attracted to S than E is, the effect is just the same as if M were drawn from E in the opposite direction. Hence, the effect of this force is the same in each semicircle CAB, BDC, or in opposite points of the circle $ABDC$. Also, $SM : ME :: \frac{SE^3}{SM^2} : \frac{SE^3}{SM^3} \times ME = ME$ very nearly, the disturbing force of S upon M in the direction ME; this is called the *addititious* force, because it tends to draw M *to* E. Hence, the addititious force : the ablatitious force :: $ME : Mr$, or $3MK$, :: rad. : three times the sine of the angle MEC, the distance of M from quadratures. Hence, in syzygies, $Mr = 3EM$ or $3EA$, from which, if we take EA the addititious force, there remains $2EA$ for the whole force with which M is drawn from E in syzygies.

844. The disturbing force ME compounded with the attraction of M to E, which varies as $\frac{1}{ME^2}$, make a force which does not vary as $\frac{1}{ME^2}$, and therefore by altering the law of force it must alter the form of the orbit; but because the force ME is directed to E, it will not (805) destroy the equal description of areas about E in equal times. But as the disturbing force Mr, or $3MK$, neither varies as $\frac{1}{ME^2}$, nor is it directed from M to E, it will both destroy the form of the orbit and the equal description of areas in equal times. Therefore M will not continue to describe a circle about E. Resolve the addititious force ME into MK, KE, then MK acting in opposition to Mr $(=3MK)$, we get $2MK$ for the whole force with which M is drawn from CB in the direction KM; hence, we may consider M as acted upon by two disturbing

forces, one of which $= 2MK$ in the direction KM, and the other $= KE$ in a direction perpendicular to KM, the force of S to E being represented by SE. Hence, the addititious force at M, and these two disturbing forces, will always be as ME, $2MK$ and KE.

845. Hitherto we have supposed the plane of the orbit of M to coincide with that of E, but if it do not, then as the force Mr acts *out* of the plane of the orbit of M (except when the nodes lie in the line SAD) it must continually draw M *from* the plane of its orbit; the plane of the orbit therefore continually changing will cause a constant motion of the nodes, and a variation of the inclination of the orbit, the method of computing which will be afterwards shown. These are, in general, the consequences of the disturbing forces, the particular effects of which we shall now proceed to consider.

846. Resolve the force Mr into Ms in the direction of the tangent to the point M, and Mw in the direction EM. Then as the body moves from C to A, the force Ms acting in the direction in which the body moves, must accelerate the body; hence, the velocity of the body is accelerated from quadratures at C to syzygies at A. But whilst the body moves from A to D, the force Ms acts in a direction contrary to the motion of the body, and therefore it retards the body as much from A to B as it accelerated it from C to A; and the same is true for the other semicircle BDC. Hence, from quadratures to syzygies the body is accelerated, and from syzygies to quadratures it is as much retarded; consequently the velocity is greatest in syzygies and least in quadratures.

847. Hence, also, the areas will be accelerated from quadratures to syzygies, and retarded from syzygies to quadratures; for (809) when the force tends in *consequentia* the areas are accelerated, and when it tends in *antecedentia* they are retarded. Hence, the areas described by M about E are greatest in syzygies and least in quadratures.

848. The ablatitious force Mr ($= 3MK$) vanishes in quadratures, and the addititious force remaining, the whole force of M to E in quadratures is there the greatest, and therefore the whole gravity of M to E is increased; and in syzygies, Mr is the greatest, and therefore the force of M to E is there the least, and the whole gravity of M to E is diminished; for as Mr is there $= 3AE$, if we take from it the addititious force $= AE$, there remains $2AE$ for the whole force with which M is drawn from E. The gravity therefore of M to E is twice as much diminished in syzygies as increased in quadratures; and in a whole revolution, the gravity of M to E is diminished.

849. As the force of M to E is greatest in quadratures the sagitta vt, in a given time, is greatest in those points, and as the velocity is there least, the arc Mt described must be the least; hence, the curvature of the orbit is there the greatest, the curvature being as the sagitta directly and the square of the arc

inversely. And as the force, and consequently the sagitta, is least in syzygies, and the velocity, and consequently the arc, is greatest, the curvature of the orbit must there be the least. Hence, the orbit must put on the form of an oval, whose longest axis passes through quadratures, and the shortest through syzygies; consequently the body M must recede further from E in quadratures than in syzygies. It will afterwards be shown, that the orbit is very nearly an ellipse. This elliptic form of the orbit, and the above-mentioned (846) acceleration and retardation of velocity, cause, when applied to the moon, an inequality in its motion, called its *variation*.

850. If ES be diminished, the action of S on E and M will be increased, and their difference, or the disturbing forces, will be increased; and as the gravity of M to E is, in a whole revolution, diminished by the disturbing forces, that diminution must be increased as E approaches to T, consequently the radius ME will be increased. Now in different circles, the periodic time varies in the sesquiplicate ratio of the radius directly, and the square root of the absolute force inversely (818); hence, as, when E approaches to S, the radius is increased and the force diminished, the periodic time must increase. Therefore if E revolve about S in an ellipse having S in its focus, when E is in the higher apside, the radius ME and the periodic time will be the least; and when E is in the lower apside, ME and the periodic time will be the greatest. How the periodic time is altered by the alteration of the orbit of E from a circle to an ellipse, will be afterwards shown. These are the general effects of the disturbing forces of S upon M, on supposition that the orbit of M would (independent of the disturbing forces) have been a circle.

851. As the moon's orbit is very nearly a circle, similar effects will take place upon its orbit, from the action of the sun; that is, when the earth is nearest to the sun, the moon's orbit will be dilated, and the periodic time increased; and when the earth is furthest from the sun, the orbit will be contracted, and the periodic time diminished. Hence, the moon's periodic time and distance is least in summer, and greatest in winter. We come now to consider the effect of the disturbing forces upon an elliptic orbit $CABD$.

852. Let M move in an ellipse about E in its focus; put $a = ME$, and let a represent the addititious force, and let the natural gravity of M to $E = \frac{b}{a^2}$. Now in quadratures, the whole force (845) of M to $E = \frac{b}{a^2} + a = \frac{ba + a^4}{a^3}$; hence, with that force (830) the distance of the apsides $= 180° \times \sqrt{\frac{b+1}{b+4}}$, which is less than 180°, because $\sqrt{\frac{b+1}{b+4}}$ is less than unity; therefore the ap-

sides are regressive when the body is in quadratures. Now in syzygies, the whole force of M to $E=\frac{b}{a^2}-2a=\frac{ba-2a^4}{a^3}$ (845); therefore (830) the distance of the apsides $=180^\circ \times \sqrt{\frac{b-2}{b-8}}$, which is greater than 180°, because $\sqrt{\frac{b-2}{b-8}}$ is greater than unity; hence, the apsides are progressive. But as the force $2a$ which causes the progressive motion in syzygies is double of the force a which causes the regressive motion in quadratures, the progressive motion in syzygies is greater than the regressive motion in quadratures. In the points between quadratures and syzygies, it is manifest that these principles cannot be applied to find the motion of the apsides, because the force does not vary as any power of the distance, inasmuch as there is a force Ms acting perpendicularly to the radius, which also produces a motion of the apsides; we shall afterwards show upon what principles we may compute the whole motion.

853. As the attractive force varies inversely as the square of the distance, we may represent the attractive force of E to S by $\frac{1}{SE^2}$, and of M to S by $\frac{1}{SM^2}$; hence, $SM : SE :: \frac{1}{SM^2}$: that part of the force of S upon M which acts parallel to $ES=\frac{SE}{SM^3}=\frac{SE}{\overline{SE-MK}^3}=\frac{1}{SE^2}+\frac{3MK}{SE^3}$, omitting the other terms of the series on account of their smallness; hence, the difference of the forces of S upon M and E in a direction perpendicular to CB, or the ablatitious force, is $=\frac{3MK}{SE^3}$; if therefore the position of M be given, and SE vary, the ablatitious force varies as $\frac{1}{SE^3}$; but if the position of M be given, the ratio of the ablatitious to the addititious force is given (845). Hence, if the position of M be given, and SE vary, the disturbing forces will vary as $\frac{1}{SE^3}$; and if the absolute force (A) of S should vary, the disturbing forces will vary as $\frac{A}{SE^3}$. But if P=the periodic time of E about S, then (818) $\frac{1}{P^2}$ varies as $\frac{A}{SE^3}$. Also, if $d=$ the diameter of the body S, and $m=$ its density, then A varies as $d^3 \times m$; hence, the disturbing forces vary as $\frac{d^3}{SE^3} \times m$, or as the cube of the apparent diameter of S seen from E, and its density conjointly. Hence, if the absolute force of S and the distance SE vary, the disturbing forces, and consequently the errors produced by them, vary as $\frac{1}{P^2}$; or they vary as the

cube of the apparent diameter of S seen from E, and the density of S conjointly. These are the *linear* errors of M seen from E, and as ME is constant, the *angular* errors will vary in the same ratio.

854. Now let us suppose ME only to vary, or which is the same, conceive two orbits to be described by M, and M to be similarly situated in them; then the addititious force varies as ME (845); and in any given position of M, the addititious force being to the ablatitious in a given ratio (845), both the disturbing forces, and consequently the linear errors generated by them in a given time, will vary as ME. Hence, considering two different radii ME, if in any given position of M, we suppose the bodies to describe a given indefinitely small angle about E, the linear errors generated in that time will be as the force $ME \times$ the square of the time; but the times of describing equal angles about E will be as the whole periodic times (p); hence, the linear errors will be as $ME \times p^2$; and as the same is true for every indefinitely small similar parts of the circles, the linear errors in a whole revolution will vary as $ME \times p^2$; but the angular errors are as the linear errors directly and the radius ME inversely; therefore the angular errors vary as p^2. Hence, from this and the last Article, if ME, SE, and the absolute force of S vary, the angular errors will vary as $\frac{p^2}{P^2}$. Or if for $\frac{1}{P^2}$ we substitute $\frac{A}{SE^3}$, the angular errors vary as $\frac{A \times p^2}{SE^3}$, and if A be given, they vary as $\frac{p^2}{SE^3}$. Now the error generated in any given time $\times$ the number of those times in a revolution, or $\times p$, (for that number is in proportion to the time of a revolution,) must be as the whole error in the time of a revolution, or as $\frac{p^2}{P^2}$; hence, by dividing by p, the errors in a given time will vary as $\frac{p}{P^2}$. Hence, the mean motion of the apsides of the orbit described by M will vary as the mean motion of the nodes, and each will vary as $\frac{p}{P^2}$, the excentricity and inclination being small, and remaining the same.

855. By Art. 845, the addititious force ME : the force of E to S :: ME : SE; and the forces of bodies revolving in different circles being as their radii directly and the squares of the periodic times inversely*, we have, the force of E to S : the mean force of M to E :: $\frac{SE}{P^2} : \frac{ME}{p^2}$; hence, by compounding

* For (812) F varies as $\frac{V^2}{R}$, because in this case $R =$ half the chord of curvature; but if P be the periodic time, V varies as $\frac{\text{circum.}}{P}$ which varies as $\frac{R}{P}$; hence, by substitution, F varies as $\frac{R}{P^2}$.

these two proportions, the addititious force : the mean force of M to $E :: p^2 : P^2$. Hence, if $p = 27d.\ 7h.\ 43'$, and $P = 365d.\ 6h.\ 9'$, we get the addititious force of the moon : its mean force towards the earth :: 1 : 178,725. Now (845) the ablatitious force : the addititious :: 3 $MK : ME$; hence, this, compounded with the last proportion, gives the ablatitious force : the mean force of the moon to the earth :: 3 $MK : ME \times 178{,}725 :: MK : ME \times 59{,}575$. But Ms : the ablatitious force $Mr :: KE : ME$; hence, Ms : the mean force of the moon to the earth :: $MK \times KE : ME^2 \times 59{,}575$. If $P : p :: 1 : n$, we have the addititious force : the mean force of the moon to the earth :: $n^2 : 1$, and the ablatitious force : that force :: $3n^2 MK : ME$.

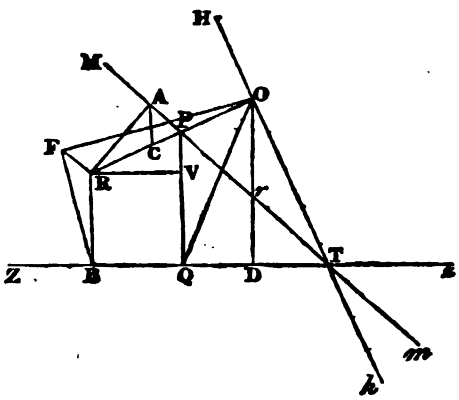
856. It will be proved in the next Chapter, that the nodes of the orbit described by the body M, have a retrograde motion. Hence, if instead of one body M, we suppose the whole circumference of the circle to be filled with bodies, the same effect will be produced on each, and the nodes of the orbit of each will have a retrograde motion; if therefore we suppose these bodies to be connected together, so as to form a solid ring, the nodes of this ring will have a retrograde motion. Hence, if this ring were joined to a spherical body, so that its plane might pass through its center, the nodes of that plane would have a retrograde motion, but less than before, in as much as the force which causes that motion would have a greater quantity of matter to move, and therefore the motion would be diminished as the inertia was increased. Now the earth, from the rotation about its axis, has its equatorial diameter greater than its polar, from the centrifugal force of its parts; hence, the excess of the quantity of matter in the earth above that in the sphere whose diameter is the earth's polar axis, answers to the aforementioned ring; consequently the attraction of the sun and moon upon this excess of matter must cause a motion of the equinoctial points upon the ecliptic, called the *Precession of the Equinoxes*, the quantity of which will be afterwards investigated.

857. Now let us suppose $CABD$ to be a globe revolving about an axis perpendicular to that section, and conceive a canal to be cut upon its surface in this section, and to be filled with a fluid. Then this fluid will be accelerated from quadratures to syzygies, and retarded from syzygies to quadratures, by the force Ms. Hence, there will be an accumulation of the fluid in syzygies, which will cause it to rise higher in those points, and the fluid being drawn from quadratures, it will there be depressed. But in consequence of the motion acquired, the water will flow beyond syzygies, and will continue to rise, until its motion be destroyed by the force Ms in the next quadrant; the fluid will therefore continue to rise to some distance beyond the syzygies, and the highest point of the fluid will be beyond the syzygies, and the lowest point beyond the quadratures. Thus the fluid will be highest and lowest at the same time, at the distance of about 90°. If we suppose the body $CABD$ to represent the

earth, and S the sun or moon, it is manifest that their attraction will cause the same effect upon the water which covers its surface; and the syzygies being that meridian on which the sun or moon is, it follows that the time of the high tide from the sun or moon will be some time after they have passed the meridian, and the low water about six hours after. When the sun and moon are in conjunction or opposition, it is manifest that they will both tend to raise the water at the same place, and therefore there will then be the highest tide; and when they are in quadratures, one tends to raise the water where the other tends to depress it, and therefore the tides will then be the least. These are the general principles of the tides, which will be further explained in Chapter XXXVIII.

858. In Art. 843, &c. we have supposed that M revolves about E at rest, whereas it is proved (840) that two bodies revolving and attracting each other, will revolve about their center of gravity; but the motions of M and E, (which FIG.
we will suppose to represent the moon and earth) about that center are disturbed by similar forces by S representing the sun; and if the sum of these forces 197.
be referred to the moon, and the earth be supposed at rest, their effect in disturbing the relative situation of the earth and moon will be the same as if the respective effects on each took place, and they revolved about their center of gravity G. For here the addititious force of S on M to the center G is MG, GS representing the force of G to S; also, EG will represent the addititious force of E to G. Hence, the sum of these forces $= ME$, the whole addititious force by which the tendency of M and E towards each other is increased, which is the same quantity as when E was at rest. Also, the sum of the ablatitious forces in the former case is equal to the ablatitious force in the latter; for the distance of each body from quadratures being the same, the ratio (845) of the ablatitious to the addititious forces are the same, and therefore the *sum* of the ablatitious forces of M and E when revolving about G (for they act in opposite directions) is equal to the ablatitious force of M when revolving about E at rest. In the theory of the moon therefore, we consider the moon to revolve about the earth at rest, and refer all the disturbing force to the moon. If E and M represent the masses of the earth and moon, then $\frac{E}{EM^2}$ represents the attraction of M to E, and $\frac{M}{EM^2}$ represents the attraction of E to M; hence, M is supposed to be attracted to E at rest by the force $\frac{M+E}{EM^2}$.

859. If a plane $ZHzh$ be acted upon by two forces, one of which would make it revolve about the axis ZTz, and the other about HTh, to find the axis MTm lying in the same plane, about which it will revolve. Let $c:1$ as the an-

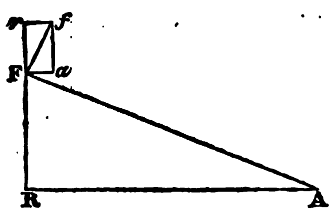


gular velocity about Zz : that about Hh. Assume any point R, and draw RO perpendicular to Hh, and RB to Zz; divide the angle HTZ by MT, so that sin. MTZ : sin. MTH :: 1 : c, and MTm will be the axis required. Let MT cut RO in P, and draw RB, PQ, OD perpendicular to Zz, RA to Mm, and RV to PQ, and join OQ. Now $PQ : PO :: 1 : c \mp \frac{FO}{PQ}$. Also, the linear velocity is as the angular velocity $\times$ radius; therefore the linear velocity of R about Zz, Hh, are as $c \times BR$, or $c \times VQ$, and RO; we have therefore to consider what may be the joint effect of these two velocities. Now $RO \mp c \times VQ = RO \mp VQ \times \frac{PO}{PQ} = RO \mp \overline{PQ - PV} \times \frac{PO}{PQ} = RP + PV \times \frac{TO}{PQ}$; but by sin. tri. $PR = \frac{TP \times RA}{TO}$, and $PV = \frac{DT \times PR}{TO} = \frac{DT}{TO} \times \frac{TP \times RA}{TO}$; also, $\frac{DT}{TO}$ $=$ sin. $DOT =$ sin. $(DrT - PTO) =$ sin. $(90° - DTO - PTO) =$ cos. $(DTO + PTO) =$ (Trig. Art. 102.) $\frac{TO \times TQ - PO \times PQ}{TP}$; in PV substitute this for $\frac{DT}{TO}$, and for PR substitute its value, and by Reduction we get $PR + PV \times \frac{PO}{PQ}$ (the joint effect of the two velocities) $= \frac{TQ \times PO + TO \times PQ}{TP} \times \frac{1}{PQ} \times RA = \frac{\text{sin. } OTZ}{\text{sin. } PTZ} \times RA$, which varies as RA; that is, the velocity of every point about Mm varies as its perpendicular distance from MTm; hence, MTm is the axis about which the plane revolves. But if the point F at which the two forces act to give the plane a rotatory motion about Zz, Hh, lie out of that plane, draw FR perpendicular to the plane, and join FO; FB. Now the velocity of F about Hh is as FO and that velocity is perpendicular to FO; and if we resolve it into two, are in the direction FR and the other parallel to OB, the velocity in the direction $FR = RO$ as before, and the velocity in the di-

rection parallel to the plane, $=FR$. For the same reason, the velocity of F about Zz gives a velocity in the direction FR, $=c\times FR$ as before, and in a direction parallel to BR, $=c\times BR$. Hence, the axis Mm will still continue the axis of rotation, and F has a motion parallel to the plane, or it gives the plane a motion in its own plane.

860. The velocities FR, $c\times FR$ are as $1:c$ or as $PQ:PO$. Draw AC parallel to PQ. Now $PQTO$ would be inscribed in a circle, the angles O, Q, being right ones; hence, the angle $PQO=PTO$; but the angle $ARP=$ comp. $APR=$comp. $OPT=PTO=PQO$, $ACR=QPO$, and AC is parallel to PQ; hence, the triangles RAC, QPO are similar, and $RC:CA::PQ:QO::$ vel. FR : vel. $c\times FR$, and let these express the velocities. Now the triangles ACR, OPQ are similar, and OQ would be the compound velocities of OP, PQ, and $PQ:OQ::FR$ (represented by RC) : RA (representing the compound velocities of FR, $c\times FR$, in the direction parallel to RA) $=FR\times\frac{OQ}{PQ}$. Hence, the two velocities of F about Zz, Hh, are equivalent to $\frac{\sin.\ OTZ}{\sin.\ PTZ}\times RA$ which is perpendicular to the plane, and $FR\times\frac{OQ}{PQ}$ parallel to RA.

861. If on any body two forces be impressed upon any two points, of which one would cause the body to revolve about one axis, and the other about another, separately; when compounded they would cause the body to revolve about a third axis; of which the lines of its deviation from the first two axes, will be reciprocally as the angular velocities. By the last article, the two velocities of F about Zz, Hh, are equivalent to $\frac{\sin.\ OTZ}{\sin.\ PTZ}\times RA$ perpendicular to the plane $ZHhz$, and $FR\times\frac{OQ}{PQ}$ parallel to RA. But by sin. tri. $OQ=\frac{TP\times OD}{OT}$; hence, the velocity parallel to $RA=(\frac{OD}{OT}\div\frac{PQ}{TP})\times FR=\frac{\sin.\ OTZ}{\sin.\ PTZ}\times FR$. Draw Fa parallel to RA and $=\frac{\sin.\ OTZ}{\sin.\ PTZ}\times FR$, produce RF to r, and take $Fr=$



$\frac{\sin.\ OTZ}{\sin.\ PTZ}\times RA$, and complete the parallelogram $aFrf$, and the diagonal Ff will represent the *direction* of F's motion; and compounding these two motions, we

get the *velocity* $= \frac{\sin. OTZ}{\sin. PTZ} \times FA$ proportional to FA; and as the triangles Frf, FRA are similar, Ff is perpendicular to FA. Hence, F revolves about A. The same is true for every point F.

If we divide $\frac{\sin. OTZ}{\sin. PTZ} \times RA$ by RA, we get $\frac{\sin. OTZ}{\sin. PTZ}$ the angular velocity about Mm; and as the angular velocity about Hh is 1, and about Zz is c, or $\frac{PO}{PQ}$ $= \frac{\sin. OTP}{\sin. PTZ}$, the angular velocities about Mm, Hh, Zz, are as the sines of the angles OTZ, PTZ, OTP. Hence we see, that the composition of motions of rotation take place as in that of free motions; we compound velocities not momenta; in a sphere, however, the latter are in proportion to the former.

CHAP. XXXII.

ON THE THEORY OF THE MOON.

Art. 862. IN the last Chapter we explained the general principles of the disturbing force of the sun upon the moon, and the nature of the effects produced by it; but to enter into a computation of all these effects, would require an investigation of the nature of the curve described by a body attracted to two points, called the *Problem of the Three Bodies*. This problem has been solved by M. CLAIRAUT, in a Treatise, entitled, *Theorie de la Lune;* by M. EULER, in his *Theoria Motuum Lunæ;* by M. D'ALEMBERT in his *Recherches sur differens Points importans du Systême du Monde;* by FRISI in his *Cosmographiæ Physicæ et Mathematicæ, pars prior;* and by T. MAYER in his *Theoria Lunæ, juxta Systema Newtonianum.* Mr. T. SIMPSON began a *Theory of the Moon*, but left it unfinished. M. CLAIRAUT at first objected to Sir I. NEWTON's law of gravity, that it would not account for the motion of the apsides of the moon's orbit; he afterwards, however, discovered his error, and found that it would account, not only for that motion, but for all the lunar irregularities; and he was the first person who gave a complete Theory of the moon. Sir I. NEWTON first gave a Theory of the moon from the principles of gravity, and by very ingenious artifices, he found some of the principal equations; but his indirect method did not carry him to many of the smaller equations, so that his computations could not be depended upon to give the moon's place nearer than 5′ or 6′. Others have attempted the same by indirect methods, of whom FRISI has been the most successful. As it would not be consistent with the plan of this Work to give a complete Theory of the moon, we shall, in this Chapter, explain such parts thereof as can properly be here introduced, in which we shall principally follow the indirect methods of NEWTON and FRISI; and although the conclusions thus deduced are not always so accurate as those which are derived from a direct solution of the problem, yet they give the true arguments, and their coefficients to a very considerable degree of accuracy. This method of treating the subject has this advantage, that it points out more clearly the particular causes of the several equations so deduced, which are not obvious in the equations derived from the general solution. In Chap. XXXVII. we shall explain the direct solution of this problem.

863. As the attractive force varies inversely as the square of the distance, let $\frac{1}{SM^2}$ represent the attraction of the moon M towards the sun S, then $\frac{1}{SE^2}$ FIG. 196.

will be the attraction of the earth E to S. Resolve the force $\frac{1}{SM^2}$ into the directions ME, ES; then, $SM : SE :: \frac{1}{SM^2} : \frac{SE}{SM^3} = \frac{SE}{SE-MK^3} = \frac{1}{SE^2} + \frac{3MK}{SE^3}$, the force with which S acts upon M in a direction parallel to ES, from which subtract $\frac{1}{SE^2}$, the force of E to S, and we have $\frac{3MK}{SE^3}$ $(=Mr)$ the force with which S draws M from E in the direction KM. Hence, by similar triangles, rMs, MEK, $ME : KE :: \frac{3MK}{SE^3}$ $(=Mr)$: $\frac{3MK \times KE}{ME \times SE^3} = Ms$ the part of the force Mr which acts perpendicularly to EM; also, $ME : MK :: \frac{3MK}{SE^3} : \frac{3MK^2}{ME \times SE^3} = Mw$ the part of the force Mr which acts in the direction EM; but $SM : EM :: \frac{1}{SM^2}$ (the force of S upon M) : $\frac{EM}{SM^3}$ the additional force, or the force of S upon M in the direction ME; hence, the whole disturbing force of S upon M in the direction ME is $\frac{EM}{SM^3} - \frac{3MK^2}{ME \times SE^3} =$ (as $\frac{MK^2}{EM} = EM \times \frac{MK^2}{EM^2} = EM \times \overline{\text{sin. } MEC}^2$) $\frac{EM}{SM^3} - \frac{3EM}{SE^3} \times \overline{\text{sin. } MEC}^2 = \frac{EM}{SM^3} - \frac{3EM}{SE^3} \times \overline{\tfrac{1}{2} - \tfrac{1}{2}\text{ cos. } 2MEC} = -\frac{EM}{2SE^3} + \frac{3EM}{2SE^3} \times \text{cos. } 2MEC$ very nearly.

864. Let the periodic time of M about E be to that of E about S as $n : 1$; then the orbits being supposed to be very nearly circular, we have (855) the addititious force $\frac{EM}{SM^3}$: the mean force of M to $E :: n^2 : 1$. Let the mean force of M to E, at the mean distance unity, be represented by unity, and we get $n^2 = \frac{1}{SM^3} = \frac{1}{SE^3}$ nearly; hence, the disturbing force of S upon M in the direction ME is $-\frac{1}{2} n^2 \times EM + \frac{3}{2} n^2 \times EM \times \text{cos. } 2MEC$. Now in a whole revolution, the last term is destroyed by the opposition of its signs; and the mean value of EM being unity, we get $-\frac{1}{2} n^2$ for the mean disturbing force of S upon M in the direction ME. Now in order to obtain the force of gravity of the moon towards the earth, upon the supposition already made, we must consider that when we assume the periodic time of the moon to be n, it is the periodic time corresponding to the *mean* force of the moon towards the earth, which force is equal to the natural gravity of the moon towards the earth, diminished by the mean disturbing force of S upon M in the direction EM; that is, the mean force of gravity of the moon towards the earth $-\frac{1}{2}n^2 = 1$, the mean force of M to E; hence, the mean force of gravity of the moon towards the earth, at the mean distance unity, $= 1 +$

$\frac{1}{2}n^2$; consequently that force at any other distance EM is $\frac{1+\frac{1}{2}n^2}{EM^2}$.* Hence, the whole force of the moon towards the earth $=\frac{1+\frac{1}{2}n^2}{EM^2}-\frac{1}{2}n^2\times EM+\frac{3}{2}n^2\times EM\times\cos.\ 2MEC$. Put $\frac{n^2}{1+\frac{1}{2}n^2}=m$, and the whole force becomes proportional to $\frac{1}{EM^2}-\frac{1}{2}m\times EM+\frac{3}{2}m\times EM\times\cos.\ 2MEC$, in which expression, unity represents the force of gravity of the moon towards the earth at the mean distance, and the other quantities represent the proper proportional disturbing forces.

865. If the disturbing force be assumed $\frac{EM}{SE^3}-\frac{3EM}{SE^3}\times 27\ \overline{\sin.\ MEC}^2$ then, upon the same supposition, the whole force will be $\frac{1}{EM^2}+m\times EM-3m\times EM\times\overline{\sin.\ MEC}^2$. Also, the quantity $\frac{3MK\times KE}{ME\times SE^3}=\frac{3n^2MK\times KE}{ME}$, which represented the force acting in the direction Mr, must now be represented by $\frac{1}{1+\frac{1}{2}n^2}\times\frac{3n^2MK\times KE}{ME}=\frac{3m\times MK\times KE}{ME}$.

866. If the velocity with which the moon was projected at the mean distance unity, be u; then at any other distance EM, it would be $\frac{u}{ME}$ nearly, in an orbit very nearly a circle, supposing that there was no tangential force. Now the force at the mean distance $=1-\frac{1}{2}m$, and (825) a body must fall down half that distance ($\frac{1}{2}$) to acquire the velocity in the circle; hence, by the laws of falling bodies, $u=\sqrt{2\times\overline{1-\frac{1}{2}m}\times\frac{1}{2}}=\sqrt{1-\frac{1}{2}m}=1-\frac{1}{4}m$ nearly. Let v be the velocity of the moon at M; then as the force (865) which acts upon the moon at M in the direction of the tangent is $\frac{3m\times MK\times KE}{ME}$, we have, by the principles of motion, $\dot{v}v=\frac{3m\times MK\times KE}{ME}\times\dot{CM}=$ (if we put $MK=x$, and assume $ME=1$, which we may here consider constant, without producing any sensible error) $3mx\dot{x}$, hence $v^2=3mx^2+Cor.$ But (866) at the mean distance, the velocity $v=1-\frac{1}{4}m=u$; hence, $v^2=\frac{u^2}{ME^2}+3mx^2=\frac{u^2}{ME^2}+3m\times ME^2\times\frac{x^2}{ME^2}=\frac{u^2}{ME^2}+3m\times ME^2\times\overline{\sin.\ MEC}^2=$ (as $\overline{\sin.\ MEC}^2=\frac{1}{2}-\frac{1}{2}\cos.\ 2MEC$) $\frac{u^2}{ME^2}+\frac{3}{2}m\times ME^2-\frac{3}{2}m\times ME^2\times\cos.\ 2MEC$.

* Frisi, and other Writers, make this force $=\frac{1}{EM^2}$. For this correction we are indebted to Dr. Maskelyne.

867. Thus far we have considered the velocity of M in respect to S as fixed: but as S is in motion, let us put $d : 1 ::$ synodic revolution of the moon : its periodic time; then at any angular distance CEM from quadratures, the moon has actually described the angle $d\times CEM$ from the time it was in quadratures. Hence, we must write $d\times\dot{\overline{CM}}$ for $\dot{\overline{CM}}$, consequently $v^2=\frac{u^2}{ME^2}+3m\times d\times ME^2\times\overline{\sin. MEC}^2=$ (if $s=$ sine of MEC) $\frac{u^2}{ME^2}+3m\times d\times ME^2\times s^2$; and (as the second term is very small compared with the first) $v=\frac{u}{ME}+\frac{3}{2}m\times d\times ME^2\times s^2$ very nearly.

On the Radius Vector of the Moon's Orbit, and the Equation of its Center.

FIG. 198. 868. Let AGQ be a semi-ellipse, F its center, E, L, its foci, FG its semi-axis minor, and draw EM, EG, and MN perpendicular to AQ. Put $FA=1$, $\angle AEM=z$, $FG=c$, $FE=w$. By the property of the ellipse, $AF^2 : FG^2 = EG^2-EF^2=AF^2-EF^2 :: AN\times NQ=\overline{AF-FN}\times\overline{AF+FN}=AF^2-FN^2 : NM^2 :: AF^2-\overline{EN\mp EF}^2 : NM^2$; to the value of NM^2 found from hence, add EN^2, and extract the square root, and we get $EM=FA\pm\frac{EF\times EN}{FA}-\frac{EF^2}{FA}$; but $\pm EN=EM\times\cos. MEA$; hence, $EM-\frac{EF\times EM\times\cos. MEA}{FA}=\frac{FA^2-EF^2}{FA}=\frac{FG^2}{FA}$; therefore $EM=\frac{FG^2}{FA-FE\times\cos. MEA}=\frac{c^2}{1-w\times\cos. z}=$ (by division) $c^2\times\overline{1+w\times\cos. z+w^2\times\overline{\cos. z}^2+w^3\times\overline{\cos. z}^3+\&c.}$ but (see my Trig.) $\overline{\cos. z}^2=\frac{1}{2}+\frac{1}{2}\cos. 2z$, $\overline{\cos. z}^3=\frac{3}{4}\cos. z+\frac{1}{4}\cos. 3z$, &c. also, $c^2=1-w^2$; hence, $EM=1-\frac{1}{2}w^2+\overline{w-\frac{1}{4}w^3}\times\cos. z+\frac{1}{2}w^2\times\cos. 2z+\frac{1}{4}w^3\times\cos. 3z+$ &c. If the excentricity be very small, $EM=1+w\times\cos. z$ nearly; and the variation of EM is nearly in proportion to cos. AEM.

869. Draw Em indefinitely near to EM; and with radius $Ez=\sqrt{AF\times FG}$, describe the circle zx, and take the area $zEw=AEM$, and suppose a body to revolve from A to M about E; then the angle AEM is the true anomaly, and AEw (Note to Art. 227.) is the mean anomaly; hence, the mean anomaly : the true anomaly $:: \angle zEw : \angle zEv ::$ sector zEw : sector zEv :: area AEM : area zEv; therefore the increment of the mean anomaly (Z) : the increment of the true anomaly (z) :: area EMm : area Evu $:: EM^2 : Ev^2 = Ez^2 = c$; hence, $\dot{Z}=\frac{EM^2}{c}\times\dot{z}=$ (by substituting for EM its value $\overline{1+w\times\cos. z+w^2\times\overline{\cos. z}^2+w^3\times\overline{\cos. z}^3+\&c.}^2$ by Article 868.) $c^3\times\dot{z}\times$

$\overline{1 + 2w \times \cos. z + 3w^2 \times \cos. z^2 + 4w^3 \times \cos. z^3 + \&c.}$ = (by substituting for c^3 its value $1 - \frac{3}{2} w^2$ nearly, and putting for $\overline{\cos. z}^2$, $\overline{\cos. z}^3$, &c. their values as before) $\dot{z} + 2w \times \cos. z \times \dot{z} + \frac{3}{2} w^2 \times \cos. 2z \times \dot{z} + w^3 \times \cos. 3z \times \dot{z}$ + &c. whose fluent is $Z = z + 2w \times \sin. z + \frac{3}{4} w^2 \times \sin. 2z + \frac{1}{3} w^3 \times \sin. 3z$ + &c. the mean anomaly in terms of the true.

870. Given the mean anomaly, to find the true. By the last Article, $z = Z - 2w \times s. z - \frac{3}{4} w^2 \times s. 2z - \frac{1}{3} w^3 \times s. 3z$, omitting the terms which come after; hence, $z = Z - 2w \times s. z$ nearly, $= Z - 2w \times s. Z$ nearly; also, $z = Z - 2w \times s. z - \frac{3}{4} w^2 \times s. 2z$ more nearly; substitute in the second term of this equation, the above value of z, and in the third term, substitute Z for z, and $z = Z - 2w \times \sin. (Z - 2w \times s. Z) - \frac{3}{4} w^2 \times s. 2Z$. Now in the given equation, substitute in the second term $(-2w \times s. z)$ the last value of z; and in the third term $(-\frac{3}{4} w^2 s. 2z)$ substitute $Z - 2w \times \sin. Z$ for z; also, in the last term $(-\frac{1}{3} w^3 \times s. 3z)$ substitute Z for z, and we get $z = Z - 2w \times \sin. (Z - 2w \times s. \overline{Z - 2w \times s. Z - \frac{3}{4} w^2 \times s. 2Z}) - \frac{3}{4} w^2 \times \sin. (2Z - 4w \times s. Z) - \frac{1}{3} w^3 \times \sin. 3Z$. Now as $4w \times \sin. Z$ is very small compared with $2Z$, we have $-\frac{3}{4} w^2 \times \sin. (2Z - 4w \times s. Z) = -\frac{3}{4} w^2 \times \sin. \overline{2Z - 4w \times \sin. Z \times \cos. 2Z} = -\frac{3}{4} w^2 \times \sin. 2Z + 3w^3 \times \sin. Z \times \cos. 2Z =$ (as $\sin. Z \times \cos. 2Z = \frac{1}{2} \sin. 3Z - \frac{1}{2} \sin. Z) - \frac{3}{4} w^2 \times \sin. 2Z + \frac{3}{2} w^3 \times \sin. 3Z - \frac{3}{2} w^3 \times \sin. Z$. In like manner, $-2w \times \sin. (Z - 2w \times \sin. Z) =$ (by considering $\cos. 2w \times \sin. Z = 1) - 2w \times \sin. Z + 4w^2 \times \sin. Z \times \cos. Z = -2w \times \sin. Z + 2w^2 \times \sin. 2Z$. Hence, $-2w \times \sin. (Z - 2w \times \sin. \overline{Z - 2w \times \sin. Z} - \frac{3}{4} w^2 \times \sin. 2Z) = -2w \times \sin. (Z - 2w \times \sin. Z + \frac{5}{4} w^2 \times \sin. 2Z) = -2w \times \sin. (Z - [2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z]) = -2w \times$
$\overline{\sin. Z \times \cos. (2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z) - \cos. Z \times \sin. (2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z)}$
$=$ [as $\cos. (2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z) = 1 - 2w^2 \times \overline{\sin. Z}^2$ nearly, and $\sin. (2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z) = 2w \times \sin. Z - \frac{5}{4} w^2 \times \sin. 2Z$ nearly] $- 2w \times \sin. Z + 4w^3 \times \overline{\sin. Z}^3 + 4w^2 \times \sin. Z \times \cos. Z - \frac{5}{2} w^3 \times \cos. Z \times \sin. 2Z =$ [as $\sin. Z^3 = \frac{3}{4} \sin. Z - \frac{1}{4} \sin. 3Z$, $\sin. Z \times \cos. Z = \frac{1}{2} \sin. 2Z$, and $\cos. Z \times \sin. 2Z = \frac{1}{2} \sin. 3Z + \frac{1}{2} \sin. Z$] $- 2w \times \sin. Z + \frac{7}{4} w^3 \times \sin. Z + 2w^2 \times \sin. 2Z - \frac{9}{4} w^3 \times \sin. 3Z$. These quantities being substituted into the first equation, and the like terms being collected together, we get
$$z = Z - 2w \times \sin. Z + \frac{5}{4} w^2 \times \sin. 2Z - \frac{13}{12} w^3 \times \sin. 3Z + \frac{1}{4} w^3 \times \sin. Z$$
the true anomaly. Thus we may find the true anomaly from the mean, for all the planets; but this rule is not so well adapted for calculation, as those given in the *tenth* Chapter. If the excentricity be very small, the equation of the center becomes nearly $= -2w \times \sin. Z = -2w \times \sin. z$ nearly; hence, the equation is in proportion to the sine of the true anomaly nearly.

871. The second term becomes a maximum at 90° from the apsides; the third term at the octants from the apsides; the fourth term at 30° from the apsides. But as the first term is the principal one, the first of these equations be-

comes a maximum at 90° from the apsides; the second, at the octants from the apsides; and the third, at the distance of 30° from the apsides. As the first equation is the principal one, the whole equation must be a maximum when Z is nearly 90°, or $90°+e$, e being very small; hence, sin. $Z=1-\frac{1}{2}e^2$ nearly; sin. $2Z=-2e$ nearly; and sin. $3Z=-1$ very nearly; by substitution therefore we get the equation when a maximum $=-2w+we^2+\frac{w^3}{4}-\frac{5w^2e}{2}-\frac{13w^3}{12}$ very nearly; make the fluxion of this $=0$, and we find $e=\frac{5w}{4}$; hence, the greatest equation is $-2w-\frac{115w^3}{48}$. For the moon, $w=,05505$; hence, the greatest equation is $-6°.\ 18'.\ 30''-1'.\ 22''=-6°.\ 19'.\ 52''$.

872. For w substitute 0,05505 the mean excentricity of the moon's orbit, and the true anomaly of the moon $z=Z-6°.\ 18'.\ 30''\times\sin.\ Z+13'\times\sin.\ 2Z-$
$+8''\times\sin.\ Z$
$37''\times\sin.\ 3Z=Z-6°.\ 18'.\ 22''\times\sin.\ Z+13'\times\sin.\ 2Z-37''\times\sin.\ 3Z$.

873. By Art. 868. the radius vector $=1-\frac{1}{2}w^2+w\times\cos.\ z+\frac{1}{2}w^2\times\cos.\ 2z+$ &c. in which, if we substitute for z its value $Z-2w\times\sin.\ Z$ nearly, and neglect all those terms where any powers of w greater than the second enter; and for cos. z, we put cos. $(Z-2w\times\sin.\ Z)=$ (as $2w\times\sin.\ Z$ is very small) cos. $Z+2w\times\overline{\sin.\ Z}^2=\cos.\ Z+w-w\times\cos.\ 2Z$, we get the radius vector $=1+\frac{1}{2}w^2+w\times\cos.\ Z-\frac{1}{2}w^2\times\cos.\ 2Z$.

On the Effect of increasing or diminishing by a very small Quantity, the Force or Velocity of a Body moving in an Ellipse about the Focus.

FIG. 199. 874. Let APM be an ellipse described by a body P revolving about the focus S; H the other focus; and conceive the force at P to be augmented by a very small quantity, in the ratio of $1:1+r$, and let it still continue to vary inversely as the square of the distance; to find the new ellipse which it describes, and its major axis; the excentricity of the ellipse being supposed to be very small. Let PE, Pe be the spaces through which a body must fall with the forces $1+r$ and 1 at P, to acquire the velocity at P; then $PE:Pe::1:1+r$, therefore $PE:Ee::1:r$, and $Ee=r\times PE$. Now let PL, Pl be the spaces fallen through to acquire the velocity at P in these respective cases; then (conics) $SP^2=SE\times SL=Se\times Sl$; hence, $SE:Se::Sl:SL$, and $SE:Ee$, or $r\times PE$, $::Sl$, or SL nearly, $:Ll=\frac{r\times PE\times SL}{SE}$. But (868) $SP=\frac{c^2}{1-w\times\cos.\ z}=\frac{1-w^2}{1-w\times\cos.\ z}$, and $SE=\frac{SP^2}{SL}=\frac{1}{2}SP^2$; therefore $PE=$

$SP - SE = SP - \frac{1}{2}\,SP^2$; hence, $Ll = \frac{4r}{SP} - 2r = \frac{4 - 4w \times \cos. z}{1 - w^2} \times r - 2r =$ (by dividing and neglecting the powers of w above the first) $2r - 4rw \times \cos. z$ the variation of the major axis. Now in a whole revolution, $4rw \times \cos. z$ is destroyed by the opposition of its signs; hence, the major axis is diminished by $2r$; therefore the semi-major axis $= 1 - r$, or the semi-major axis is diminished in the ratio of $1 : 1 - r$. If the same force be subtracted instead of added, the semi-major axis will be increased in the ratio of $1 : 1 + r$. If the ellipse become a circle, its radius must vary in the same ratio, by the addition or subtraction of this small new force. Now if this new force r be constant, instead of varying inversely as the square of the distance, and be supposed to be very small, and the orbit nearly a circle, the whole force will still vary nearly in the inverse square of the distance*, and therefore P will still describe an ellipse very nearly, the variation of whose major axis will be very nearly the same.

875. As r is very small, the periodic time will (818) vary as $1 : \frac{\overline{1 \mp r}\,^{\frac{3}{2}}}{\overline{1 \pm r}\,^{\frac{1}{2}}}$ $= 1 \mp 2r$ very nearly. But the mean motion is inversely as the periodic time; hence, when the force varies in the ratio of $1 : 1 \pm r$, the mean motion varies as $1 : \frac{1}{1 \mp 2r}$, or as $1 : 1 \pm 2r$ and the first mean motion : the difference of the mean motions :: $1 : \pm 2r$.

876. If by the continual addition of some small new force which varies inversely as the square of the distance, the ellipse continually changes its figure into a new ellipse, but that, upon account of the small variation of the ellipse, the computation of the variation of the transverse axis may be considered as made for the same ellipse, then, if r represent the sum of all the forces added in a whole revolution, after a whole revolution $Ll = 2r$, in an orbit nearly circular; hence, the semi-axis major $= 1 - r$ very nearly. If the forces be subtracted, the major axis $= 1 + r$. If the orbit be a circle, the radius of the circle will be increased or diminished in the ratio of $1 : 1 \mp r$.

877. If by the addition or subtraction of this new force, the axis major be diminished or increased by Ll, and we take $Pv = Pl$, v will be the focus of the new ellipse; if the force be increased, then Pv is less than PH, and the apsides are progressive in the descent of the body from the higher to the lower

* That this is true, appears from hence. Let x and r be very small when compared with unity; then $\frac{1}{\overline{1+x}\,^2} = 1 - 2x$ very nearly; add r to each, and $\frac{1}{\overline{1+x}\,^2} + r = 1 - 2x + r = \frac{1}{\overline{1 - \frac{1}{2}r + x}\,^2}$ very nearly. The same is true, if r be subtracted. Hence, with the force $\frac{1}{\overline{1+x}\,^2} \pm r$, a body will describe an ellipse very nearly.

apside; but from the lower to the higher apside, they will be as much regressive. If the force be diminished, the contrary takes place. Hence, in a whole revolution, the addition or subtraction of any new force which varies inversely as SP^2, will cause no motion of the apsides. The progress or regress of the apsides therefore depend upon the increment or decrement of the force being in a greater or less ratio than the inverse square of the distance.

878. With the center S describe the circular arc vr, and $\frac{1}{2}rH$ will be the variation of the excentricity. Now $\frac{1}{2}rH = \frac{1}{2}vH \times \cos. z$ nearly $=(874)\ r \times \cos. z - 2rw \times \overline{\cos. z}^2 = r \times \cos. z + rw \times \cos. 2z - rw$, and of these three quantities, the two first will be destroyed in a whole revolution by the opposition of their signs, and the third is constant. If therefore r denote the sum of all the forces added, the variation of excentricity in a whole revolution $= -rw$. And if r and w be very small, the variation of excentricity becomes extremely small; hence, the variation of the excentricity principally depends upon the increment or decrement of the force being in a greater or less ratio than the inverse square of the distance.

879. If the gravity remain the same, and the velocity be increased in the ratio of $1 : 1+v$, v being very small, then $PE : Pe :: 1 : \overline{1+v}^2 :: 1 : 1+2v$, therefore $PE : Ee :: 1 : 2v$; hence, $Ll = 4v - 4vw \times \cos. z$. The major axis is therefore increased in the ratio of $1 : 1+2v$; hence, the periodic time is increased in the ratio of $1 : 1+3v$. If the velocity be diminished in the ratio of $1 : 1-v$, then the major axis will be diminished in the ratio of $1 : 1-2v$; and the periodic time in the ratio of $1 : 1-3v$. And in respect to the motion of
FIG. 200. the apsides, whilst P moves from the higher to the lower apside, or through MCA; if the velocity be increased or diminished, the distance Pw, Px from the other focus will be increased or diminished, and the apsides will move backwards or forwards to m or n; but in the ascent from the lower apside A to the higher M, if the velocity be increased or diminished, then the distance Pv, Py from the other focus will be increased or diminished, and the apsides will move forwards or backwards.

On the Alteration of the Figure of the Moon's Orbit, supposed to have no Excentricity; and the Variation *of the Moon.*

FIG. 196. 880. The velocity in quadratures : velocity in syzygies $:: \frac{u}{ME} : \frac{u}{ME} + \frac{3}{2} m \times d \times ME^2$ (867) $:: 1 : 1 + \frac{3}{2} \times \frac{m}{u} \times d$ nearly :: (as $m = 0{,}00557959$, $u = 0{,}9986095$, $d = 1{,}080853$) $1 : 1 + \frac{100}{11039}$; hence, if we assume the mean velocity $= 11039$, the velocity in quadratures $= 10989$, and in syzygies $= 11089$.

Also, from quadratures to syzygies, the increment of the velocity, and consequently of the area, in a circular orbit, varies as s^2.

881. Let $ABDC$ be an ellipse, whose major axis CB is very nearly equal to its minor axis AD, E its center; draw any diameter MET, and QEK its conjugate diameter, on which let fall a perpendicular MO, and draw MH perpendicular to CB. Now by the property of the ellipse, $HE^2 = \frac{EC^2}{EA^2} \times \overline{EA^2 - MH^2}$; hence, $ME^2\ (=HE^2 + MH^2) = EC^2 - \overline{EC^2 - EA^2} \times \frac{HM^2}{EA^2} =$ (as the ellipse is very nearly a circle) $EC^2 - \overline{EC^2 - EA^2} \times \frac{HM^2}{EM^2}$ nearly, and by taking the square root, $ME = EC - \overline{EC - EA} \times \frac{HM^2}{EM^2}$ nearly, $= EC - \overline{EC - EA} \times \overline{\sin. CEM^2}$ nearly; hence, in going from C to A, the diminution of EM is in proportion to the square of the sine of the angle CEM. FIG. 201.

882. If the mean distance be represented by unity, and e be the difference between the mean and the greatest or least distances; then $EC = EA + 2e$, and $EM = EA + 2e - 2e \times \overline{\sin. CEM}^2 =$ (as $EA + e = 1$, and $\overline{\sin. CEM}^2 = \frac{1}{2} - \frac{1}{2}\cos. 2CEM$) $1 + e \times \cos. 2CEM = 1 - e \times \cos. 2AEM$.

883. If we suppose the moon's orbit at first to have been a circle, the disturbing forces will make the orbit an oval (849), whose major axis CB lie in quadratures, and the minor axis AD in syzygies; and as the minor axis DEA is always directed to the sun, whilst the earth E revolves about the sun, we must conceive this oval figure to revolve in a moveable plane about E, so as to keep DEA always directed to the sun. This oval is very nearly an ellipse. For take a very small arc Mm, draw the ordinate mr, and mt perpendicular to ME. Now if v be the absolute velocity of M, then (867) $v^2 = \frac{1 - \frac{1}{2}m}{ME^2} + 3m \times d \times ME^2 \times s^2$; but the absolute velocity of M : its velocity in respect to the revolving plane :: d : 1; therefore in respect to the revolving plane, we have the square of the velocity $= \frac{1}{d^2} \times \overline{\frac{1 - \frac{1}{2}m}{ME^2} + 3m \times d \times ME^2 \times s^2} = Mm^2$. Also, the force (865) at $M = \frac{1}{ME^2} + m \times ME - 3m \times ME \times s^2 = 2Mr$, the force being represented by the space through which the body is drawn by that force, or by twice the sagitta (808). But by similar triangles, $mr^2 : mt^2 :: ME^2 : MO^2$; hence, $\frac{mr^2 \times MO^2}{ME^2} = mt^2 = Mm^2$ very nearly, as the orbit is nearly a circle; also, by the property of the ellipse, $MO = \frac{EC \times CA}{EK}$, and $Mr = \frac{rm^2 \times ME^2}{2ME^2 \times EK^2}$; hence,

$\frac{Mr}{Mm^2} = \frac{\frac{1}{2}EM^3}{EC^2 \times EA^2} = \frac{1}{2}EM^3$ very nearly. Let $EM = x$, $EC = 1 + e$, $EA = 1 - e$, and neglecting those terms where all the powers of e above the first enter, and substituting for Mr and Mm^2 their before-mentioned values, we get from the last equation, $x^3 = d^2 \times \frac{1 + mx^3 - 3mx^3s^2}{1 - \frac{1}{2}m + 3mdx^4s^2} = \frac{d^2}{1 - \frac{1}{2}m} \times \overline{1 + mx^3 - 3mx^3s^2 \times \overline{1 + \frac{dx}{1 - \frac{1}{2}m}}}$, and putting $\frac{d^2}{1 - \frac{1}{2}m} = t^2$, we get $x^3 = t^2 + mt^4 \times \overline{1 - 3s^2 \times \overline{1 + \frac{dx}{1 - \frac{1}{2}m}}}$, and taking the cube root, we get $x = t^{\frac{2}{3}} + \frac{1}{3}mt^{\frac{8}{3}} \times \overline{1 - 3s^2 \times \overline{1 + \frac{dx}{1 - \frac{1}{2}m}}} =$ (as $x = t^{\frac{2}{3}}$ nearly) $t^{\frac{2}{3}} + \frac{1}{3}mt^{\frac{8}{3}} - mt^{\frac{8}{3}} \times s^2 \times \overline{1 + \frac{dt^{\frac{2}{3}}}{1 - \frac{1}{2}m}}$; hence it appears, that the diminution of EM (from the force and velocity) is very nearly in proportion to the square of the sine of CEM, consequently (881) the point M describes very nearly an ellipse upon the moveable plane, the major axis of which : the difference of the axes :: $t^{\frac{2}{3}} + \frac{1}{3}mt^{\frac{8}{3}} : mt^{\frac{8}{3}} \times \overline{1 + \frac{dt^{\frac{2}{3}}}{1 - \frac{1}{2}m}}$ :: (as $m = 0{,}0055796$, $d = 1{,}080853$) 71,6 : 1; hence, the ratio of the diameters is as 71,6 : 70,6, but as we have here neglected small quantities, this ratio is not so correct as that which we shall now proceed to determine.

884. By Art. 865. the force at A : the force at C :: $\frac{1}{AE^2} - 2m \times AE : \frac{1}{CE^2} + m \times CE$; also, (880) from the acceleration of velocity by the ablatitious force, the velocity at A : velocity at C :: 1,0090588 (v) : 1; but independent of this acceleration, the velocity at A : the velocity at C :: $CE : AE$ (806); hence, the velocity at A : velocity at C :: $v \times CE : AE$. Now the increment of the arc described in a given time is as the velocity, and the sagitta is as the force; also the curvature is as the sagitta directly and the square of the arc inversely; hence, the curvature at A : curvature at C :: $\frac{\frac{1}{AE^2} - 2m \times AE}{v^2 \times CE^2} : \frac{\frac{1}{CE^2} + m \times CE}{AE^2}$ :: $1 - 2m \times AE^3 : v^2 + v^2m \times CE^3$.

FIG. 202. 885. When the moon is in quadratures at C with the sun, S, it will describe above 90° before it comes into conjunction, owing to the motion SS' of the sun in the same time. Now for the *mean* motions, the angle $CEA : CEa$ :: 27*d.* 7*h.* 43′ : 29*d.* 12*h.* 44′, or as the periodic : the synodic revolution of the moon, which put as $1 : d$, and take $Ea = EA$, and a is the place of the moon at the next conjunction, and the curve described by the moon will be found, by taking $Em = EM$, and the angle $CEm : CEM :: d : 1$. To find the curvature of the

orbit Ca at C and a, with the center E describe the circle Cu, draw the tangent Ch, and $Ervt$, and take $Es=Er$, and the angle $CEs \cdot CEr :: d : 1$, and produce Es to y, and consider the angles CEy, CEt in their evanescent state; now as $Er=Es$, and $Ev=Ez$, therefore $rv=sz$; and as the sagitta represents the curvature when the tangent is the same, we have at the point C, curv. of Cr : curv. of $Cv :: tr : tv$, and curv. of Cs : curv. of $Cv :: ys : yz$; hence, FIG. 203.

Curv. of Cr – curv. of Cv : curv. of $Cv :: rv$ or $zs : vt$
Curv. of Cv : curv. of Cs – curv. of $Cv :: zy : zs$

$\therefore$ curv. of Cr – curv. of Cv : curv. of Cs – curv. of $Cv :: zy : vt :: cz^2 : cv^2 :: d^2 : 1$.

Now the curvature being inversely as the radius of curvature, the curvature of Cu may be presented by $\frac{1}{CE}$, and then (by the property of the ellipse) the curvature of $Cr=\frac{CE}{AE^2}$; hence, $\frac{C}{AE^2}-\frac{1}{CE}$: curv. of $Ca-\frac{1}{CE} :: d^2 : 1$; consequently the curvature of Ca at $C=\frac{CE^2+rAE^2}{d^2AE^2\times CE}$, putting $r=d^2-1$. For the same reason, the curvature of Ca at $a=\frac{AE^2+rCE^2}{d^2CE^2\times AE}$. Hence, the curvature in conjunction : the curvature in quadratures :: $\frac{AE^2+rCE^2}{d^2CE^2\times AE}$: $\frac{CE^2+rAE^2}{d^2AE^2\times CE} :: AE^3+rCE : CE^3+rAE$, assuming $AE\times CE=1$, because the mean distance is unity, and the orbit is very nearly a circle. Hence, from this and the last Article, we have, $1-2m\times AE^3 : v^2+v^2m\times CE^3 :: AE^3+r\times CE : CE^3+r\times AE$. Put $1+x=CE$ and $1-x=AE$, and substituting for CE and AEt hese values, and neglecting all the powers of x above the first, on account of the smallness of x, and multiplying extremes and means, we shall get a simple equation, from which $x=,00716$; hence $CE=1,00716$ and $CA=,99284$, consequently $CE : AE :: 1,00716 : ,99284 :: 70 : 69$ in whole numbers. This therefore is the variation of the form of the orbit arising from the force of the sun, supposing that the orbit would have been a circle without that disturbing force. And as the orbit of the moon is an ellipse having the earth in its focus, and this ellipse is nearly a circle, the same cause must produce very nearly the same effect in the moon's orbit. Dr. HALLEY first took notice of this contraction of the lunar orbit in syzygies, from the phænomena of the moon's motion, and made the ratio of the diameters as 44,5 : 45,5, from observation. See his remarks upon the lunar Theory, at the end of his catalogue of the southern stars. Hence (882), $EM=1-\frac{1}{140}\times\cos. 2AEM$. FIG. 201.

883. From the alteration of the form of the orbit, as explained in the last

Article, and from the acceleration of the areas (880), there will arise two corrections to be applied to the mean motion of the moon, in order to give the true motion; the joint effect of these two, constitute an equation called the *Variation*. We shall consider each separately, beginning with that arising from the figure of the orbit. Let CA be the quadrant of a circle which the moon would have described from quadratures to syzygies without any disturbing force;
FIG. 204. take $Ea : EA :: 69 : 70$, and describe the ellipse aC; draw MmK perpendicular to EC, and join EM, Em. Then the moon in the quadrant CA revolving uniformly, the angle CEM will represent the mean motion; but its true place will be in the ellipse at m; for not here considering the acceleration of the areas, the true and mean place must be both in the same perpendicular to EC; for the area CEA : area CEa :: area CEM : area CEm, and as CEA, CEa are supposed to be described in the same time, and the areas are (805) proportional to the times, therefore CEM, CEm are described in the same time. Hence, the angle MEm (being the difference of the two angles described by the true and mean motion of the moon about E) is the correction from this cause.

887. Now to consider the effect from the acceleration of the areas, it appears by Art. 880. that if the mean area, or the area described in the octants, be 11039, the areas at a and C will be 11089 and 10989 in the same time. Take therefore $Ea : E\alpha :: \sqrt{11089} : \sqrt{10989}$, or as $11039 : 10989$, or as $69 : 68{,}6875$, and describe the ellipse αnC, and draw Enr, and r will be the true place of the moon corresponding to the mean place M. For as the area $CEm : CEn$ in the constant ratio of $mK : nK$, $C\dot{E}m : C\dot{E}n$ in a given ratio; but $C\dot{E}m$ is constant, therefore $C\dot{E}n$ is constant. Now $C\dot{E}r : C\dot{E}n :: Er^2 : En^2$, therefore $C\dot{E}r - C\dot{E}n : C\dot{E}n :: Er^2 - En^2 : En^2 :: 2En \times nr : En^2$ very nearly, $:: 2nr : En$; and considering En as constant on account of its very small variation, the second and fourth terms being constant, $C\dot{E}r - C\dot{E}n$ varies as $2nr$, or as nr. But at C, $C\dot{E}n = C\dot{E}r$; put therefore the fluxion of $C\dot{E}n = \dot{v}$, and $C\dot{E}r - \dot{v}$ varies as nr. Now $nr : mn :: nK : nE$, or αE nearly, and $mn : a\alpha :: nK : \alpha E$, hence, $nr : a\alpha :: nK^2 : \alpha E^2$, consequently nK^2 varies as nr, or as $C\dot{E}r - \dot{v}$; hence, the increment of the area CEr at r above the increment at C, increases as nK^2, or very nearly as the square of the sine of the angle rEC, and therefore (880) it increases as the areas described by the moon from quadratures to syzygies increase. Now bringing mnK to bew, and r to s, indefinitely near to C, the area $CEs : CEb :: we : wb :: E\alpha : Ea :: \sqrt{10989} : \sqrt{11089} :: 10989 : 11039 ::$ (880) the area described by the moon at C : the mean area described by the moon. As therefore the area CEs described by the moon at C has its proper ratio to the mean area CEb that would have been described in the same time, and it increases in its proper ratio, at any point r the moon has its true position in respect to the place m

where it would have been if there had been no acceleration of areas, and m is the place, if there had been no acceleration of areas, corresponding to the mean place M; therefore the angle MEn is the whole equation, or the *Variation.*

888. If we consider EK as radius, Kn, KM will be the tangents of the true and mean motions, and the difference of the two angles will be greatest in the octants*; therefore if the angle $MCE=45°$, $MK : Kn$ (:: 70 : 68,6875) :: tan. 45° : tan. 44°. 27′. 28″; consequently the greatest variation is 32′. 32″. This would be the case if the moon described 90° from quadratures to syzygies; but as it describes a greater angle in the proportion of a periodic to a synodic revolution, or in the proportion of 27*d.* 7*h.* 43′ to 29*d.* 12*h.* 44′; therefore if 32′. 32″ be increased in this proportion, it gives 35′. 10″ for the greatest variation. But $nE : EK :: nm : mr$ (nearly) $=\frac{EK \times nm}{nE}$ which (because nm varies as nK) varies as $EK \times nK$ very nearly, or nearly as the sine of $2rEC$, or sin. $2rEA$ twice the distance of the moon from the sun; hence, the *variation* = 35′. 10″ × sine of twice the distance ($2X$) of the sun from the moon. In the transit of the moon from quadratures to syzygies, the mean place M is *before* the true place r, and from syzygies to quadratures, the mean place is *behind* the true.

889. This is the variation at the mean distance of the sun from the earth; but at other distances it (854) varies as the square of the synodic† time of the moon directly and the cube of the distance of the earth from the sun inversely; let therefore $d : 1$ :: the time of the synodic : the time of the periodic revolution of the moon at the mean distance CS (unity) of the sun from the earth, FIG.
and let SCs be the angle described by the sun in the time $d-1$, or by which 205.
the synodic exceeds the periodic revolution; and let TCt be the angle described by the sun in the difference of those times, at any other distance CT of the sun from the earth. Now if c = the excentricity of the earth's orbit, Y the sun's true anomaly, (868) the distance of the sun from the earth $=1+c \times \cos. Y$; and the angular motion of the sun in the same time varies (820) inversely as the square of the distance, and the angles SCs, TCt described by the sun may be considered as described in the same time, the difference being only the angle described by the sun in the difference of the times of describing these angles by the moon; hence, $\frac{1}{1^2} : \frac{1}{\overline{1+c \times \cos. Y}^2} = 1-2c \times \cos. Y$ $:: d-1$: the time of describing $TCt = \overline{d-1} \times \overline{1-2c \times \cos. Y}$; therefore the difference of the times of describing SCs, TCt, is $-\overline{d-1} \times 2c \times \cos. Y$; consequently the time of a synodic revolution of the moon at its mean distance : the

* This is not accurately true, but sufficiently nearly so for the purpose for which we here want it.

† The period of the variation being a *synodic* revolution, it appears, by a like reasoning as in Art. 854. that the error will be in proportion to the square of the *synodic* time of a revolution directly, and the cube of the distance of the sun inversely.

time at any other distance :: 1 : $1-2c\times\frac{d-1}{d}\times$ cos. $Y=1-{,}1496\ c\times$ cos. Y.

Hence, the variation becomes 35′. 10″ $\times\frac{\overline{1-{,}1496\ c\times \cos. Y}^2}{\overline{1+c\times\cos. Y}^3}\times$ sin. $2X=$ 35′. 10″ $\times\ \overline{1-3{,}299\ c\times\cos. Y}\times$ sin. $2X$; and if we take the excentricity of the earth's orbit to be ,01681, when the sun is in its apogee the variation will be 33′. 13″, and in its perigee, it becomes 37′. 7″. All this is upon supposition

FIG. 204. that the moon's orbit has no excentricity, and that the ratio of CE : aE continues constant, whereas that ratio depends (885) upon the time of a synodic revolution; the orbit also has an excentricity; the variation therefore will differ a little from what is here determined. TYCHO first observed this irregularity of the moon.

890. The annual equation of the synodic revolution, is the difference of the times of describing SCs, TCt, or $-0{,}1496\times c\times$ cos. Y, the mean synodic time being unity. When cos. $Y=1$, this equation becomes $-1h$. 46′. 51″ for its maximum. Hence, the equation at any other time will be $-1h$. 46′. 51″ $\times$ cos. Y.

To find the Equation of the mean Motion of the Moon, arising from the different Distances of the Earth from the Sun.

891. As the earth approaches the sun, the moon's orbit is (851) dilated, and as the earth recedes from the sun, the orbit is contracted, and this arises from the diminution and augmentation of the disturbing force of the sun upon

FIG. 196. the moon in the direction ME; but the equal description of areas in equal times will not (805) be affected by this force; and in respect to the acceleration and retardation of velocity from the ablatitious force, the acceleration in one quadrant very nearly destroys the retardation in the next; we may therefore consider the mean area described to be always in proportion to the time. Now at the mean distance unity of the moon from the earth, the natural gravity of the moon to the earth being unity, the mean force of the moon to the earth at that distance of the moon $=1-\frac{1}{2}m$; but if $Y=$ the true anomaly of the sun, and the disturbing force of the sun at the mean distance unity be unity, the disturbing force at any other distance $1+c\times$ cos. Y will (853) be $\frac{1}{\overline{1+c\times\cos. Y}^3}=1-3c\times$ cos. Y very nearly; therefore the mean force of the moon to the earth at that distance of the sun $=1-\frac{1}{2}m\times\overline{1-3c\times\cos. Y}$. Hence, the force is altered in the ratio of $1-\frac{1}{2}m$: $1-\frac{1}{2}m+\frac{3}{2}mc\times$ cos. Y, or as 1 : 1 $+\frac{3}{2}mc\times$ cos. Y very nearly; therefore (874) the radius of the moon's orbit is changed in the ratio of 1 : $1-\frac{3}{2}mc\times$ cos. Y, and (875) the mean motion of

the moon at the mean distance of the earth from the sun : the mean motion at the distance $1 + c \times \cos. Y :: 1 : 1 + 3mc \times \cos. Y$; therefore in the time in which the sun has described the arc Y, the mean motion of the moon : the equation of the mean motion :: fluent of $\dot{Y}$: fluent of $3mc \times \cos. Y \times \dot{Y} :: Y : 3mc \times \sin. Y$; but in the time in which the sun describes Y, the mean motion of the moon is $\frac{2139}{160}$ * $\times Y$; hence, $\frac{2139}{160} \times Y$: the equation of the mean motion $:: Y : 3mc \times \sin. Y$; therefore the annual equation of the mean motion is $\frac{2139}{160} \times 3mc \times \sin. Y = 12'. 55'' \times \sin. Y$.

892. As sin. Y is positive from aphelion to perihelion, and negative from perihelion to aphelion, the annual equation is to be added to the mean motion in the former case, and subtracted in the latter. When the earth is in the perihelion, the orbit of the moon is dilated, and the moon moves slower; when the earth is in the aphelion, the orbit of the moon is contracted, and the moon moves faster; and the annual equation, by which this inequality is compensated, is nothing in aphelion and perihelion, and at the mean distance of the sun it is 12'. 55'' according to our determination. Sir I. NEWTON makes it 11'. 50''; according to MAYER it is 11'. 16''; M. D'ALEMBERT makes it 12'. 57''; and HALLEY makes it about 13'; according to M. de la LANDE, it is 11'. 8'',6. This annual equation being in proportion to sin. Y, is in proportion to the equation of the sun's center, that equation being in proportion to the same quantity (870), the excentricity of the earth's orbit being very small.

To find the Equation arising from the Inclination of the Orbit to the Ecliptic.

893. Let $CABD$ be the plane of the ecliptic, NMn the plane of the moon's orbit, Nn the line of the nodes, M the moon, E the earth, S the sun; draw Ss, Av, perpendicular to the plane NMn, also Aw perpendicular to Nn, and MK perpendicular to CB; join vw, SM, and draw svE, sM, and on EM produced let fall the perpendicular sI. Put $a = SE$, $s =$ the sine of Awv the inclination of the orbit, and let us consider the mean value of $ME = 1$, $v = Aw$ the sine of SEN, $x = MK$; then, $vA = vs$, and $Ss = avs$; hence, $Es^2 = a^2 - a^2v^2s^2$; and $Es = a \times \overline{1 - \tfrac{1}{2} v^2s^2}$; also, $sM^2 = sE^2 + EM^2 - 2sE \times MK = a^2 - a^2v^2s^2 + 1^2 - 2ax \times \overline{1 - v^2s^2}|^{\frac{1}{2}}$; hence, $SM^2 \; (= Ss^2 + sM^2) = a^2 + 1^2 - 2ax \times \overline{1 - v^2s^2}|^{\frac{1}{2}}$, therefore $SM^3 = \overline{a^2 + 1^2 - 2ax \times \overline{1 - v^2s^2}|^{\frac{1}{2}}}|^{\frac{3}{2}} = a^3 - 3a^2x \times \overline{1 - \tfrac{1}{2} v^2s^2}$ nearly, consequently $\frac{1}{SM^3}$ FIG. 207.

* Because the mean motion of the sun is to the mean motion of the moon as 160 : 2139 very nearly.

$=\frac{1}{a^3}+\frac{3x}{a^4}\times\overline{1-\frac{1}{2}v^2s^2}$ nearly. Now if $\frac{1}{ES^2}$, $\frac{1}{MS^2}$ represent the attractions of E and M to S, then $ES:Es::\frac{1}{ES^2}:\frac{Es}{ES^3}$ the attraction of E in the direction Es, and $SM:sM::\frac{1}{SM^2}:\frac{sM}{SM^3}$ the attraction of M in the direction Ms; hence, $ES:EI::EM\ (=1):MK::\frac{Es}{ES^3}:\frac{MK\times Es}{ES^3}$ the attraction of E in the direction EM, and $sM:MI::\frac{sM}{SM^3}:\frac{MI}{SM^3}$ the attraction of M in the direction EM; hence, the difference of the forces of M and E in that direction is $\frac{MI}{SM^3}-\frac{MK\times Es}{ES^3}$; but $MI=EI-1=Es\times MK-1$; hence, the disturbing force in the direction EM is $Es\times\overline{MK-1}\times\frac{1}{SM^3}-\frac{MK\times Es}{ES^3}=Es\times\overline{x-1}\times\overline{\frac{1}{a^3}+\frac{3x}{a^4}\times\overline{1-\frac{1}{2}v^2s^2}}-\frac{Es\times x}{a^3}=-\frac{1}{a^3}+\frac{3x^2}{a^3}-\frac{3x^2v^2s^2}{a^3}$; but $\frac{1}{a^3}=n^2$; and as $x^2=\frac{1}{2}-\frac{1}{2}$ cos. $2MEC$, for a whole revolution of the moon we may assume $x^2=\frac{1}{2}$; also, $v^2=\frac{1}{2}-\frac{1}{2}$ cos. $2SEN$; hence, in any given position of the nodes in respect to the sun, the mean disturbing force is $\frac{1}{2}n^2-\frac{3}{4}n^2s^2+\frac{3}{4}n^2s^2\times$ cos. $2SEN$; and as the attraction of M to $E=1+\frac{1}{2}n^2$, the mean force of M to $E=1+\frac{1}{2}n^2-\frac{1}{2}n^2+\frac{3}{4}n^2s^2-\frac{3}{4}n^2s^2\times$ cos. $2SEN=1+\frac{3}{4}n^2s^2-\frac{3}{4}n^2s^2\times$ cos. $2SEN$. Hence, the mean force : the mean force in different situations of the sun in respect to the node $::1+\frac{3}{4}n^2s^2:1+\frac{3}{4}n^2s^2-\frac{3}{4}n^2s^2\times$ cos. $2SEN::1:1-\frac{3n^2s^2}{4+3n^2s^2}\times$ cos. $2SEN=1-\frac{3}{4}n^2s^2\times$ cos. $2SEN$ nearly; therefore (874) the mean radius of the orbit will be increased in the ratio of $1:1+\frac{3}{4}n^2s^2\times$ cos. $2SEN$; and (875) the mean motion is diminished in the ratio of $1:1-\frac{3}{2}n^2s^2\times$ cos. $2SEN$; consequently the mean motion of the moon in the time the sun departed from the node through the angle SEN : the equation of the mean motion in the same time :: the fluent of $\dot{\overline{SEN}}$, or SEN, : the fluent of $-\frac{3}{2}n^2s^2\times$ cos. $2SEN\times\dot{\overline{SEN}}$, or $-\frac{3}{4}n^2s^2\times$ sin. $2SEN$. Now when the sun leaves the node, it comes to the node again after it has described an angle of about 341,3°; therefore in the time in which the sun has departed from the node through angle SEN, its motion has been $\frac{341,3}{360}\times SEN$, and the mean motion of the moon in that time has been $\frac{2139}{160}\times\frac{341,3}{360}\times SEN$; hence, $\frac{2139}{160}\times\frac{341,3}{360}\times SEN$: the equation of the mean motion :: $SEN:-\frac{3}{4}n^2s^2\times$ sin. $2SEN$, consequently the equation of the mean motion is $\frac{2139}{160}\times\frac{341,3}{360}\times-\frac{3}{4}n^2s^2\times$ sin. $2SEN=-1'.\ 28''\times$ sin. $2SEN$. But if S be the distance of the moon from the node, and X the mean distance of the moon from the sun, then $2SEN=2S-2X$ very nearly; hence, the equation is $-1'.\ 28''\times$ sin. $\overline{2S-2X}$; this equation is there-

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fore to be subtracted from the mean motion of the moon, in the transit of the sun from the nodes to quadratures from thence; and added, in the transit from quadratures to the nodes.

To find the Alteration of the Periodic Time of the Moon, by the disturbing Forces.

894. Let the mean distance of the moon from the earth be unity, $u=$ the velocity of projection at that point, and $1 : p$ the ratio of radius to the circumference of a circle; then $\frac{p}{u}=$ the periodic time of a body revolving in a circle at that distance, and (889) $\frac{dp}{u}=$ the synodic revolution. Now (867) $v=\frac{u}{ME}+\frac{3}{2}dmME^2\times\overline{\sin.\ MEC}^2$, and the orbit being without excentricity, it becomes (883) elliptical, whose minor axis lies in syzygies. Let $\dot{z}$ represent the fluxion of the arc of the angle MEC to radius unity, then $ME\times\dot{z}$ is the fluxion to radius ME, and the corresponding synodic fluxion is $d\times ME\times\dot{z}$. Hence, if $\dot{t}$ represent the corresponding fluxion of the time, we have $\dot{t}=\dfrac{d\times ME\times\dot{z}}{\frac{u}{ME}+\frac{3}{2}d\times m\times ME^2\times\overline{\sin.\ MEC}^2}=$ . . . $\dfrac{d\times ME^2\times\dot{z}}{u+\frac{3}{2}d\times m\times ME^3\times\overline{\sin.\ MEC}^2}=\frac{d}{u}\times ME^2\times\dot{z}-\frac{d^2}{u^2}\times\frac{3}{2}m\times ME^5\times\overline{\sin.\ MEC}^2\times\dot{z}$. Put $e=$ the difference between the mean distance (1) and the greatest or least distance; then (882) $ME=1+e\times\cos.\ 2MEC$; therefore $ME^2=1+2e\times\cos.\ 2MEC$, and $ME^5=1+5e\times\cos.\ 2MEC$, very nearly; by substitution therefore we get $\dot{t}=\frac{d}{u}\times\overline{\dot{z}+2e\times\cos.\ 2MEC\times\dot{z}}-\frac{d^2}{u^2}\times\overline{\frac{3}{2}m\times\overline{\sin.\ MEC}^2\times\dot{z}+\frac{15}{2}em\times\overline{\sin.\ MEC}^2\times\cos.\ 2MEC\times\dot{z}}=\frac{d}{u}\times\dot{z}+\frac{d}{u}\times 2e\times\cos.\ 2MEC\times\dot{z}-\frac{d^2}{u^2}\times\frac{3}{2}m\times\overline{\frac{1}{2}-\frac{1}{2}\cos.\ 2MEC}\times\dot{z}-\frac{d^2}{u^2}\times\frac{15}{2}\times em\times\cos.\ 2MEC\times\overline{\frac{1}{2}-\frac{1}{2}\cos.\ 2MEC}\times\dot{z}=\frac{d}{u}\times\dot{z}+\frac{d}{u}\times 2e\times\cos.\ 2MEC\times\dot{z}-\frac{d^2}{u^2}\times\frac{3}{4}m\times\dot{z}+\frac{d^2}{u^2}\times\frac{3}{4}m\times\cos.\ 2MEC\times\dot{z}$, neglecting the other two terms on account of their smallness; hence, $t=\frac{d}{u}\times z-\frac{d^2}{u^2}\times\frac{3}{4}m\times z+\frac{d}{u}\times\overline{e+\frac{d}{u}\times\frac{3}{8}m}\times\sin.\ 2MEC$; therefore FIG. 201.

the time of a *synodic* revolution $= \frac{d}{u} \times p - \frac{d^2}{u^2} \times \frac{3}{4} mp$; and the equation of the mean time is $\frac{d}{u} \times e + \frac{d}{u} \times \frac{3}{8} m \times \sin. 2MEC$. Hence, the time of a *periodic* revolution is $\frac{1}{u} \times p - \frac{3}{4} \times \frac{1}{u^2} \times m \times p$. If the moon describe a circle with the mean radius $ME = 1$, the periodic time is the same, with the same disturbing force, as appears by making $e = 0$. Now (818) the periodic time in this circle (there being no disturbing force) = the periodic in the ellipse which the moon would describe if it were not disturbed, the radius of the circle being the mean distance; and this ellipse being nearly a circle, the same disturbing forces must produce very nearly the same effect in each case; we may therefore consider the periodic time of the moon to be the same as that which we have here determined.

895. If there had been no disturbing force, the time of a synodic revolution would have been $\frac{d}{u} \times p$; consequently the time of a *synodic* revolution is diminished by the quantity $\frac{3}{4} mp \times \frac{d^2}{u^2}$ in consequence of the disturbing forces. The time of a *periodic* revolution is diminished by the quantity $\frac{3}{4} mp \times \frac{1}{u^2}$.

On the Motion of the Moon's Apogee, and the Variation of the Excentricity of its Orbit.

896. The principle upon which we here propose to find the motion of the apogee, is, to find in what time the whole force by which the moon is urged towards the earth will draw it through a space in the direction of the radius vector, equal to the difference between the greatest and least distances, the moon setting off from the apogee; and comparing twice that time with the time of a revolution.

FIG. 206. 897. To find the whole force with which the moon is urged towards E. The force of M to E (864) $= \frac{1}{EM^2} - \frac{1}{2} m \times EM + \frac{3}{2} m \times EM \times \cos. 2MEC$; and if N be the higher apside, and lie in octants, the force in that point $= \frac{1}{EN^2} - \frac{1}{2} m \times EN$, the cos. $2MEC$ being then $= 0$; and if this force at N : the centrifugal force :: c : 1, then the centrifugal force at $N = \frac{1}{c \times EN^2} - \frac{1}{2} m \times \frac{EN}{c}$;

but* the centrifugal force varies as the square of the velocity perpendicular to the radius directly, and the radius inversely; and the square of the velocity at $M = \frac{u^2}{EM^2} + \frac{3}{2} m \times EM^2 - \frac{3}{2} m \times EM^2 \times \cos. 2MEC$ (866); also, the square of the velocity at N in the octants $= \frac{u^2}{EN^2} + \frac{3}{2} m \times EN^2$; hence, the centrifugal force $\frac{1}{c \times EN^2} - \frac{1}{2} m \times \frac{EN}{c}$ at N in the octants : the centrifugal force at M:: $\frac{u^2}{EN^3} + \frac{3}{2} m \times EN : \frac{u^2}{EM^3} + \frac{3}{2} m \times EM - \frac{3}{2} m \times EM \times \cos. 2MEC :: 1 : \frac{EN^3}{EM^3} - \frac{3mNE^3}{2u^2} \times \cos. 2MEC$ nearly, therefore the centrifugal force at $M = \frac{EN}{c \times EM^3} \times \overline{1 - \frac{1}{2} m \times EN^3} - \frac{3m \times EN}{2c \times u^2} \times \cos. 2MEC$; hence, the whole force of the moon towards $E = \frac{1}{EM^2} - \frac{1}{2} m \times EM + \frac{3}{2} m \times EM \times \cos. 2MEC - \frac{EN}{c \times EM^3} \times \overline{1 - \frac{1}{2} m \times EN^3} + \frac{3m \times EN}{2c \times u^2} \times \cos. 2MEC.$

898. Now in estimating the *mean* motion of the apsides, all the terms where cos. $2MEC$ enters, may be neglected, for the following reasons. The cos. $2MEC = \cos. 2CEN \times \cos. 2MEN - \sin. 2CEN \times \sin. 2MEN$; hence, the force $\frac{3m \times EN}{2c \times u^2} \times \cos. 2MEC$ is composed of two parts, the first of which, having given the angle CEN the distance of the apogee from quadratures, varies as the cos. $2MEN$, which, in the transit of the moon from the higher apside to its octants decreases, and therefore makes the moon recede further from the focus; and thence to its quadratures it increases and makes the moon approach as much to the focus; and four times in every revolution this access and recess will destroy each other. The other part, which is as sin. $2MEN$, increases from the apogee to its octants, and decreases from its octants to its quadratures, and this also destroys itself four times in a revolution. If the excentricity be w, then (868, as $EM = 1 + w \times \cos. MEN$) $\frac{3}{2} m \times EM \times \cos. 2MEC = \frac{3}{2} m \times \overline{1 + w \times \cos. MEN} \times \cos. 2MEC$; of which, $\frac{3}{2} m \times \cos. 2MEC$ is destroyed, as before shown; also $\frac{3}{2} m \times w \times \cos. MEN \times \cos. 2MEC$ is very small, and in the opposite situation of the apsides, has a different sign; the access and recess therefore from this part of the force, will destroy each other in a whole revolution of the apsides. In estimating therefore the *mean* motion

* For in Art. 826, the centrifugal force wx varies as $\frac{x\mathfrak{Q}^2}{xS}$, and as the time is given, $x\mathfrak{Q}$ varies as the velocity in a direction perpendicular to the radius.

of the apsides, the terms in the force where cos. $2MEC$ enters, may be neglected; the whole force therefore in the direction of the radius may be here taken $= \frac{1}{EM^2} - \frac{1}{2} m \times EM - \frac{EN}{c \times EM^3} + \frac{m \times EN^4}{2c \times EM^3}$.

899. Let v be the velocity of the moon at M in the direction ME, $x = EM$; then by the principles of motion, $v\dot{v} = -\frac{\dot{x}}{x^2} + \frac{1}{2} m x\dot{x} + \frac{EN}{c} \times \overline{1 - \frac{m \times EN^3}{2}} \times \frac{\dot{x}}{x^3}$, whose fluent corrected is $\frac{1}{2} v^2 = \frac{1}{x} - \frac{1}{EN} - \frac{1}{4} m \times \overline{EN^2 - x^2} - \overline{1 - \frac{1}{2} m \times EN^3} \times \overline{\frac{EN}{2c \times x^2} - \frac{1}{2c \times EN}}$, or $v^2 = \frac{1}{x^2} \times \ldots\ldots$ $\overline{2x \times \overline{1 - \frac{x}{EN}} - \frac{1}{2} m \times \overline{EN^2 \times x^2 - x^4} - \overline{1 - \frac{1}{2} m \times EN^3} \times \frac{EN^2 - x^2}{c \times EN}}$. But if we suppose the moon to be projected from its apogee in the octants, with the velocity $\sqrt{\frac{u^2}{EN^2} + \frac{3}{2} m \times EN^2}$, and force of gravity $\frac{1}{EN^2} - \frac{1}{2} m \times EN$, and which varies from that point in the inverse duplicate ratio of the distance, then if a represent the semi-axis major of this ellipse, and w its excentricity, we have (826) $c : 1 :: a : a - w$; if therefore we put $EN - x = z$, and $EN^2 - x^2 = 2EN \times z - z^2$, we get $2x \times \overline{1 - \frac{x}{EN}} = 2z - \frac{2z^2}{a + w}$; also, $-\frac{1}{2} m \times \overline{EN^2 x^2 - x^4} = -m \times EN^3 \times z + \frac{5}{2} m \times EN^2 \times z^2$; and $-\overline{1 - \frac{1}{2} m \times EN^3} \times \frac{EN^2 - x^2}{c \times EN} = -\overline{1 - \frac{1}{2} m \times EN^3} \times \frac{2EN \times z - z^2}{c \times EN} = -2z + \frac{2wz}{a} + \frac{z^2}{a + w} + \frac{wz^2}{a^2 + aw} + m \times EN^3 \times z - \frac{mw \times EN^3 z}{a} - \frac{1}{2} m \times EN^2 \times z^2$; hence, $v^2 = \frac{1}{x^2} \times \overline{\frac{2w - mw \times EN^3}{a} \times z - \overline{\frac{1}{a} - 2m \times EN^2} \times z^2}$; and if $\frac{1}{a} - 2m \times EN^2 = d^2$, and

$$\frac{\frac{w}{a} - \frac{mw \times EN^3}{2a}}{\frac{1}{a} - 2m \times EN^2} = e,$$ we have $v = \frac{d}{x} \times \sqrt{2ez - z^2}$.

900. The body beginning to descend from its apogee, when it comes to its perigee, $v = 0$, therefore $z = 2e = \frac{2w - mw \times EN^3}{1 - 2ma \times EN^2} = 2w + 3mw$; hence, twice the excentricity of the ellipse described without any disturbing force : the whole approach of the moon to the earth in its passage from its apogee to its perigee by the disturbing forces $:: 2w : 3mw :: 1 : \frac{3}{2} m$. And as the difference between $2e$ and $2w$ is only the small quantity $3mw$, the mean distance of the moon from

the earth may be considered very nearly equal to the semi-axis major of the ellipse which would be described by the moon projected from the apogee in the octants, by a force varying in the inverse duplicate ratio of the distance.

901. Let $Er = EN - En$, then $Nr = 2e$; take the indefinitely small arc Mm, and with the center E describe the circular arcs Mt, ms, then $ts =$ the descent of the body towards E in the time of describing Mm; on Nr describe a semicircle $Nvwr$; put $Nt = z$, $ts = \dot{z}$, $\dot{t} =$ the time down ts, and draw tv, sw perpendicular to rN, and wx to tv. Then by the principles of motion, $\dot{t} = \frac{x\dot{z}}{d\sqrt{2ez - z^2}}$ $= \frac{\overline{EN - e} \times \dot{z}}{d\sqrt{2ez - z^2}} + \frac{\overline{e - z} \times \dot{z}}{d\sqrt{2ez - z^2}} = \frac{EN - e}{d} \times \frac{vw}{e} + \frac{xv}{d}$, whose fluent is $t = \frac{EN - e}{d} \times \frac{Nv}{e} + \frac{tv}{d}$, and when M comes to n, the whole time of descent from the higher to the lower apside becomes $\frac{EN - e}{d} \times \frac{Nvr}{e} =$ (if $1 : p ::$ radius : circumference) $\frac{1}{2} p \times \frac{EN - e}{d} = \frac{1}{2} p \times \dfrac{a + w - \dfrac{\frac{w}{a} - \frac{mw \times EN^3}{2a}}{\frac{1}{a} - 2m \times EN^2}}{\sqrt{\frac{1}{a} - 2m \times EN^2}} =$ $\frac{\frac{1}{2} p \times \overline{1 - 2m \times EN^3}}{\overline{\frac{1}{a} - 2m \times EN^2}|^{\frac{3}{2}}}$; and if the mean distance be unity, then $EN = 1 + e$, and e being very small, $EN^3 = 1 + 3e$, $EN^2 = 1 + 2e$ nearly; hence, the time from the higher apside till the return to the same apside becomes $\frac{p \times \overline{1 - 2m \times \overline{1 + 3e}}}{\overline{\frac{1}{a} - 2m \times \overline{1 + 2e}}|^{\frac{3}{2}}} = \frac{p \times \overline{1 - 2m}}{\overline{\frac{1}{a} - 2m}|^{\frac{3}{2}}}$; and the orbit being nearly circular, a is nearly $= 1$; hence, the time $= \frac{p}{\sqrt{1 - 2m}}$ very nearly.

902. By Art. 894. the moon's periodic time is $\frac{p}{u} - \frac{3}{4} m \times \frac{p}{u^2}$; hence, the mean time from the moon leaving its apogee till it returns to it : the mean periodic time :: $\frac{p}{\sqrt{1 - 2m}} : \frac{p}{u} - \frac{3}{4} m \times \frac{p}{u^2} :: \frac{1}{\sqrt{1 - 2m}} : \frac{1}{u} - \frac{3}{4} m \times \frac{1}{u^2}$. Now $m = ,0055796$, and $u = \sqrt{1 - \frac{1}{2} m} = ,9986041$; hence, the above ratio becomes $1 : ,9916199$; therefore $,9916199 : 1 :: 360° : 363°. 2'. 32'',3916$; consequently the motion of

the apogee in one mean periodic revolution of the moon, is $3°.2'.32'',3916$; hence, $27d.\ 7h.\ 43'$: $365d.\ 6h.\ 9'$ :: $3°.\ 2'.\ 32'',3916$: $40°.\ 40'.\ 20''$ the mean progressive motion of the apogee in a year. According to MAYER's Tables, it is $40°.\ 41'.\ 33''$. If the force perpendicular to the radius vector be neglected, the motion of the apsides comes out one half of what is here determined, and this we know from other principles. This therefore tends to confirm the legality of the method here employed.

903. If for u we put $\sqrt{1-\frac{1}{2}m}$; we have the mean interval of time between the passages of the moon through its apogee : its periodic time :: $\frac{1}{\sqrt{1-2m}}$: $\frac{1}{\sqrt{1-\frac{1}{2}m}} - \frac{3}{4}m \times \frac{1}{1-\frac{1}{2}m}$ :: $\frac{1}{1-m}$: $\frac{1}{1-\frac{1}{4}m} - \frac{3}{4}m \times \frac{1}{1-\frac{1}{2}m}$ nearly, :: $1+m$: $1+\frac{1}{4}m - \frac{3}{4}m \times \overline{1+\frac{1}{2}m}$ nearly, :: $1+m$: $1-\frac{1}{2}m$ :: $\frac{1+m}{1-\frac{1}{2}m}$: 1 :: $1+\frac{3}{2}m$: 1 very nearly. Hence, the mean motion of the moon : the mean progressive motion of the apsides :: 1 : $\frac{3}{2}m$. But this is the mean motion of the moon, and answers to the mean distance of the earth from the sun; but at any other distance $1+c \times \cos. Y$ of the earth from the sun, the disturbing force of the sun (891) is $1-3c \times \cos. Y$; hence, in general, the mean motion of the moon : the mean progressive motion of the apsides, including the annual variation, :: 1 : $\frac{3}{2} \times m \times \overline{1-3c \times \cos. Y}$; therefore the mean progressive motion of the apsides in the time the sun describes the angle Y : equation of the mean motion in that time :: fluent of $\dot{Y}$: fluent of $-3c \times \cos. Y \times \dot{Y}$ :: Y : $-3c \times \sin. Y$; but in the time the sun describes the angle Y, the mean motion of the apsides is $\frac{40°.\ 41'.\ 33''}{360°} \times Y$; hence, $\frac{40°.\ 41'.\ 33''}{360°} \times Y$: the equation of the mean motion :: Y : $-3c \times \sin. Y$, therefore the *annual* equation of the apsides $= -3c \times \frac{40°.\ 41'.\ 33''}{360°} \times \sin. Y = -19'.\ 35'' \times \sin. Y$. Sir I. NEWTON makes it $19'.\ 43''$.

904. By Art. 864. the force of M to $E = \frac{1}{ME^2} - \frac{1}{2}m \times ME + \frac{3}{2}m \times ME \times \cos. 2MEC$; and at the higher apside N, it becomes $\frac{1}{NE^2} - \frac{1}{2}m \times NE + \frac{3}{2}m \times NE \times \cos. 2NEC$; now if the force from N were to vary inversely as the square of the distance, at M it would become $\frac{1}{ME^2} - \frac{m \times NE^3}{2ME^2} \times \overline{1-3\cos. 2NEC}$; and by changing the angle NEC, the ellipse is continually changing into a new one; but if the orbit be very nearly a circle, the excen-

tricity and position of the apsides will (877, 878) not sensibly be altered in a whole revolution. Hence, the motion of the apsides, and the variation of the excentricity, depend upon $-\frac{1}{2}m \times ME \times \overline{1-3.\ \cos.\ 2MEC} + \frac{m \times NE^3}{2ME^2} \times \overline{1-3.\cos.\ 2NEC}$, which is the difference between the real force at M, and what would have been the force if it had varied inversely as the square of the distance from N.

905. The forces $-\frac{1}{2}m \times ME + \frac{m \times NE^3}{2ME^2}$ are the same, very nearly, in every situation of the apsides; also the force $\frac{3}{2}m \times ME \times \cos.\ 2MEC$ (in orbits nearly circular) nearly destroys itself in every revolution, by the opposition of its signs. Hence, the variation of the excentricity and the inequality of the motion of the apsides, so far as they depend upon the situation of the apsides, arises from the force $-\frac{3m \times NE^3}{2ME^2} \times \cos.\ 2NEC$, which varies as cos. $2NEC$ nearly, in an orbit which is very nearly a circle. Hence, the whole inequality of the motion of the apsides in their transit from quadratures C to N is nearly in proportion to the fluent of cos. $2NEC \times \dot{\overline{2NEC}}$, or to sin. $2NEC$, or sin. $2NES$ the distance of the apsides from syzygies. By observation, this greatest variation of the true from the mean place of the apside is about 12°. 18′; hence, this equation of the motion of the apsides = 12°. 18′ × sin. $2NES$ nearly.

906. Let the absolute gravity in the higher apside $N = \frac{b}{EN^2}$, and let it vary from that point in the inverse duplicate ratio of the distance, and let w be the excentricity of the ellipse so described, and a be its semi-axis major; then (826) the centrifugal force at $M = \frac{a-w}{a} \times \frac{b \times EN}{ME^3}$. Now (905) the variation of the excentricity depends upon the force $-\frac{3m \times EN^3}{2ME^2} \times \cos.\ 2NEC$. Hence, if $x = ME$, $v =$ the velocity in direction of the radius, then $v\dot{v} = \frac{-b\dot{x}}{x^2} + \frac{3m \times EN^3}{2} \times \frac{\dot{x}}{x^3} \times \cos.\ 2NEC + b \times \overline{1 - \frac{w}{a}} \times EN \times \frac{\dot{x}}{x^3}$, whose correct fluent (if $z = EN - x$) is $v^2 = \frac{b}{EM^2} \times \frac{2wz - z^2}{a} - 3m \times EN^3 \times \cos.\ 2NEC \times \overline{\frac{1}{EM} - \frac{1}{EN}}$; now the last term is equal $-\frac{3m \times EN^3}{EM^2} \times \cos.\ 2NEC \times \overline{EM - \frac{EM^2}{EN}} = -\frac{3m \times EN^3}{EM^2} \times \cos.\ 2NEC \times \overline{z - \frac{z^2}{EN}}$; hence, $v^2 = \frac{1}{EM^2} \times \Big\{\frac{2bwz - bz^2}{a} - 3m \times EN^3 \times \cos.\ 2NEC \times z + 3m \times EN^2 \times \cos.\ 2NEC \times z^2;\Big)$ but the body de-

scending from the higher to the lower apside, this velocity at the lower apside vanishes; therefore if we make $v=0$, we get $z=$

$$\frac{\frac{2bw}{a}-3m\times EN^3\times\cos.\ 2NEC}{\frac{b}{a}-3m\times EN^2\times\cos.\ 2NEC}$$

the whole space through which the body has descended in a direction of the radius vector, in its motion from the higher to the lower apside, or twice the excentricity of the orbit described. Now (864) the force of gravity $\frac{b}{EN^2}$ in the higher apside $=\frac{1}{EN^2}-\frac{1}{2}m\times EN+\frac{3}{2}m\times EN\times\cos.$ $2NEC$; and if unity represent the semi-axis major of that orbit which would be described by the mean force $\frac{1}{EN^2}-\frac{1}{2}m\times EN$ in the higher apside, and that force be increased in the ratio of $\frac{1}{EN^2}-\frac{1}{2}m\times EN:\frac{1}{EN^2}-\frac{1}{2}m\times EN+\frac{3}{2}m\times EN\times\cos.\ 2NEC$, or as $1:1+\frac{3}{2}m\times EN^3\times\cos.\ 2NEC$, then (874) by this increase of force, the semi-axis major is diminished in the ratio of $1:1-\frac{3}{2}m\times EN^3\times\cos.\ 2NEC$; therefore unity being the semi-axis major by the former force, the semi-axis major a by the latter force $=1-\frac{3}{2}m\times EN^3\times\cos.\ 2NEC$. But $b=1-\frac{1}{2}m\times EN^3+\frac{3}{2}m\times EN^3\times\cos.\ 2NEC$; hence, $\frac{b}{a}=1-\frac{1}{2}m\times EN^3+3m\times EN^3\times\cos.\ 2NEC$; consequently $z=2w-$

$$\frac{3m\times EN^3\times\cos.\ 2NEC}{1-\frac{1}{2}m\times EN^3+3m\times EN^3\times\cos.\ 2NEC-3m\times EN^2\times\cos.\ 2NEC}=2w-\frac{3m\times EN^3\times\cos.\ 2NEC}{1-\frac{1}{2}m\times EN^3}$$

nearly; therefore the variation of the excentricity $=-\frac{3m\times EN^3\times\cos.\ 2NEC}{2-m\times EN^3}$ nearly. Hence, the variation is nearly in proportion to the cosine of twice the distance of the apogee from quadratures, or twice the distance of the apogee from the sun.

907. When the apsides are in the octants, $\cos.\ 2NEC=0$, and the excentricity is there the mean. When the apsides are in syzygies, $\cos.\ 2NEC=-1$, and the variation of excentricity $=\frac{3m\times EN^3}{2-m\times EN^3}$, the mean excentricity being then increased by this quantity, on account of its being positive. When the apsides are in quadratures, $\cos.\ 2NEC=1$, and the variation $=-\frac{3m\times EN^3}{2-m\times EN^3}$, which being negative, shows that the mean excentricity is there diminished by this quantity. Hence, the excentricity is increased whilst the apsides move from quadratures to syzygies, and decreased whilst they move from syzygies to quadratures.

908. As $m=0,0055796$, and $EN=1,05506$, the mean distance being unity, and the mean excentricity $=,05505$, we have $\frac{3m \times EN^3}{2-m \times EN^3}=0,00986$ the greatest variation of the excentricity from the mean excentricity. By observation it is 0,01168.

909. Sir I. NEWTON gives the following construction for finding the excentricity of the orbit, and the equation of the apogee. Let E be the focus of the elliptic orbit described by the moon M, and $EKFI$ the mean place of the apogee, corrected by the annual equation (903); $EF=,05505$ the mean excentricity, $EK=,04387$ the least excentricity of the moon's orbit, and $EI=,06671$ the greatest; bisect KI in F, and with the center F describe the circle IHK; take HFI equal to twice the distance of the true place of the sun, from the place A of the apogee corrected by the annual equation, and EH will be the excentricity of the orbit, and HEF the equation of the apogee, nearly. Let us now compare this construction with our conclusions. The angle HEG being small, if we consider $EH=EG$, then $EH-EF=FG=\cos.\ HFI=\cos.$ twice the distance of the apogee once equated, from syzygies, to radius FI; but (906) the variation of the excentricity varies as that cosine nearly, therefore if the cosine expresses nearly the value in one case, it must always express it; now when H comes to I, FG becomes FI, and expresses the variation in that case, by construction; hence, FG expresses, in general, the variation, consequently EH will be the excentricity. Also (if we consider the variation of EH to be very small) HG may be considered as measuring the angle HEG, and HG is the sine of HFI twice the distance of the apogee from syzygies nearly, and (905) the equation of the apogee varies as that quantity; therefore if HEG express the true variation in one case, it must in all; now when G coincides with F, $GH=FD$ (FD being perpendicular to KI)$=0,01168$, and the angle $DEF=12°.\ 15'$ which is very near to $12°.\ 18'$ the greatest equation; hence, the angle HEG always expresses nearly the equation of the apogee. If X=the mean distance of the moon from the sun, $r=0,01168$, the excentricity$=w+r \times \cos.\ \overline{2Z-2X}$ very nearly. FIG. 208.

910. Produce EF to A the mean apogee of the orbit, and let L be the other focus, join MF, ML, MH, and on MF let fall the perpendicular Hv; draw Ew parallel to LM, and ES towards the sun. Now (227) $2EMF$ is the mean equation of the center, and $2EMH$ is the equation at the given time; hence, $2HMF$ is the difference of the two equations, or the *Evection*, whose sine is $\frac{2Hv}{MH}$, or nearly$=\frac{2Hv}{MF}=\frac{2HF}{MF} \times \sin.\ HFM$. Let $Z=$ the angle MLA the mean anomaly of the moon, $X=$the distance of the mean place of the moon from the sun, or the angle SEw, and $z=LME=MEw$; then $Z-z=AEM$ the true anomaly of the moon, and $X-z=SEM$; and as the moon moves

$Z - \frac{r}{w} \times \sin. \overline{2Z - 2X}$, as we do in the correction of the third term, it would produce the evection, which we have already determined (910). Next, in the third term ($\frac{3}{4} w^2 \times \sin. 2Z$), for w put $w + r \times \cos. \overline{2Z - 2X}$ (908), and for sin. $2Z$ write the value of it corrected for the principal equation of the apogee, that is, $\sin. (2Z - \frac{2r}{w} \times \sin. \overline{2Z - 2X}) = \sin. 2Z - \frac{2r}{w} \times \sin. \overline{2Z - 2X} \times \cos. 2Z$ nearly, $= \sin. 2Z + \frac{r}{w} \times \sin. 2X - \frac{r}{w} \times \sin. \overline{4Z - 2X}$, and we get these new terms $\frac{3}{4} rw \times \sin. 2X - \frac{3}{4} rw \times \sin. \overline{4Z - 2X} + \frac{3}{2} rw \times \sin. 2Z \times \cos. \overline{2Z - 2X}$ nearly, $=$(as the last term $= \frac{3}{4} rw \times \sin. \overline{4Z - 2X} + \frac{3}{4} rw \times \sin. 2X$) $\frac{3}{2} rw \times \sin. 2X$. The preceding correction $Z + 32'. 20'' \times \sin. Y$ of Z was not here introduced, on account of the smallness of the equations which it would have produced. Hence, we have these new equations; $-32'. 30'' w \times \sin. \overline{Z + Y} + 32'. 30'' w \times \sin. \overline{Z - Y} + \frac{3}{2} rw \times \sin. 2X = -1'. 47'' \times \sin. \overline{Z + Y} + 1'. 47'' \times \sin. \overline{Z - Y} + 5'. 31'' \times \sin. 2X$.

To correct the Equation of the Variation of the Moon.

914. The variation of the moon as already determined (889) is $35'. 10'' \times \overline{1 - 3{,}299c \times \cos. Y} \times \sin. 2X = 35'. 10'' \times \sin. 2X - 3{,}299c \times \cos. Y \times \sin. 2X$. Now (870) the difference of the true and mean places of the moon is $-2w \times \sin. Z$, omitting the other terms of the series; but this is without considering the annual equation $+ 12'. 55'' \times \sin. Y$, or $,003758 \times \sin. Y$, of the moon; hence, the difference becomes $-2w \times \sin. Z + ,003758 \times \sin. Y$; and considering X as the mean distance of the moon from the sun, we have $X - 2w \times \sin. Z + ,003758 \times \sin. Y$ for a corrected distance of the sun from the moon; hence, sin. $2X$ becomes $\sin. (2X - 4w \times \sin. Z + ,007516 \times \sin. Y) = \sin. 2X - 4w \times \sin. Z \times \cos. 2X + ,007516 \times \sin. Y \times \cos. 2X = \sin. 2X - 2w \times \sin. \overline{2X + Z} + 2w \times \sin. \overline{2X - Z} + ,003758 \times \sin. \overline{2X + Y} - ,003758 \times \sin. \overline{2X - Y}$. Hence, the corrected variation of the moon becomes $35'. 10'' \times (\sin. 2X - 2w \times \sin. \overline{2X + Z} + 2w \times \sin. \overline{2X - Z} + ,003758 \times \sin. \overline{2X + Y} - ,003758 \times \sin. \overline{2X - Y}) - 3{,}299 \times 35'. 10'' \times c \times \cos. Y \times (\sin. 2X - 2w \times \sin. \overline{2X + Z} + 2w \times \sin. \overline{2X - Z})$ nearly, omitting the other two terms on account of their smallness. But the last term, after actually multiplying each part of it into cos. Y, may be further resolved into $-3{,}299 \times 35'. 10'' \times c \times (\frac{1}{2} \sin. \overline{2X + Y} + \frac{1}{2} \sin. \overline{2X - Y} - w \times \sin. \overline{2X + Z + Y} - w \times \sin. \overline{2X + Z - Y} + w \times \sin. \overline{2X - Z + Y} + w \times \sin. \overline{2X - Z - Y}$. Hence, besides the equation $35'. 10'' \times 2X$, we get the following, $35'. 10'' \times$

$(-2w \times \sin. \overline{2X+Z}+2w \times \sin. \overline{2X-Z}+,003758 \times \sin. \overline{2X+Y}-,003758 \times \sin. \overline{2X-Y})-3,299 \times 35'. 10'' \times c \times (\frac{1}{2} \sin. \overline{2X+Y}+\frac{1}{2} \sin. \overline{2X-Y}-w \times \sin. \overline{2X+Z+Y}-w \times \sin. \overline{2X+Z-Y}+w \times \sin. \overline{2X-Z+Y}+w \times \sin. \overline{2X-Z-Y}) = -3'. 52'' \times \sin. \overline{2X+Z}+3'. 52'' \times \sin. \overline{2X-Z}+7'',9 \times \sin. \overline{2X+Y}-7'',9 \times \sin. \overline{2X-Y}-58'',4 \times \sin. \overline{2X+Y}-58'',4 \times \sin. \overline{2X-Y}+6'',4 \times \sin. \overline{2X+Z+Y}+6'',4 \times \sin. \overline{2X+Z-Y}-6'',4 \times \sin. \overline{2X-Z+Y}-6'',4 \times \sin. \overline{2X-Z-Y} = -3'. 52'' \times \sin. \overline{2X+Z}+3'. 52'' \times \sin. \overline{2X-Z}-50'',5 \times \sin. \overline{2X+Y}-1'. 6'',3 \times \sin. \overline{2X-Y}+6'',4 \times \sin. \overline{2X+Z+Y}+6'',4 \times \sin. \overline{2X+Z-Y}-6'',4 \times \sin. \overline{2X-Z+Y}-6'',4 \times \sin. \overline{2X-Z-Y}$.

915. The principal part of the variation is $35'. 10'' \times \sin. 2X$. Now let $2X$ (double the distance of the moon from the sun) be corrected, by taking two terms (870) of the equation of the center of the moon, that is, $-2w \times \sin. Z + \frac{5}{4} w^2 \times \sin. 2Z$, and then $2X$ becomes $2X-4w \times \sin. Z+\frac{5}{2} w^2 \times \sin. 2Z$; substitute this therefore for $2X$ into $35'. 10'' \times \sin. 2X$, and we get $35'. 10'' \times \sin. (2X-4w \times \sin. Z+\frac{5}{2} w^2 \times \sin. 2Z)=35'. 10'' \times (\sin. 2X-4w \times \sin. Z \times \cos. 2X+\frac{5}{2} w^2 \times \sin. 2Z \times \cos. 2X)$; and the two first having been already considered, there arises a new equation $35'. 10'' \times \frac{5}{2} w^2 \times \sin. 2Z \times \cos. 2X=35'. 10'' \times \frac{5}{4} w^2 \times \sin. \overline{2Z+2X}+35'. 10'' \times \frac{5}{4} w^2 \times \sin. \overline{2Z-2X}=8'' \times \sin. \overline{2Z+2X}+8'' \times \sin. \overline{2Z-2X}$.

916. The principal part of the variation may also be corrected, if X be corrected by the equation of the evection, or if for X we substitute $X-2r \times \sin. \overline{2X-Z}$ (910), in which case we get $35'. 10'' \times \sin. (2X-4r \times \sin. \overline{2X-Z})=35'. 10'' \times \sin. 2X - 35'. 10'' \times 4r \times \cos. 2X \times \sin. \overline{2X-Z}$; hence, there arises a new equation $-35'. 10'' \times 4r \times \cos. 2X \times \sin. \overline{2X-Z}=-35'. 10'' \times 2r \times \sin. \overline{4X-Z}+35'. 10'' \times 2r \times \sin. Z = -49'',3 \times \sin. \overline{4X-Z}+49'',3 \times \sin. Z$.

917. The variation will also vary in proportion to the disturbing force of the sun. Now (893) the mean disturbing force : that disturbing force which arises from the different situations of the node of the lunar orbit :: $1 : \frac{3}{2} s^2 \times \cos. \overline{2S-2X}$; hence there arises another correction of the variation $=35'. 10'' \times \frac{3}{2} s^2 \times \sin. 2X \times \cos. \overline{2S-2X}=35'. 10'' \times \frac{3}{4} s^2 \times \sin. 2S+35'. 10'' \times \frac{3}{4} s^2 \times \sin. \overline{4X-2S}=12'',5 \times \sin. 2S-12'',5 \times \sin. \overline{2S-4X}$.

To correct the Equation of the Evection of the Moon.

918. The evection (910) is $-2r \times \sin. \overline{2X-Z}$, and $r=HF$, which (906) is proportional to m, or to the disturbing force of the sun; therefore, in

general, the evection $= -\overline{1 - 3c \times \cos. Y} \times 2r \times \sin. \overline{2X - Z}$; there arises therefore a new equation $= 6cr \times \cos. Y \times \sin. \overline{2X - Z} = 3cr \times \sin. \overline{2X - Z + Y} + 3cr \times \sin. \overline{2X - Z - Y} = + 2'. 1'' \times \sin. \overline{2X - Z + Y} + 2'. 1'' \times \sin. \overline{2X - Z - Y}$.

919. On account of the various situations of the node, the disturbing force (893) varies as $1 : 1 + \frac{3}{2} s^2 \times \cos. \overline{2S - 2X}$; hence, the evection becomes $- \overline{1 + \frac{3}{2} s^2 \times \cos. \overline{2S - 2X}} \times 2r \times \sin. \overline{2X - Z}$; there arises therefore a new equation $= -3rs^2 \times \cos. \overline{2S - 2X} \times \sin. \overline{2X - Z} = -\frac{3}{2} rs^2 \times \sin. \overline{2S - Z} - \frac{3}{2} rs^2 \times \sin. \overline{4X - 2S - Z} = -29'' \times \sin. \overline{2S - Z} - 29'' \times \sin. \overline{4X - 2S - Z}$.

920. If double the distance of the moon from the sun ($2X$) be corrected by the annual equation (891), it becomes $2X + 25'. 50'' \times \sin. Y$, and if the mean motion Z of the moon from the apogee be corrected by the same equation $12'. 55'' \times \sin. Y$, and by the annual equation $-19'. 35'' \times \sin. Y$, then Z becomes $Z + 32'. 30'' \times \sin. Y$; hence, $\overline{2X - Z}$ becomes $\overline{2X - Z - 6'. 40'' \times \sin. Y}$; therefore the evection becomes $-2r \times \sin. (\overline{2X - Z - 6'. 40'' \times \sin. Y}) = -2r \times \sin. \overline{2X - Z} - 2r \times 6'. 40'' \times \sin. Y \times \cos. \overline{2X - Z}$; hence there arises a new equation $-2r \times 6'. 40'' \times \sin. Y \times \cos. \overline{2X - Z} = -r \times 6'. 40'' \times \sin. \overline{2X - Z + Y} + r \times 6'. 40'' \times \sin. \overline{2X - Z - Y} = -41'' \times \sin. \overline{2X - Z + Y} + 41'' \times \sin. \overline{2X - Z - Y}$.

921. Hence, we have the following equations of the moon's longitude (reckoned upon its orbit), to be applied to the mean in order to obtain the true longitude; having collected into one term, all those which have the same argument.

I. . . .	$-6^\circ. 17'. 32'',7$		$\sin. Z.$
II. . . .	$+$	13. 0	$\sin. 2Z.$
III. . . .	$-$	0. 37	$\sin. 3Z.$
IV. . . .	$-1.$	16. 26	$\sin. \overline{2X - Z}.$
V. . . .	$-$	3. 52	$\sin. \overline{2X + Z}.$
VI. . . .	$+$	40. 41	$\sin. 2X.$
VII. . . .	$+$	12. 55	$\sin. Y.$
VIII. . .	$+$	1. 47	$\sin. \overline{Z - Y}.$
IX. . . .	$-$	1. 47	$\sin. \overline{Z + Y}.$
X. . . .	$-$	1. 6,3	$\sin. \overline{2X - Y}.$
XI. . . .	$-$	0. 50,5	$\sin. \overline{2X + Y}.$
XII. . . .	$-$	0. 12	$\sin. \overline{2Z - 2X}.$
XIII. . .	$+$	0. 2,1	$\sin. \overline{4Z - 4X}.$

XIV.	- -	−	0. 49,3	sin. $\overline{4X-Z}$.
XV.	- - -	+	0. 12,5	sin. $2S$.
XVI.	- -	−	1. 28	sin. $\overline{2S-2X}$.
XVII.	- -	−	0. 29	sin. $\overline{2S-Z}$.
XVIII.	- -	+	1. 59,3	sin. $\overline{2X-Z-Y}$.
XIX.	- - -	+	1. 49,9	sin. $\overline{2X-Z+Y}$.
XX.	- -	+	0. 8	sin. $\overline{2Z+2X}$.
XXI.	- -	−	0. 12,5	sin. $\overline{2S-4X}$.
XXII.	- -	−	0. 29	sin. $\overline{4X-2S-Z}$.
XXIII.	- -	+	0. 6,4	sin. $\overline{2X+Z-Y}$.
XXIV.	- -	+	0. 6,4	sin. $\overline{2X+Z+Y}$.

922. These are the equations of the moon's longitude upon its orbit, deduced from the principles which Sir I. NEWTON has given in his *Principia;* and it is manifest, that we might have proceeded in the same manner, and found many more smaller equations. By this method of treating the subject, the sources of all the equations become manifest, and every one is calculated directly from the cause which produces it. Comparing these with the equations deduced from the direct method, no greater difference is found, than what is observed to take place amongst those which are computed from the direct solution by different Authors; and MAYER after his most laborious calculations, founded upon a very elegant theory, was obliged to correct many of his equations from observations. We shall afterwards show the reduction necessary to give the longitude upon the ecliptic.

923. The equation $1'.\ 49'',9 \times$ sin. $\overline{2X-Z+Y}$, Sir I. NEWTON thus represents. The construction of FIG 208. remaining, make the angle $MHr = 2X$ FIG.
$-Z+Y$, and take $Hr = ,000352$, which quantity subtends an angle of $1'.\ 12\frac{1}{2}''$ 208.
at M, the whole equation being $2'.\ 25''$ according to Sir I. NEWTON, and not $1'.\ 49'',9$ as we make it; and Hmr will be the correction for this equation. For conceive the center of the lunar orbit to be transported from H to r, and to describe a circle about H, and draw rs perpendicular to MH; then $rs = ,000352 \times$ sin. $\overline{2X-Z+Y}$, therefore the angle $rMH = 1'.\ 12\frac{1}{2}'' \times$ sin. $\overline{2X-Z+Y}$, which varies as sin. $\overline{2X-Z+Y}$, and the whole quantity of variation is $2'.\ 25''$, taking Hr on opposite sides of H, and at right angles to MH. As therefore the whole variation of rMH is (from this construction) the true quantity, and it varies in its proper ratio, it must always represent the true correction.

On the Equations of the horizontal Parallax of the Moon.

924. By Art. 868. the distance of the moon from the center of the earth is $1+\frac{1}{2}w^2+w\times\cos. Z-\frac{1}{2}w^2\times\cos. 2Z$; but the horizontal parallax of the moon is inversely as the distance, or as $\frac{1}{1+\frac{1}{2}w^2+w\times\cos. Z-\frac{1}{2}w^2\times\cos. 2Z}$ $=1-\frac{1}{2}w^2-w\times\cos. Z+w^2\times\overline{\cos. Z}^2+\frac{1}{2}w^2\times\cos. 2Z=$ (as $\overline{\cos. Z}^2=\frac{1}{2}\cos. 2Z+\frac{1}{2}$) $1-w\times\cos. Z+w^2\times\cos. 2Z$. Now (909) $w+r\times\cos. \overline{2Z-2X}$ $=EH$ the true excentricity at the given time, and (909) the sine of HEG (the aberration of the line of the apsides from the mean place) $=\frac{r\times\sin. \overline{2Z-2X}}{w+r\times\cos. \overline{2Z-2X}}$ $=\frac{r}{w}\times\sin. \overline{2Z-2X}$ nearly. Hence, if in the above expression for the radius vector, we put $w+r\times\cos. \overline{2Z-2X}$ for w, and $Z-\frac{r}{w}\times\sin. \overline{2Z-2X}$ for Z; and if for $\cos. (Z-\frac{r}{w}\times\sin. \overline{2Z-2X})$ we write $\cos. Z+\frac{r}{w}\times\sin. Z\times\sin. \overline{2Z-2X}$ (the cos. of $\frac{r}{w}\times\sin. \overline{2Z-2X}$ being considered unity), we get the horizontal parallax of the moon proportional to $1-w\times\cos. Z-r\times\sin. Z\times\sin. \overline{2Z-2X}-r\times\cos. Z\times\cos. \overline{2Z-2X}-\frac{r^2}{w}\times\sin. Z\times\sin. \overline{2Z-2X}\times\cos. \overline{2Z-2X}+w^2\times\cos. 2Z+2wr\times\cos. 2Z\times\cos. \overline{2Z-2X}$. But $\sin. Z\times\sin. \overline{2Z-2X}=\frac{1}{2}\cos. \overline{2X-Z}-\frac{1}{2}\cos. \overline{3Z-2X}$; also, $\cos. Z\times\cos. \overline{2Z-2X}=\frac{1}{2}\cos. \overline{2X-Z}+\frac{1}{2}\cos. \overline{3Z-2X}$; lastly, $\sin. Z\times\sin. \overline{2Z-2X}\times\cos. \overline{2Z-2X}=\frac{1}{2}\sin. Z\times\sin. \overline{4Z-4X}=\frac{1}{4}\cos. \overline{4X-3Z}-\frac{1}{4}\cos. \overline{5Z-4X}$; hence, by substitution, we get the horizontal parallax of the moon proportional to $1-w\times\cos. Z+w^2\times\cos. 2Z+wr\times\cos. 2X-\frac{r^2}{4w}\times\cos. \overline{4X-3Z}-r\times\cos. \overline{2X-Z}+wr\times\cos. \overline{4Z-2X}+\frac{r^2}{4w}\times\cos. \overline{5Z-4X}$, the mean horizontal parallax being unity.

925. This would be the horizontal parallax of the moon in an elliptic orbit having the earth in its focus; but on account of the change of the form of the orbit, at the distance X of the moon from the sun, the radius (882) varies in the ratio of $1-\frac{1}{110}\times\cos. 2X:1$; hence, the parallax varies in the ratio of $\frac{1}{1-\frac{1}{110}\times\cos. 2X}:1$, or as $1+\frac{1}{110}\times\cos. 2X:1$; there arises therefore another equation $+\frac{1}{110}\times\cos. 2X$ of the parallax. The horizontal parallax of the

moon is therefore $1 - w \times \cos. Z + w^2 \times \cos. 2Z + \overline{wr + \frac{1}{16}} \times \cos. 2X - \frac{r^2}{4w} \times$ $\cos. \overline{4X - 3Z} - r \times \cos. \overline{2X - Z} + wr \times \cos. \overline{4Z - 2X} + \frac{r^2}{4w} \times \cos. \overline{5Z - 4X}$, the mean horizontal parallax being unity. If therefore we assume $r = ,01168$, $w = ,05505$, and the *mean* equatorial parallax to be $57'. 11'',4$, as MAYER makes it, we have the equatorial horizontal parallax $= 57'. 11'',4 - 3'. 8'',9 \times \cos. Z +$ $10'',4 \times \cos. 2Z + 26'',7 \times \cos. 2X - 2'',1 \times \cos. \overline{4X - 3Z} - 40'',1 \times \cos. \overline{2X - Z}$ $+ 2'',2 \times \cos. \overline{4Z - 2X} + 2'',1 \times \cos. \overline{5Z - 4X}$. Hence, we find (173) the horizontal parallax for any other latitude.

On the Motion of the Nodes of the Moon's Orbit.

926. If z be the cosine of any angle x, then $z^2 = \frac{1}{2} + \frac{1}{2} \cos. 2x$; $z^4 = \frac{3}{8} + \frac{1}{2} \cos. 2x + \frac{1}{8} \cos. 4x$; $z^6 = \frac{5}{16} + \frac{15}{32} \cos. 2x + \frac{3}{16} \cos. 4x + \frac{1}{32} \cos. 6x$; &c. radius being unity. See my Treatise on plane and spherical Trigonometry.

927. Let E be the earth, $CADB$ be the plane of the ecliptic, NMn the moon's orbit inclined to it, and supposed to be a circle, Nn the line of the nodes, AEB the line of the syzygies perpendicular to CED, MK perpendicular to CD, Av a perpendicular to the plane of the moon's orbit, and vt, At perpendicular to NEn. Now the ablatitious force acting on the moon at M acts parallel to AE, and consequently it makes the same angle with the plane of the moon's orbit as AE does; and if we conceive AE to be a force, Av is that part of it which acts perpendicular to the plane of the moon's orbit; and if we put $s =$ the sine of the angle Atv the inclination of the orbit, to radius unity, then $1 : s :: At : Av = s \times At$; hence, AE : the ablatitious force $3MK$ (843) $:: s \times At : \frac{3s \times At \times MK}{AE}$* that part of the ablatitious force which acts perpendicular to the plane of the moon's orbit. Let the periodic time of the sun : that of the moon $:: 1 : n$; then the part of the ablatitious force acting perpendicular to the moon's orbit : the ablatitious force $:: \frac{3s \times At \times MK}{AE} : 3MK ::$ $s \times At : AE$, and (855) the ablatitious force : the gravity of the moon $:: 3n^2MK$ $: ME$ or AE; hence, the force acting perpendicular to the moon's orbit : the moon's gravity $:: 3n^2s \times At \times MK : AE^2$. Now the velocity of the moon being represented by the small arc Mw described in a given time, the velocity gene- FIG. 209.

* On account of the inclination of the moon's orbit, $3MK$ does not accurately express the ablatitious force; the difference however which this consideration would make in the result is so very small, that we shall here omit it.

rated by the gravity of the moon to the earth in the same time is represented by twice the sagitta of Mw which $= \frac{Mw^2}{ME} = \frac{Mw^2}{AE}$; hence, the velocity generated in a given time being as the force, $AE^2 : 3n^2s \times At \times MK :: \frac{Mw^2}{AE} : \frac{3n^2s \times Mw^2 \times At \times MK}{AE^3}$ the velocity generated by the ablatitious force in a direction perpendicular to the moon's orbit; draw therefore wz perpendicular to the orbit and equal to this quantity, and describe the circle $N'Mzn'$, and NN', or nn', will be the corresponding motion of the nodes. Draw np perpendicular to Mn. Now $Mw : wz ::$ sin. Mp or Mn, which is Mk, : $n'p = \frac{wz \times Mk}{Mw}$, and sin. pnn' $(s) : n'p = \frac{wz \times Mk}{Mw} :: 1 : nn' = \frac{wz \times Mk}{s \times Mw}$; hence the angle $nEn' = \frac{wz \times Mk}{s \times ME \times Mw} = 3n^2 \times \frac{Mw}{ME} \times \frac{At \times MK \times Mk}{AE^3} = 3n^2 \times \frac{Mw}{ME} \times$ sin. $AEN \times$ sin. $MEC \times$ sin. MEN the motion of the node, whilst the moon describes Mw, $\frac{Mw}{ME}$ denoting the angle which Mw subtends at E. This is the case when the orbit is a circle. Let us consider therefore what will be the motion of the nodes, if we diminish the diameter AB by a very small quantity, and suppose the curve to become an ellipse, the periodic time remaining the same.

FIG. 210. 928. Let $CA'DB'$ be the ellipse; draw MQK perpendicular to CD, and mqk parallel and indefinitely near to it; join EQ, EM, Eq, Em; with the center E describe the circular arc qct; let nEN be the line of the nodes, and draw Mr and Qs perpendicular to En. Let one body describe the circle and the other the ellipse, and let them set out together from C, then they will come to M, Q, at the same time; for (805) the area $CEM : CEA ::$ the time through CM : the time through CA, and $CEQ : CEA' ::$ the time through CQ : the time through CA'; but $CEM : CEA :: CEQ : CEA'$, therefore the time through CM : time through $CA ::$ time through CQ : the time through CA'; but by supposition, the second and fourth terms are equal, therefore the first and third are equal, and the bodies come to M and Q at the same time, and Mm, Qq will be the cotemporary arcs described; and the cotemporary areas described about E being in proportion to the whole areas, we have $Mm \times ME : qc \times QE :: EA : EA' :: ME : EA'$; therefore $Mm : qc :: QE : EA'$. But (843) the disturbing forces at M and Q are as $MK : QK$ very nearly*, and therefore (927) the motions perpendicular to the planes of the orbits (represented in Art. 927.

* They are not accurately so, as before observed, because the orbits are a little inclined to the ecliptic $CABD$.

by wz) will be as $MK : QK$. Now the force acting perpendicular to the plane of the orbit at Q turns the plane about QE, and thereby produces a motion of the nodes; and the motion produced by this force is compounded with the motion Qq, or with the two motions Qc, cq, of which Qc lies in the line about which the plane turns; therefore the motion of the nodes is the same as it would be in a body describing the circular arc cq in the same time. Hence, from the first expression in the last Article for the angle nEn', we get the motion of the nodes of the circle : the cotemporary motion of the nodes of the ellipse $:: \frac{MK \times Mr}{ME \times Mm} : \frac{QK \times Qs}{QE \times qc} :: \frac{MK \times Mr}{ME \times QE} : \frac{QK \times Qs}{QE \times EA'}$; but $\frac{Mr}{ME} = \sin. nEM = \sin. \overline{nEC + CEM} = \sin. nEC \times \cos. CEM + \sin. CEM \times \cos. nEC$; also $\frac{Qs}{QE} = \sin. nEQ = \sin. \overline{nEC + CEQ} = \sin. nEC \times \cos. CEQ + \sin. CEQ \times \cos. nEC$; now in a whole revolution, cos. CEM, and cos. CEQ are destroyed by the opposition of their signs, and therefore to get the *mean* motions, the terms where they enter may be neglected; also, $\sin. CEM = \frac{MK}{ME}$, and $\sin. CEQ = \frac{QK}{QE}$; hence, the *mean* motion of the nodes of the circle : that of the ellipse $:: \frac{MK^2}{ME \times QE} : \frac{QK^2}{EA' \times QE} :: \frac{AE^2}{AE} : \frac{EA'^2}{EA'} :: AE : EA'$.

929. With the center E and radius Ea describe the circle ab, and let that be the circle which the moon would have described if there had been no disturbing forces, supposing $CA'BD'$ to be the ellipse which it would describe without excentricity, from the disturbing forces; and let the periodic time in this circle be equal to the periodic time in the circle $CADB$, and consequently equal to the periodic time in the ellipse, according to the foregoing supposition. Now the periodic time in the circles being the same, the mean motion of the nodes (927) are the same; hence, the motion of the nodes in the circular orbit ab : the motion of the nodes in the elliptic orbit $CA'BD' :: EA : EA' ::$ (885) 70 : 69.

930. Now let us suppose the moon to describe its true orbit $CADB$, in which FIG.
CD is the line of quadratures, AEB that of syzygies, aEb the major axis, E 211.
the earth in the focus, Fe the semi-minor axis; M the place of the moon, Mm a very small arc; and with the center E, describe the circles mn, Ft, cutting the line Nn of the nodes in n, t; draw MK, Qk, perpendicular to CD, and Mr, Qs, perpendicular to Nn; and let a body revolving in the circle Ft, describe Qq in the time in which Mm is described. Now (818) the periodic time in the circle tF is equal to the periodic time in the ellipse; hence (805), on account of the equality of the periodic times, the area $MEm : QEq$, or $ME \times md : QE \times Qq, ::$ the whole area of the ellipse : the whole area of the circle $:: eF : ea$; therefore $md : Qq :: QE \times eF : ME \times ea$. But (843) the disturbing

forces at M and Q are as MK, Qk very nearly, and therefore (927) the motions perpendicular to the plane of the orbits will be in the same ratio; hence, (as explained in the last Article) the motion of the nodes of the ellipse in the time Mm is described : the motion of the nodes in the circle whilst Qq is described :: $\frac{MK \times Mr}{ME \times md} : \frac{Qk \times Qs}{QE \times Qq} :: \frac{ME}{md} : \frac{QE}{Qq} :: \frac{ME^2}{\text{area } EMm} : \frac{QE^2}{\text{area } EQq} :: \frac{ME^2}{\text{area ellip.}} : \frac{QE^2}{\text{area circ.}} :: \frac{\text{area } MEm}{eF} : \frac{\text{area } QEc}{ea}$; hence, the mean motion of the nodes of the ellipse : the mean motion of the nodes of the circle :: $\frac{\textit{sum of all the areas } MEm}{eF} : \frac{\textit{sum of all the areas } QEc}{ea} :: \frac{\text{area of ellipse}}{eF} : \frac{\text{area of circle}}{ea} :: \frac{eF}{eF} : \frac{ea}{ea}$; therefore the mean motion of the nodes of the ellipse is equal to the mean motion of the nodes of the circle. Hence, if (929) we find the mean motion of the nodes for a circular orbit, and diminish it in the ratio of 70 : 69, we get the mean motion of the nodes of the true lunar orbit. It appears therefore that the motion of the nodes is not affected by the excentricity of the orbit, as Sir I. NEWTON supposed.

931. The true motion of the nodes of the ellipse at any time : the cotemporary motion of the nodes of the circle :: $\frac{ME^2}{eF} : \frac{QE^2}{ea} :: ME^2 : eF \times ea$; hence, when ME is a mean proportional between the semi-major and semi-minor axes, the true motion of the nodes of the ellipse and circle are equal.

FIG. 209. 932. If we take the nodes in quadratures and the moon in syzygies, At, MK, Mk become each equal to AE, and the motion of the nodes $= 3n^2 \times \frac{Mw}{ME}$, and they are regressive. Hence, if we take $\frac{Mw}{ME} = 32'.\ 56''.\ 27'''.\ 12\frac{1}{2}''''$ the mean horary motion of the moon, the horary motion of the nodes in this situation will be $33''.\ 10'''.\ 33''''.\ 12'''''$ for a circular orbit; and if we diminish this in the ratio of 70 : 69 on account of the elliptic form of the orbit, we have (930) $32''.\ 42'''.\ 7''''$ for the true horary motion of the nodes when the moon is in syzygies and the nodes in quadratures. And the horary motion of the nodes at any other time : this quantity :: sin. AEN × sin. MEC × sin. MEN : the cube of radius.

933. As often as any of the sines of AEN, MEC, MEN changes from positive to negative, the regressive motion of the nodes becomes progressive, and the contrary. Let the situation of the nodes be given; then sin. MEC becomes negative when the moon passes through quadratures D, and the motion of the nodes becomes progressive; when the moon passes the node n, the sin. MEN becomes negative, and the nodes become again regressive. Thus it ap-

pears, that the nodes are progressive when the moon is passing between quadratures and the nearest node, and regressive for the other part of the revolution. Hence, in one revolution of the moon the nodes are regressive. When the sine of either of these three angles AEN, MEC, MEN, becomes $=0$, the nodes are stationary. Hence, they are stationary when the sun is in the node, when the moon is in quadratures, and when the moon is in the node.

934. The sine of $MEN = \sin. \overline{MEC \mp CEN} = \sin. MEC \times \cos. CEN \mp \cos. MCE \times \sin. CEN = \sin. MEC \times \sin. NEA \mp \cos. ME \times \cos. NEA$; hence, if $ME=1$, the true horary motion of the nodes $= 3n^2 \times Mw \times \overline{\sin. MEC}^2 \times \overline{\sin. NEA}^2 \mp 3n^2 \times Mw \times \sin. MEC \times \cos. MEC \times \sin. NEA \times \cos. NEA$. Now in a whole revolution of the moon, the position of the sun and node remaining the same, the effect of the last term will be destroyed by the opposition of the signs of sin. MEC and cos. MEC, the positive and negative signs of their product destroying one another in a whole synodic revolution; hence, we get the mean horary motion of the node in one synodic revolution of the moon $= 3n^2 \times Mw \times \overline{\sin. MEC}^2 \times \overline{\sin. NEA}^2$. Now if the position of the node be given, and we substitute $\frac{1}{2} - \frac{1}{2} \cos. 2MEC$ for the $\overline{\sin. MEC}^2$, we have the mean horary motion of the node $= 3n^2 \times Mw \times \overline{\sin. NEA}^2 \times \overline{\frac{1}{2} - \frac{1}{2} \cos. 2MEC}$; and neglecting $-\frac{1}{2} \cos. 2MEC$, the effect of which will be destroyed in one revolution of the moon by the opposition of its signs, we get the mean horary motion of the nodes in one revolution of the moon $= \frac{3}{2} n^2 \times Mw \times \overline{\sin. NEA}^2$. Now when the moon is in syzygies, the $\overline{\sin. MEC}^2 = 1$, and $\cos. MEC = 0$, therefore the horary motion of the nodes is then $3n^2 \times Mw \times \overline{\sin. NEA}^2$. Hence, the mean horary motion of the nodes in one synodic revolution of the moon is equal to half their horary motion when the moon is in syzygies, whatever be the position of the node. Now when the nodes are in quadratures and the moon in syzygies, their horary motion (932) is 32′. 42″. 7‴; hence, the mean horary motion of the nodes when in quadratures is 16′. 21″. 3½‴ in the elliptic orbit. In the circular orbit it is 16′. 35″. 16‴. 36⁗.

935. As $\overline{\sin. NEA}^2 = \frac{1}{2} - \frac{1}{2} \cos. 2NEA$; the mean horary motion of the nodes in one synodic revolution of the moon $= \frac{3}{2} n^2 \times Mw \times \overline{\frac{1}{2} - \frac{1}{2} \cos. 2NEA} = \frac{3}{4} n^2 \times Mw - \frac{3}{4} n^2 \times Mw \times \cos. 2NEA$; and as in one revolution of the sun in respect to the nodes, the last term will be destroyed by the opposition of its signs, we get the mean horary motion of the nodes $= \frac{3}{4} n^2 \times Mw$. Hence, the mean motion of the moon : the mean motion of the nodes :: $Mw : \frac{3}{4} n^2 \times Mw :: 1 : \frac{3}{4} n^2$.

936. To get the mean annual motion of the nodes, we must first get their mean motion, upon supposition that they had been fixed for one revolution of the sun, and then we shall get nearly the motion of the sun in respect to the node; and by repeating the operation, we may apply corrections and arrive at

… $\overline{\tfrac{3}{4}n - \tfrac{27}{32}n^2 + \tfrac{135}{128}n^3}$, where w' represents
…ce, if we make $w' = 360°$, we
…of the nodes between two
…nother term of the
…this time, the
…41°. 21′:
…mon

…),
…true
…d have
…n will di-
…fore suppose

… the Nodes.

…, we have the fluxion of the
…n. MEN × sin. MEC; but sin.
… $\mp \overline{MEA}$ = sin. MEN × cos. MEA
…e motion of the nodes $= -3n^2\dot{z}$ ×
…EN × cos. MEN × sin. MEA × cos.
× $\overline{\tfrac{1}{2} + \tfrac{1}{2}\ \text{cos.}\ 2\,MEA} \mp \tfrac{1}{2}$ sin. $2MEN \times \tfrac{1}{2}$
…$\overline{MEN} \times \overline{1 + \text{cos.}\ 2MEA} \mp$ sin. $2MEN$ × sin.
… $2MEA$ − cos. $2MEN$ — cos. $2MEN$ ×
× sin. $2MEA$). But − cos. $2MEN$ × cos.
…n. $2MEA$ = − cos. $\overline{2MEN \mp 2MEA}$ = − cos.
…n of the nodes $= -\tfrac{3}{4}n^2\dot{z} \times (1 +$ cos. $2MEA$ − cos.
Now the mean motion of the moon is to that of the
…e mean motion of the moon be to that of the node as 1 :
moon describes the angle Z, the sun describes nZ, and
as the node is retrograde, $Z + mZ$ expresses the moon's
…ode, or the angle MEN; also, $Z - nZ$ = the motion of the
…un, or the angle MEA; and $nZ + mZ$ = the motion of the sun
…, or the angle AEN. Hence, the motion of the nodes $= -\tfrac{3}{4}n^2\dot{Z}$
… $\overline{-2nZ}$ − cos. $\overline{2Z + 2mZ}$ − cos. $\overline{2nZ + 2mZ}) = -\tfrac{3}{4}n^2\dot{Z} - \tfrac{3}{4}n^2\dot{Z} \times$ cos.
$n^2\dot{Z} \times$ cos. $\overline{2Z + 2mZ} + \tfrac{3}{4}n^2\dot{Z} \times$ cos. $\overline{2nZ + 2mZ}$, whose fluent is—

$\frac{3}{4} n^2 Z - \frac{3}{8} \times \frac{n^2}{1-n} \times \sin. \overline{2Z - 2nZ} + \frac{3}{8} \times \frac{n^2}{1+m} \times \sin. \overline{2Z + 2mZ} + \frac{3}{8} \times \frac{n^2}{n+m} \times \sin. \overline{2nZ + 2mZ}$ the motion of the nodes. And if $X =$ the distance $Z - nZ$ of the sun from the moon, and for the distance $Z + mZ$ of the moon for the node we put S, then $2nZ + 2mZ = S - X$; hence, the motion of the nodes becomes $- \frac{3}{4} n^2 Z + \frac{3}{8} \times \frac{n^2}{n+m} \times \sin. \overline{2S - 2X} + \frac{3}{8} \times \frac{n^2}{1+m} \times \sin. 2S - \frac{3}{8} \times \frac{n^2}{1-n} \times \sin. 2X$. But (935) $m = \frac{3}{4} n^2$; and the first term expresses the mean motion of the nodes in a circular orbit; hence, we get these equations of the mean motion of the nodes; $\frac{3}{8} \times \frac{n}{1 + \frac{3}{4} n} \times \sin. \overline{2S - 2X} + \frac{3}{8} \times \frac{n^2}{1 + . n^2} \times \sin. 2S - \frac{3}{8} \times \frac{n^2}{1-n} \times \sin. 2X = 1°. 31'. 18'' \times \sin. \overline{2S - 2X} + 7'. 11'' \times \sin. 2S - 7'. 49'' \times \sin. 2X$; and if these be diminished in the ratio of 70 : 69, we get the true equations, $1°. 30'. 1'' \times \sin. \overline{2S - 2X} + 7'. 5'' \times \sin. 2S - 7'. 42'' \times \sin. 2X$.

940. The moon's motion being here assumed Z, if for Z we substitute (870) $Z - 2w \times \sin. Z$, which is very nearly the true motion, and for $\dot{Z}$ we put $\dot{Z} - 2w \times \cos. Z \times \dot{Z}$; then the fluxion of the mean motion $- \frac{3}{4} n^2 \dot{Z}$ becomes $- \frac{3}{4} n^2 \dot{Z} + \frac{3}{2} n^2 w \times \cos. Z \times \dot{Z} =$ (as $Y = nZ$) $- \frac{3}{4} n \times \dot{Y} + \frac{3}{2} n^2 w \times \cos. Z \times \dot{Z}$; and to find the annual effect, we must (891) multiply this quantity by $1 - 3c \times \cos. Y$; hence, the fluxion of the motion of the nodes from these causes $= - \frac{3}{4} n \dot{Y} + \frac{3}{2} n^2 w \times \cos. Z \times \dot{Z} + \frac{9}{4} nc \times \cos. Y \times \dot{Y}$ nearly, whose fluent $= - \frac{3}{4} n Y + \frac{3}{2} n^2 w \times \sin. Z + \frac{9}{4} nc \times \sin. Y$; and the mean motion being $- \frac{3}{4} nY$, the equations become $\frac{3}{2} n^2 w \times \sin. Z + \frac{9}{4} nc \times \sin. Y = 1'. 35'' \times \sin. Z + 9'. 43''\frac{1}{2} \times \sin. Y$; and these diminished in the ratio of 70 : 69, give $1'. 34'' \times \sin. Z + 9'. 35'' \times \sin. Y$ for the true equations.

941. To correct the three equations found in Art. 939. *First*, to correct $\frac{3}{4} n^2 \dot{Z} \times \cos. \overline{2nZ + 2mZ}$. For Z write $Z - 2w \times \sin. Z$, and for $\dot{Z}$ substitute $\dot{Z} - 2w \times \cos. Z \times \dot{Z}$; also, for the motion of the sun nZ substitute a more correct value $nZ - 2c \times \sin. nZ$; and multiply the whole by $1 - 3c \times \cos. nZ$, being the disturbing force at any distance $1 + c \times \cos. Y$ (891); hence, the above equation becomes $\frac{3}{4} n^2 \dot{Z} \times \overline{1 - 2w \times \cos. Z} \times \overline{1 - 3c \times \cos. nZ} \times \cos. \overline{2nZ + 2mZ - 4c \times \sin. nZ}$; multiply these quantities together, and neglect those terms in which the product wc enters as being very small, and we shall get the following quantity, $\frac{3}{4} n^2 \dot{Z} \times \overline{1 - 2w \times \cos. Z - 3c \times \cos. nZ} \times \cos. \overline{2nZ + 2mZ - 4c \times \sin. nZ}$;

but cos. $\overline{2nZ+2mZ-4c\times\sin. nZ}=$ cos. $\overline{2nZ+2mZ}+4c\times$ sin. $nZ\times$ sin. $\overline{2nZ+2mZ}$ nearly, because the cos. of $4c\times$ sin. nZ is nearly unity; hence, (neglecting $\frac{3}{4}n^2\dot{Z}\times$ cos. $\overline{2nZ+2mZ}$ found in Art. 939.), the correction becomes $\frac{3}{4}n^2\dot{Z}\times\overline{-2w\times\cos. Z-3c\times\cos. nZ}\times$ cos. $\overline{2nZ+2mZ}+3n^2c\dot{Z}\times$ sin. $nZ\times$ sin. $\overline{2nZ+2mZ}$ nearly, $=-\frac{3}{2}n^2w\times\dot{Z}\times$ cos. $Z\times$ cos. $\overline{2nZ+2mZ}-\frac{9}{4}n^2c\times\dot{Z}\times$ cos. $Z\times$ cos. $\overline{2nZ+2mZ}+3n^2c\times\dot{Z}\times$ sin. $nZ\times$ sin. $\overline{2nZ+2mZ}$; but cos. $Z\times$ cos. $\overline{2nZ+2mZ}=\frac{1}{2}$ cos. $\overline{2nZ+2mZ+Z}+\frac{1}{2}$ cos. $\overline{2nZ+2mZ-Z}$; and cos. $nZ\times$ cos. $\overline{2nZ+2mZ}=\frac{1}{2}$ cos. $\overline{3nZ+2mZ}+\frac{1}{2}$ cos. $\overline{nZ+2mZ}$; also, sin. $nZ\times$ sin. $\overline{2nZ+2mZ}=\frac{1}{2}$ cos. $\overline{nZ+2mZ}-\frac{1}{2}$ cos. $\overline{3nZ+2mZ}$; substitute these quantities, and collect the like terms together, and the fluxional correction becomes, $-\frac{3}{4}n^2w\dot{Z}\times$ cos. $\overline{2nZ+2mZ+Z}-\frac{3}{4}n^2w\dot{Z}\times$ cos. $\overline{2nZ+2mZ-Z}-\frac{21}{8}n^2c\dot{Z}\times$ cos. $\overline{3nZ+2mZ}+\frac{3}{8}n^2c\times\dot{Z}\times$ cos. $\overline{nZ+2mZ}$, the fluent of which is $-\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2nZ+2mZ+Z}}{2n+2m+1}$
$+\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2nZ+2mZ-Z}}{1-2n-2m}-\frac{21}{8}n^2c\times\dfrac{\sin. \overline{3nZ+2mZ}}{3n+2m}+\frac{3}{8}n^2c\times\dfrac{\sin. \overline{nZ+2mZ}}{n+2m}$
$=$(as $Y=nZ$) $-\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2S-2X+Z}}{1+2n}+\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2S-2X-Z}}{1-2n}-\frac{7}{8}nc\times$ sin. $\overline{2S-2X+Y}+\frac{3}{8}nc\times$ sin. $\overline{2S-2X-Y}$ very nearly, $=-41''\times$ sin. $\overline{2S-2X+Z}+55''\times$ sin. $\overline{2S-2X-Z}-3'.\ 48''\times$ sin. $\overline{2S-2X+Y}+1'.\ 38''\times$ sin. $\overline{2S-2X-Y}$, which two last diminished in the ratio of 70 : 69, give $-3'.\ 45''\times$ sin. $\overline{2S-2X+Y}+1'.\ 37''.\times$ sin. $\overline{2S-2X-Y}$; the smaller equations it is unnecessary thus to reduce.

942. *Secondly*, to correct $\frac{3}{4}n^2\dot{Z}\times$ cos. $\overline{2Z+2mZ}$. For Z write $Z-2w\times$ sin. Z, and for $\dot{Z}$ substitute $\dot{Z}-2w\times$ cos. $Z\times\dot{Z}$, and multiply the whole by $1-3c\times$ cos. nZ, and the equation becomes $\frac{3}{4}n^2\dot{Z}\times\overline{1-2w\times\cos. Z}\times\overline{1-3c\times\cos. nZ}\times$ cos. $\overline{2Z+2mZ-4w\times\sin. Z}$; proceed now exactly the same as in the last operation, and we shall get the correction $=-\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2S+Z}}{3+2m}$
$+\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2S-Z}}{1+2m}-\frac{9}{8}n^2c\times\dfrac{\sin. \overline{2S+Y}}{2+n+2m}-\frac{9}{8}n^2c\times\dfrac{\sin. \overline{2S-Y}}{2-n+2m}=-45''\times$ sin. $\overline{2S+Z}+45''\times$ sin. $\overline{2S-Z}-11''\times$ sin. $\overline{2S+Y}-11''\times$ sin. $\overline{2S-Y}$.

943. *Thirdly*, to correct $-\frac{3}{4}n^2\dot{Z}\times$ cos. $\overline{2Z-2nZ}$. For Z write $Z-2w\times$ sin. Z, and for $\dot{Z}$ substitute $\dot{Z}-2w\times$ cos. $Z\times\dot{Z}$, and multiply the whole by $1-3c\times$ cos. nZ, and the equation becomes $-\frac{3}{4}n^2\dot{Z}\times\overline{1-2w\times\cos. Z}\times\overline{1-3c\times\cos. nZ}\times$ cos. $\overline{2Z-2nZ-4w\times\sin. Z}$; proceed now in the same manner as before, and we get the correction $=-\frac{3}{4}n^2w\times\dfrac{\sin. \overline{2X-Z}}{1-2n}+\frac{3}{4}n^2w\times$

$\frac{\sin. \overline{2X+Z}}{2-3n} + \frac{9}{8} n^2 c \times \frac{\sin. \overline{X+Z}}{2-n} + \frac{9}{8} n^2 c \times \frac{\sin. \overline{X-Z}}{2-3n} = -53'' \times \sin. \overline{2X-Z} +$ $47'' \times \sin. \overline{2X+Z} + 12'' \times \sin. \overline{X+Z} + 12'' \times \sin. \overline{X-Z}$.

944. The principal equation of the nodes is $\frac{3}{8} n \times \sin. \overline{2nZ + 2mZ}$ (939); therefore a more correct value of $2mZ$ (twice the motion of the nodes) is $2mZ - \frac{3}{4} n \times \sin. \overline{2nZ+2mZ}$, the equation being subtracted, because the motion $2nZ$ of the nodes is retrograde, and this equation is additive, or to be applied according to the order of the signs. Substitute therefore this value for $2mZ$ in $\frac{3}{4} n^2 \dot{Z} \times \cos. \overline{2nZ + 2mZ}$ (which is the fluxion of the principal equation) and it becomes $\frac{3}{4} n^2 \dot{Z} \times \cos. (2nZ + 2mZ - \frac{3}{4} n \times \sin. \overline{2nZ+2mZ})$, and omitting the equation already found, the result is $\frac{3}{4} n^2 \dot{Z} \times \frac{3}{4} n \times \sin. \overline{2nZ+2mZ}|^2 = \frac{9}{16} n^3 \dot{Z} \times \overline{\frac{1}{2} - \frac{1}{2} \cos. \overline{4nZ + 4mZ}} = \frac{9}{32} n^3 \dot{Z} - \frac{9}{32} n^3 \dot{Z} \times \cos. \overline{4nZ+4mZ}$, whose fluent is $\frac{9}{32} n^3 Z - \frac{9}{32} n^3 \times \frac{\sin. \overline{4nZ+4mZ}}{4n+4m} = \frac{9}{32} n^3 Z - \frac{9}{128} n^2 \times \sin. \overline{4nZ + 4mZ}$; hence, the equation is $-\frac{9}{128} n^2 \times \sin. \overline{4nZ+4mZ} = -1'.\ 23'' \times \sin. \overline{4S-4X}$, which diminished in the ratio of 70 : 69, gives $-1'.\ 22'' \times \sin. \overline{4S-4X}$.

945. The quantity $\frac{9}{32} n^3 Z$, found in deducing the last equation, continually increases as Z increases, and is therefore a correction of the mean motion $-\frac{3}{4} n^2 Z$ before found; hence, a more correct value of the mean motion is $-\frac{3}{4} n^2 Z + \frac{9}{32} n^3 Z$.

Hence, the equations of the mean motion of the nodes are,

I.	- -	+	$1^\circ.\ 30'.\ 1'' \times \sin. \overline{2S-2X}$.
II.	- -	+	$7.\ 5 \times \sin. 2S$.
III.	- -	−	$7.\ 42 \times \sin. 2X$.
IV.	- -	+	$1.\ 34 \times \sin. Z$.
V.	- -	+	$9.\ 35 \times \sin. Y$.
VI.	- -	−	$0.\ 41 \times \sin. \overline{2S-2X+Z}$.
VII.	- -	+	$0.\ 55 \times \sin. \overline{2S-2X-Z}$.
VIII.	- -	−	$3.\ 45 \times \sin. \overline{2S-2X+Y}$.
IX.	- -	+	$1.\ 37 \times \sin. \overline{2S-2X-Y}$.
X.	- -	−	$0.\ 45 \times \sin. \overline{2S+Z}$.
XI.	- -	+	$0.\ 45 \times \sin. \overline{2S-Z}$.
XII.	- -	−	$0.\ 11 \times \sin. \overline{2S+Y}$.
XIII.	- -	−	$0.\ 11 \times \sin. \overline{2S-Y}$.
XIV.	- -	−	$0.\ 53 \times \sin. \overline{2X-Z}$.
XV.	- -	+	$0.\ 47 \times \sin. \overline{2X+Z}$.

XVI.	+	0'. 12"	$\times \sin. \overline{X+Z}$.
XVII.	+	0. 12	$\times \sin. \overline{X-Z}$.
XVIII.	−	1. 22	$\times \sin. \overline{4S-4X}$.

On the Variation *of the Inclination of the Moon's Orbit.*

946. Let NN' be the horary motion of the moon's nodes; draw Mk perpendicular to Nn; and Mx perpendicular to the ecliptic, and draw xks, join Ms, and draw ku perpendicular to Ms; then the angle sMk is the cotemporary variation of the inclination. Now the angle $NEN' : sMk :: \frac{sk}{sE} : \frac{ku}{kM} :: \frac{1}{sE} : \frac{s}{kM}$ :: sin. MEN : $s \times$ cos. MEN. But if the angle $MEw = \dot{Z}$, the motion of the nodes (927) is $3n^2 \times \dot{Z} \times$ sin. $AEN \times$ sin. $MEC \times$ sin. MEN; hence, the cotemporary variation of the inclination is $3n^2 s\dot{Z} \times$ sin. $AEN \times$ cos. $MEN \times$ cos. MEA. Now cos. $MEN =$ cos. $\overline{MEC \mp CEN} =$ cos. $MEC \times$ cos. $CEN \pm$ sin. $MEC \times$ sin. $CEN \pm$ cos. $MEC \times$ sin. $AEN \pm$ sin. $MEC \times$ cos. AEN; hence, the fluxion of the variation of the inclination is $3n^2 s\dot{Z} \times$ sin. $AEN \times$ sin. $MEC \times$ (cos. $MEC \times$ sin. $AEN \pm$ sin. $MEC \times$ cos. AEN); but in one synodic revolution of the moon, cos. MEC will be destroyed; therefore the fluxion of the mean variation for that time $= 3n^2 s\dot{Z} \times$ sin. $AEN \times$ cos. $AEN \times \overline{\text{sin. } MEC}^2 =$ $3n^2 \dot{Z} \times$ sin. $AEN \times$ cos. $AEN \times \overline{\tfrac{1}{2} - \tfrac{1}{2} \text{ cos. } 2MEC} =$ (for the mean of one synodic revolution) $\tfrac{3}{2} n^2 s\dot{Z} \times$ sin. $AEN \times$ cos. AEN*. Now the mean motion of the moon ($\dot{Z}$) : that of the sun :: 2139 : 160, and (938) the mean motion of the sun : its mean motion $\dot{\overline{AN}}$ in respect to the nodes :: 341,3 : 360; hence, $\dot{Z} = \frac{2139 \times 341,3}{160 \times 360} \times \dot{\overline{AN}} = 12,68 \times \dot{\overline{AN}}$; therefore the fluxion of the mean variation of the inclination, as the nodes move from quadratures to syzygies, is $\tfrac{3}{2} \times 12,68 \times n^2 s \times \dot{\overline{AN}} \times$ sin. $AEN \times$ cos. $AEN = 19,02 \times n^2 s \times$ sin. $AEN \times$ $\dot{\overline{\text{sin. } AEN}}$, whose fluent is $9,51 \times n^2 s \times \overline{\text{sin. } AEN}^2 = 16'. 24'' \times \overline{\text{sin. } AEN}^2$, taking $s = 0,896$ its mean value. This is for a circular orbit; diminish it in the ratio of 70 : 69, and it becomes $16'. 10'' \times \overline{\text{sin. } AEN}^2$. Hence, the mean variation of the inclination varies as the square of the sine of the sun's distance from the node. When $AEN = 90°$, the mean diminution is 16'. 10". This is

* Hence, the mean variation of the inclination is $s \times$ sin. $AEN \times$ cos. AEN, or as $s \times$ sin. $2AEN$, from which Sir I. Newton, Pr. 35. Lib. 3. deduces his very elegant construction for finding the inclination at any time.

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the mean for one revolution of the moon when the nodes come into quadratures; its half therefore 8′. 5″ is the mean variation from the mean inclination of the orbit. Now $\overline{\sin. AEN}^2 = \frac{1}{2} - \frac{1}{2}\cos. 2AEN$. Hence, the mean variation $= 8'. 5'' - 8'. 5'' \times \cos. 2AEN$, which subtracted from 8′. 5″ the mean variation from the mean inclination gives $8'. 5'' \times \cos. 2AEN$, the quantity to be applied to the mean inclination in order to get the mean inclination for one revolution of the moon at any time.

947. Now when the nodes are in quadratures, the fluxion of the variation is $3n^2 s\dot{Z} \times \sin. MEC \times \cos. MEC$ (the sin. AEN being unity) $= 3n^2 s \times \sin. MEC \times \dot{\overline{\sin. MEC}}$, whose fluent is $\frac{3}{2} n^2 s \times \overline{\sin. MEC}^2 =$ (assuming $s = ,0871$ the sine of the inclination at this time) $2'. 31'' \times \overline{\sin. MEC}^2$; but this must be increased in the ratio of the periodic to the synodic revolution for the reason in Art. 867. hence, the variation $= 2'. 43'' \times \overline{\sin. MEC}^2$ for a circular orbit; which diminished in the ratio of 70 : 69, gives $2'. 40'',7 \times \overline{\sin. MEC}^2$ for the variation of the inclination. Hence, when the moon is in syzygies, the variation $= 2'. 40'',7$ is the diminution of the inclination in the transit of the moon from the nodes (in quadratures) to syzygies; the half of which 1′. 20″ is the variation from the mean inclination in that time. Hence (946), in the transit of the nodes from syzygies to quadratures, when the moon is in quadratures the variation of the inclination has been $16'. 10'' - 1'. 20'' = 14'. 50''$, and when the moon is in syzygies, the variation has been $16'. 10'' + 1'. 20'' = 17'. 30''$; therefore if the inclination be 5°. 17′. 20″ when the nodes are in syzygies, the least inclination becomes 4°. 59′. 50″, and the mean $= 5°. 8'. 35''$.

To find the Equations of the Variation of the Inclination of the Lunar Orbit.

948. By Art. 927. the fluxion of the motion of the nodes is $3n^2\dot{Z} \times \sin. AEN \times \sin. MEN \times \cos. MEA$; but (946) the motion of the nodes : the cotemporary variation of the inclination :: sin. $MEN : s \times \cos. MEN$; hence, the fluxion of the variation of the inclination is $3n^2 sZ \times \sin. AEN \times \cos. MEN \times \cos. MEA$. Now, writing the sines and cosines with their proper signs, when the inclination is diminishing this fluxion is positive, and when the inclination is increasing the fluxion is negative; therefore that it may be properly applied, its sign must be changed. But $AEN = MEN \mp MEA$, therefore $\sin. AEN = \sin. MEN \times \cos. MEA \mp \sin. MEA \times \cos. MEN$; hence, the fluxion of the variation of the inclination $= -3n^2 s\dot{Z} \times \overline{\cos. MEA}^2 \times \sin. MEN \times \cos. MEN \mp \overline{\cos. MEN}^2 \times \sin. MEA \times \cos. MEA = -3n^2 s\dot{Z} \times (\overline{\frac{1}{2} + \frac{1}{2}\cos. 2MEA} \times \frac{1}{2}\sin. 2MEN \mp \overline{\frac{1}{2} + \frac{1}{2}\cos. 2MEN} \times \frac{1}{2}\sin. 2MEA) = -\frac{3}{4} n^2 s\dot{Z} \times (\sin. 2MEN \mp \sin.$

$2MEA + \text{sin.}\ \overline{2MEN \mp 2MEA}) = -\frac{3}{4}\, n^2 s\dot{Z} \times (\text{sin.}\ 2MEN \mp \text{sin.}\ 2MEA + \text{sin.}\ 2AEN)$. Now this is the same expression as that for the motion of the nodes, except that we have here the sines of the angles instead of the cosines; therefore making the same substitution here as in Art. 939. we get the fluxion of the variation of the inclination $= -\frac{3}{4}\, n^2 s\dot{Z} \times (\text{sin.}\ \overline{2nZ + 2mZ} + \text{sin.}\ \overline{2Z + 2mZ} - \text{sin.}\ \overline{2Z - 2nZ})$, whose fluent is $-\frac{3}{4}\, n^2 s \times \left\{\frac{\text{ver. sin.}\ \overline{2nZ + 2mZ}}{2n + 2m} + \frac{\text{ver. sin.}\ \overline{2Z + 2mZ}}{2 + 2m} - \frac{\text{ver. sin.}\ \overline{2Z - 2nZ}}{2 - 2n}\right\} = -\frac{3}{4}\, n^2 s \times \left\{\frac{1 - \text{cos.}\ \overline{2nZ + 2mZ}}{2n + 2m} + \frac{1 - \text{cos.}\ \overline{2Z + 2mZ}}{2 + 2m} - \frac{1 - \text{cos.}\ \overline{2Z - 2nZ}}{2 - 2n}\right\} = (\text{as } m = \frac{3}{4}\, n^2) - \frac{3ns}{8 + 6n} - \frac{3n^2 s}{8 + 6n^2} + \frac{3n^2 s}{8 - 8n} + \frac{3ns}{8 + 6n} \times \text{cos.}\ \overline{2S - 2X} + \frac{3n^2 s}{8 + 6n^2} \times \text{cos.}\ 2S - \frac{3n^2 s}{8 - 8n} \times \text{cos.}\ 2X$; and the three equations are $8'.\ 12'' \times \text{cos.}\ \overline{2S - 2X} + 39'' \times \text{cos.}\ 2S - 42'' \times \text{cos.}\ 2X$, which diminished in the ratio of 70 : 69, gives $8'.\ 5'' \times \text{cos.}\ \overline{2S - 2X} + 38\frac{1}{2}'' \times \text{cos.}\ 2S - 41\frac{1}{2}'' \times \text{cos.}\ 2X$ for the true equations. And the three constant quantities show that the mean inclination of the orbit is less than it would have been if there had been no disturbing force.

949. To correct these three equations. *First*, let us take $-\frac{3}{4} n^2 s\dot{Z} \times \text{sin.}\ \overline{2nZ + 2mZ}$; in which, for $\dot{Z}$ write $\dot{Z} - 2w \times \text{cos.}\ Z \times \dot{Z}$, for nZ write $nZ - 2c \times \text{sin.}\ nZ$, and for $2mZ$ write $2mZ - \frac{3}{4} n \times \text{sin.}\ \overline{2nZ + 2mZ}$, which is the correction of the motion of the node by its principal equation (939), and multiply the whole by $1 - 3c \times \text{cos.}\ nZ$, for the reasons already given in Art. 941. and the equation becomes $-\frac{3}{4}\, n^2 s\dot{Z} \times \overline{1 - 2w \times \text{cos.}\ Z} \times \overline{1 - 3c \times \text{cos.}\ nZ} \times \text{sin.}\ (2nZ + 2mZ - 4c \times \text{sin.}\ nZ - \frac{3}{4}\, n \times \text{sin.}\ \overline{2nZ + 2mZ})$; but $\text{sin.}\ (2nZ + 2mZ - 4c \times \text{sin.}\ nZ - \frac{3}{4}\, n \times \text{sin.}\ \overline{2nZ + 2mZ}) = \text{sin.}\ \overline{2nZ + 2mZ} - \text{cos.}\ \overline{2nZ + 2mZ} \times \overline{4c \times \text{sin.}\ nZ + \frac{3}{4}\, n \times \text{sin.}\ \overline{2nZ + 2mZ}}$ nearly, because $4c \times \text{sin.}\ nZ + \frac{3}{4}\, n \times \text{sin.}\ \overline{2nZ + 2mZ}$ being small, its cosine is nearly $= 1$, and we may put that quantity itself for its sine; make therefore this substitution, and actually multiply the quantities, and reject $-\frac{3}{4} n^2 s\dot{Z} \times \text{sin.}\ \overline{2nZ + 2mZ}$ which is the equation to be corrected, and we get the fluxional correction $= \frac{3}{2}\, n^2 sw\dot{Z} \times \text{cos.}\ Z \times \text{sin.}\ \overline{2nZ + 2mZ} + \frac{9}{4}\, n^2 sc\dot{Z} \times \text{cos.}\ nZ \times \text{sin.}\ \overline{2nZ + 2mZ} + 3n^2 sc\dot{Z} \times \text{sin.}\ nZ \times \text{cos.}\ \overline{2nZ + 2mZ} + \frac{9}{16} n^3 s\dot{Z} \times \text{sin.}\ \overline{2nZ + 2mZ} \times \text{cos.}\ \overline{2nZ + 2mZ}$ nearly, $= \frac{3}{4} n^2 sw\dot{Z} \times \text{sin.}\ \overline{2nZ + 2mZ + Z} + \frac{3}{4} n^2 sw\dot{Z} \times \text{sin.}\ \overline{2nZ + 2mZ - Z} + \frac{21}{8}\, n^2 sc\dot{Z} \times \text{sin.}\ \overline{3nZ + 2mZ} - \frac{3}{8}\, n^2 sc\dot{Z} \times \text{sin.}\ \overline{nZ + 2mZ} + \frac{9}{32}\, n^3 s\dot{Z} \times \text{sin.}\ \overline{4nZ + 4mZ}$, whose fluent is $\frac{3}{4}\, n^2 sw \times \frac{\text{ver. sin.}\ \overline{2nZ + 2mZ + Z}}{1 + 2n + 2m} + \frac{3}{4}\, n^2 sw \times \frac{\text{ver. sin.}\ \overline{2nZ + 2mZ - Z}}{2n + 2m - 1} + \frac{21}{8}\, n^2 sc$

$\times \dfrac{\text{ver. sin. } \overline{3nZ + 2mZ}}{3n+2m} - \frac{3}{8} n^2 sc \times \dfrac{\text{ver. sin. } \overline{nZ + 2mZ}}{n+2m} + \frac{9}{32} n^3 s \times$ $\dfrac{\text{ver. sin. } \overline{4nZ + 4mZ}}{4n+4m} =$ (by substituting $1 - \cos.$ for ver. sin.) $\frac{3}{4} \times \dfrac{n^2 sw}{1+2n} - \frac{3}{4} \times \dfrac{n^2 sw}{1-2n} + \frac{7}{8} nsc - \frac{3}{8} nsc + \frac{9}{128} n^2 s - \frac{3}{4} \times \dfrac{n^2 sw}{1+2n} \times \cos. \overline{2nZ + 2mZ + Z} + \frac{3}{4} \times \dfrac{n^2 sw}{1-2n} \times \cos. \overline{2nZ + 2mZ - Z} - \frac{7}{8} nsc \times \cos. \overline{3nZ + 2mZ} + \frac{3}{8} nsc \times \cos. \overline{nZ + 2mZ}$ $- \frac{9}{128} n^2 s \times \cos. \overline{4nZ + 4mZ}$; but the five first terms are constant; and therefore only affect the mean inclination; and the last five express the corrections of the assumed equation. Now $2nZ + 2mZ + Z = 2S - 2X + Z$, $2nZ + 2mZ - Z = 2S - 2X - Z$, $3nZ + 2mZ = 2S - 2X + Y$, $nZ + 2mZ = 2S - 2X - Y$, and $4nZ + 4mZ = 4S - 4X$; and by substituting for n, w, c and s their values, we get these equations, $- 3\frac{3}{8}'' \times \cos. \overline{2S - 2X + Z} + 5'' \times \cos. \overline{2S - 2X - Z} - 20\frac{1}{3}'' \times \cos. \overline{2S - 2X + Y} + 8\frac{3}{4}'' \times \cos. \overline{2S - 2X - Y} - 7\frac{1}{3}'' \times \cos. \overline{4S - 4X}$. It is unnecessary to reduce these small equations in the ratio of 70 : 69.

950. *Secondly*, to correct $-\frac{3}{4} n^2 s\dot{Z} \times \sin. \overline{2Z + 2mZ}$. For Z put $Z - 2w \times \sin. Z$, and for $\dot{Z}$ write $\dot{Z} - 2w \times \cos. Z \times \dot{Z}$, omitting the other corrections, as the effects which they here produce are very small; hence, the quantity becomes $-\frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times \sin. (2Z + 2mZ - 4w \times \sin. Z) = -\frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times (\sin. \overline{2Z + 2mZ} - 4w \times \sin. Z \times \cos. \overline{2Z + 2mZ}) = -\frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times (\sin. \overline{2Z + 2mZ} - 2w \times \sin. \overline{3Z + 2mZ} + 2w \times \sin. \overline{Z + 2mZ})$, and neglecting $-\frac{3}{4} n^2 s\dot{Z} \times \sin. \overline{2Z + 2mZ}$ which is the given quantity, and also those terms where w^2 enters as being very small, we get the fluxional correction $= \frac{3}{2} n^2 wsZ \times \sin. \overline{3Z + 2mZ} - \frac{2}{3} n^2 wsZ \times \sin. \overline{Z + 2mZ} + \frac{3}{2} n^2 ws\dot{Z} \times \cos. Z \times \sin. \overline{2Z + 2mZ} = \frac{3}{2} n^2 ws\dot{Z} \times \sin. \overline{3Z + 2mZ} - \frac{3}{2} n^2 ws\dot{Z} \times \sin. \overline{Z + 2mZ} + \frac{3}{4} n^2 ws\dot{Z} \times \sin. \overline{3Z + 2mZ} + \frac{3}{4} n^2 ws\dot{Z} \times \sin. \overline{Z + 2mZ}$, whose fluent is $\frac{3}{2} n^2 ws \times \dfrac{1 - \cos. \overline{3Z + 2mZ}}{3+2m} - \frac{3}{2} n^2 ws \times \dfrac{1 - \cos. \overline{Z + 2mZ}}{1+2m} + \frac{3}{4} n^2 ws \times \dfrac{1 - \cos. \overline{3Z + 2mZ}}{3+2m}$ $+ \frac{3}{4} n^2 ws \times \dfrac{1 - \cos. \overline{Z + 2mZ}}{1+2m} = \frac{1}{2} n^2 ws - \frac{3}{2} n^2 ws + \frac{1}{4} n^2 ws - \frac{3}{4} n^2 ws - \frac{1}{2} n^2 ws \times \cos. \overline{3Z + 2mZ} + \frac{3}{2} n^2 ws \times \cos. \overline{Z + 2mZ} - \frac{1}{4} n^2 ws \times \cos. \overline{3Z + 2mZ} - \frac{3}{4} n^2 ws \times \cos. \overline{Z + 2mZ} =$ (omitting the constant quantities which affect only the mean inclination) $- \frac{3}{4} n^2 ws \times \cos. \overline{3Z + 2mZ} + \frac{3}{4} n^2 ws \times \cos. \overline{Z + 2mZ} = -4\frac{1}{4}'' \times \cos. \overline{3Z + 2mZ} + 4\frac{1}{4}'' \times \cos. \overline{Z + 2mZ} = -4\frac{1}{4}'' \times \cos. \overline{2S + Z} + 4\frac{1}{4}'' \times \cos. \overline{2S - Z}$, the corrections required.

951. *Thirdly*, to correct $\frac{3}{4} n^2 s\dot{Z} \times \sin. \overline{2Z - 2nZ}$. For Z put $Z - 2w \times$

sin. Z, and for $\dot{Z}$ put $\dot{Z} - 2w \times \cos. Z \times \dot{Z}$; and the given quantity becomes $\frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times \sin. (2Z - 2nZ - 4w \times \sin. Z) = \frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times (\sin. \overline{2Z - 2nZ} - 4w \times \sin. Z \times \cos. \overline{2Z - 2nZ}) = \frac{3}{4} n^2 s\dot{Z} \times \overline{1 - 2w \times \cos. Z} \times (\sin. \overline{2Z - 2nZ} - 2w \times \sin. \overline{3Z - 2nZ} + 2w \times \sin. \overline{Z - 2nZ})$, and neglecting $\frac{3}{4} n^2 ws\dot{Z} \times \sin. \overline{2Z - 2nZ}$ which is the given quantity, and also those terms where w^2 enters on account of their smallness, and collecting the like terms, we get the fluxional correction $= -\frac{9}{4} n^2 ws\dot{Z} \times \sin. \overline{3Z - 2nZ} + \frac{3}{4} n^2 ws\dot{Z} \times \sin. \overline{Z - 2nZ}$, whose fluent is $-\frac{9}{4} n^2 ws \times \frac{1 - \cos. \overline{3Z - 2nZ}}{3 - 2n} + \frac{3}{4} n^2 ws \times \frac{1 - \cos. \overline{Z - 2nZ}}{1 - 2n} = -\frac{9}{4} \times \frac{n^2 ws}{3 - 2n} + \frac{3}{4} \times \frac{n^2 ws}{1 - 2n} + \frac{9}{4} \times \frac{n^2 ws}{3 - 2n} \times \cos. \overline{3Z - 2nZ} - \frac{3}{4} \times \frac{n^2 ws}{1 - 2n} \times \cos. \overline{Z - 2nZ}$, and omitting the constant terms as affecting only the mean inclination, we get the corrections $= \frac{9}{4} \times \frac{n^2 ws}{3 - 2n} \times \cos. \overline{2X + Z} - \frac{3}{4} \times \frac{n^2 ws}{1 - 2n} \times \cos. \overline{2X - Z} = 4\frac{1}{2}'' \times \cos. \overline{2X + Z} - 5'' \times \cos. \overline{2X - Z}$ the equations required.

Hence, the variation of the inclination from the mean quantity is,

I. - - - -	$+8'.\ 5''$	$\times \cos. \overline{2S - 2X}.$
II. - - -	$+0.\ 38\frac{1}{2}$	$\times \cos. 2S.$
III. - - -	$-0.\ 41\frac{1}{2}$	$\times \cos. 2X.$
IV. - - -	$-0.\ 3\frac{3}{4}$	$\times \cos. \overline{2S - 2X + Z}.$
V. - - - -	$+0.\ 5$	$\times \cos. \overline{2S - 2X - Z}.$
VI. - - -	$-0.\ 20\frac{1}{3}$	$\times \cos. \overline{2S - 2X + Y}.$
VII. - - -	$+0.\ 8\frac{3}{4}$	$\times \cos. \overline{2S - 2X - Y}.$
VIII. - - -	$-0.\ 7\frac{1}{3}$	$\times \cos. \overline{4S - 4X}.$
IX. - - -	$-0.\ 4\frac{1}{4}$	$\times \cos. \overline{2S + Z}.$
X. - - - -	$+0.\ 4\frac{1}{4}$	$\times \cos. \overline{2S - Z}.$
XI. - - -	$+0.\ 4\frac{1}{2}$	$\times \cos. \overline{2X + Z}.$
XII. - - -	$-0.\ 5$	$\times \cos. \overline{2X - Z}.$

These arguments are, so far as they go, the same as those for the equations of the motion of the nodes, only in terms of the cosines instead of the sines; and the co-efficients of these arguments are respectively equal to those in the

equations of the motion of the nodes, multiplied by ,0897 the sine of the mean inclination.

952. The constant quantities which have arisen in deducing the variation of the inclination in the four last Articles, show how much the mean inclination is affected by the disturbing forces. Now these quantities are the same as the respective co-efficients of the equations with a contrary sign; therefore the sum of all the constant quantities is $-7'.\ 43\frac{2}{3}''$, which being negative shows that the mean inclination of the lunar orbit is $7'.\ 43\frac{2}{3}''$ less than it would have been, if there had been no disturbing forces. Now if we take the situation of the sun, moon and node such, that all the arguments may become $=0$, their cosines will be unity, and the whole variation from the mean will become $=+7'.\ 43\frac{2}{3}''$; at that time therefore the inclination of the orbit is the same as it would have been if the motion of the moon had not been disturbed by the sun's attraction. When all the arguments become 90° or 270°, the cosine of each vanishes, and the inclination of the orbit becomes the mean inclination.

FIG. 212. 953. If NM be the moon's orbit, NV the ecliptic, MV perpendicular to NV, and M the place of the moon, whose distance MN from the node is known; then if the true inclination MNV of the orbit be found by applying the above equations to the mean inclination, we have sin. $MV\times$ sin. $MNV=$ sin. MV the latitude of the moon.

954. Let the moon be in the node, and the node in quadratures of the sun, then $S=0$, $X=90°$; and neglecting all the equations except the three first, we get the equation $=-8'.\ 5''+38\frac{1}{2}''+41\frac{1}{2}''=-6'.\ 45''$, the quantity by which the mean inclination is diminished. Now let the moon have moved up to the sun in quadratures, then $S=90°$, $X=0$, and the equation $=-8'.\ 5''-38\frac{1}{2}''-41\frac{1}{2}''=-9'.\ 25''$ the quantity by which the mean inclination is diminished, and at which time the inclination is the least possible, the principal equations being all negative. Hence, in the passage of the moon from the node to the sun in quadratures, the inclination has diminished $9'.\ 25''-6'.\ 45''=2'.\ 40''$. In Art. 947. we found it to be $2'.\ 40'',7$.

955. Let the sun and moon be in the node, then $S=0$, $X=0$; hence, the equation $=8'.\ 5''+38\frac{1}{2}''-41\frac{1}{2}''=8'.\ 2''$, the quantity by which the mean inclination is increased, and the inclination is now the greatest, omitting the smaller equations.

956. Now let us trace the progress of the variation of the inclination from the time the sun leaves the node till it comes to the next node. For one revolution of the moon, the mean variation from the mean inclination (948) is $8'.\ 5''$ cos. $\overline{2S-2X}$, or as the cosine of double the distance of the sun from the node. When the sun is in the node, the cosine of $\overline{2S-2X}$ is $+1$, the greatest possible; therefore the inclination is the greatest. As the sun recedes from the node, the cosine of $\overline{2S-2X}$ decreases till the sun comes into quadratures of

the nodes, at which time the cosine of $\overline{2S-2X}$ is -1, the least possible; therefore the inclination is the least possible; consequently the mean inclination has decreased from the time the sun leaves the nodes till it comes into quadratures. Now from the time the sun leaves quadratures till it comes to the next node, the cosine of $\overline{2S-2X}$ increases from -1 to $+1$, consequently the inclination increases for that time, and then becomes the same as it was when the sun left the preceding node. We have here considered the inclination without any reference to the place of the moon in its orbit, which, as appears from the other equations, will affect the inclination; the inclination here mentioned must therefore be considered as nearly the mean inclination for a revolution of the moon. Hence, the whole variation in the transit of the sun from the nodes to quadratures is the difference between $-8'.\ 5''$ and $+8'.\ 5''$, or $16'.\ 10''$ as before stated. Sir I. NEWTON makes it $16'.\ 23\frac{1}{2}''$ for a circular orbit, which diminished in the ratio of 70 : 69, gives $16'.\ 9\frac{1}{2}''$. The other small equations have also their periods, in which they return to the same quantity. Hence, the mean inclination remains constant.

To reduce the Place of the Moon in its Orbit, to the Ecliptic.

957. By spherical Trigonometry we have, tan. $NV =$ cos. $N \times$ tan. $NM = \sqrt{1-s^2} \times$ tan. $NM = \overline{1-\frac{1}{2}s^2} \times$ tan. NM; hence, tan. $\overline{NV-NM} = \dfrac{-\frac{1}{2}s^2 \times \text{tan. } NM}{1+\overline{1-\frac{1}{2}s^2} \times \overline{\text{tan. } NM}^2} =$ (putting $\dfrac{\text{sin. } NM}{\text{cos. } NM}$ for tan. NM) $- \dfrac{\frac{1}{2}s^2 \times \text{sin. } NM \times \text{cos. } NM}{1-\frac{1}{2}s^2 \text{ sin. } \overline{NM}^2} = -\frac{1}{2}s^2 \times$ sin. $NM \times$ cos. NM nearly, $= -\frac{1}{4}s^2 \times$ sin. $2NM$; and as $NV-NM$ is very small, we may consider this quantity as expressing the difference of the two arcs. Now the moon's distance from the node $= S - \frac{3}{8}n \times$ sin. $\overline{2S-2X}$ nearly, the latter term being (939) the principal equation of the nodes; and if we correct the value of S by the principal equation of the center $-2w \times$ sin. Z, we get a more correct value of $NM = S - 2w \times$ sin. $Z - \frac{3}{8}n \times$ sin. $\overline{2S-2X}$; also, if we correct the value of s by the principal equation $\frac{3}{8}sn \times$ cos. $\overline{2S-2X}$ (948) of the inclination, it becomes $s + \frac{3}{8}sn \times$ cos. $\overline{2S-2X}$; hence, $NV-NM$ becomes $=$ sin. $(2 - 4w \times$ sin. $Z - \frac{3}{4}n \times$ sin. $\overline{2S-2X}) \times (-\frac{1}{4}s^2 - \frac{3}{16}sn \times$ cos. $\overline{2S-2X})$ nearly, $=($sin. $2S - 4w \times$ sin. $Z \times$ cos. $2S - \frac{3}{4}n \times$ sin. $\overline{2S-2X} \times$ cos. $2S) \times (-\frac{1}{4}s^2 - \frac{3}{16}sn \times$ cos. $\overline{2S-2X}) = -\frac{1}{4}s^2 \times$ sin. $2S + s^2w \times$ sin. $Z \times$ cos. $2S - \frac{3}{16}s^2n \times$ sin. $2S \times$ cos. $\overline{2S-2X} + \frac{3}{16}s^2n \times$ cos. $2S \times$ sin. $\overline{2S-2X}$ nearly, $= -\frac{1}{4}s^2 \times$ sin. $2S + \frac{1}{2}s^2w \times$ sin. $\overline{2S+Z} - \frac{1}{2}s^2w \times$ sin. $\overline{2S-Z} - \frac{3}{16}s^2n \times$ sin. $2X = -6'.\ 54'' \times$ sin. $2S + 46'' \times$ sin. $\overline{2S+Z} - 46'' \times \overline{2S-Z} - 23'' \times$ sin. $2X$ the reduction.

958. Hence, we may reduce the equations (921) for the longitude upon the moon's orbit, to the longitude upon the ecliptic, by applying the first, third and fourth equations to the equations XV. XVII. and VI. having the same arguments, and adding the third equation; hence, we get the following equations of the moon's longitude upon the ecliptic.

I.	$-6^\circ.\ 17'.\ 32'',7$	sin.	$Z.$
II.	$+\ 13.\ 0$	sin.	$2Z.$
III.	$-\ 0.\ 37$	sin.	$3Z.$
IV.	$-1.\ 16.\ 26$	sin.	$\overline{2X-Z}.$
V.	$-\ 3.\ 52$	sin.	$\overline{2X+Z}.$
VI.	$+\ 40.\ 18$	sin.	$2X.$
VII.	$+\ 12.\ 55$	sin.	$Y.$
VIII.	$+\ 1.\ 47$	sin.	$\overline{Z-Y}.$
IX.	$-\ 1.\ 47$	sin.	$\overline{Z+Y}.$
X.	$-\ 1.\ 6,3$	sin.	$\overline{2X-Y}.$
XI.	$-\ 0.\ 50,5$	sin.	$\overline{2X+Y}.$
XII.	$-\ 0.\ 12$	sin.	$\overline{2Z-2X}.$
XIII.	$+\ 0.\ 2,1$	sin.	$\overline{4Z-4X}.$
XIV.	$-\ 0.\ 49,3$	sin.	$\overline{4X-Z}.$
XV.	$-\ 6.\ 41,5$	sin.	$2S.$
XVI.	$-\ 1.\ 28$	sin.	$\overline{2S-2X}.$
XVII.	$-\ 1.\ 15$	sin.	$\overline{2S-Z}.$
XVIII.	$+\ 0.\ 46$	sin.	$\overline{2S+Z}.$
XIX.	$+\ 2.\ 12$	sin.	$\overline{2X-Z-Y}.$
XX.	$+\ 2.\ 3$	sin.	$\overline{2X-Z+Y}.$
XXI.	$+\ 0.\ 8$	sin.	$\overline{2Z+2X}.$
XXII.	$-\ 0.\ 12,5$	sin.	$\overline{2S-4X}.$
XXIII.	$-\ 0.\ 29$	sin.	$\overline{4X-2S-Z}.$
XXIV.	$-\ 0.\ 6,4$	sin.	$\overline{2X+Z-Y}.$
XXV.	$-\ 0.\ 6,4$	sin.	$\overline{2X+Z+Y}.$

On the true Place and Motion of the Moon's Apogee.

FIG. 213. **959.** Let N be the *mean* place of the moon's apogee corrected by its annual equation, Nn the major axis of the moon's orbit, E the earth, F the center of the orbit, EI, EK the greatest and least excentricity, and with the center F

and radius FI describe a circle $ICKB$, and draw AED perpendicular to Nn; draw ES towards the sun, and make the angle $IFH=2NES$, and draw EHN', and it gives (909) the situation of the apogee corrected by its annual and its great equation. Hence, when the sun is in the apogee N, N' coincides with N; when EN' becomes a tangent to the circle at C, FHE becomes a right angle, and N' is then got to its limit; when the sun comes to quadratures at A, H coincides with K, and N' with N; as the sun moves from quadratures A to the perigee n, when EN' becomes a tangent to the circle at B, N' gets to its limit on that side of N; when the sun comes into syzygies at n, H then coincides with I, and N' with N, and the period of the equation is completed. The same takes place whilst the sun moves from syzygies at n to N.

960. By Art. 909. the principal equation of the apogee is the angle HEF; now in that triangle, we know EF, FH and the angle EFH, which is the supplement of HFI, or twice the distance of the sun from the mean place of the apogee corrected by the annual equation (903); hence, $EF+FH : EF-FH ::$ tan. $\frac{1}{2} . \overline{FHE+FEH}$: tan. $\frac{1}{2} . \overline{FHE-FEH}$; and if we take $EF : FH :: 5505 : 1168$ according to Sir I. Newton, the log. of $\overline{EF-FH}$ − log. of $\overline{EF+FH} = -0,1871317$; hence, if from the log. tan. of $\frac{1}{2} . \overline{FHE+FEH}$ we subtract 0,1871317, we have the log. tan. of $\frac{1}{2} . \overline{FHE-FEH}$, and this subtracted from the third term leaves the angle HEF, and thus a Table of this equation may be very readily constructed. This equation of the apogee *added* to its place as corrected above, whilst the sun moves from syzygies to quadratures of the apsides, or *subtracted* whilst it moves from quadratures to syzygies, gives the place of the apogee corrected by the great equation. The following Table of this equation and of the excentricity of the moon's orbit is taken from Dr. Halley's *Astronomical Tables;* the argument, called the *Annual Argument*, is the distance of the sun from the mean place of the apogee corrected by its annual equation.

A TABLE OF THE GREAT EQUATION OF THE MOON'S APOGEE, AND OF THE ECCENTRICITY OF ITS ORBIT.

	Sig. O. VI. +		Sig. I. VII. +		Sig. II. VIII. +		
Ann. Arg. Deg.	Equation of ☽'s Apogee D. M. S.	Excentricity of the Moon's Orbit.	Equation of ☽'s Apogee D. M. S.	Excentricity of the Moon's Orbit.	Equation of ☽'s Apogee D. M. S.	Excentricity of the Moon's Orbit.	Ann. Arg. Deg.
[illegible]	0. 0. 0	,066777	9. 27. 57	,061754	11. 40. 0	,050224	30
1	0. 21. 4	,066771	9. 42. 12	,061434	11. 30. 39	,049838	29
2	0. 42. 8	,066754	9. 55. 58	,061107	11. 20. 14	,049457	28
3	1. 3. 10	,066724	10. 9. 14	,060772	11. 8. 44	,049082	27
4	1. 24. 9	,066683	10. 21. 58	,060429	10. 56. 8	,048714	26
5	1. 45. 5	,066630	10. 34. 9	,060080	10. 42. 26	,048354	25
6	2. 5. 57	,066566	10. 45. 47	,059725	10. 27. 38	,048001	24
7	2. 26. 44	,066489	10. 56. 49	,059363	10. 11. 45	,047656	23
8	2. 47. 25	,066402	11. 7. 15	,058995	9. 54. 47	,047321	22
9	3. 8. 0	,066302	11. 17. 4	,058621	9. 36. 44	,046995	21
10	3. 28. 27	,066192	11. 26. 14	,058243	9. 17. 37	,046679	20
11	3. 48. 46	,066070	11. 34. 43	,057860	8. 57. 25	,046374	19
12	4. 8. 55	,065936	11. 42. 31	,057472	8. 36. 11	,046081	18
13	4. 28. 54	,065792	11. 49. 36	,057080	8. 13. 56	,045800	17
14	4. 48. 42	,065636	11. 55. 57	,056684	7. 50. 42	,045531	16
15	5. 8. 19	,065469	12. 1. 33	,056285	7. 26. 29	,045275	15
16	5. 27. 43	,065292	12. 6. 22	,055884	7. 1. 21	,045033	14
17	5. 46. 53	,065103	12. 10. 23	,055479	6. 35. 19	,044805	13
18	6. 5. 48	,064905	12. 13. 35	,055073	6. 8. 26	,044592	12
19	6. 24. 27	,064695	12. 15. 56	,054666	5. 40. 45	,044394	11
20	6. 42. 50	,064476	12. 17. 24	,054257	5. 12. 18	,044212	10
21	7. 0. 56	,064246	12. 17. 59	,053848	4. 43. 10	,044046	9
22	7. 18. 44	,064006	12. 17. 40	,053438	4. 13. 23	,043896	8
23	7. 36. 12	,063757	12. 16. 25	,053030	3. 43. 1	,043763	7
24	7. 53. 20	,063498	12. 14. 13	,052622	3. 12. 9	,043647	6
25	8. 10. 6	,063230	12. 11. 2	,052215	2. 40. 49	,043548	5
26	8. 26. 29	,062952	12. 6. 52	,051811	2. 9. 7	,043467	4
27	8. 42. 29	,062665	12. 1. 42	,051409	1. 37. 6	,043404	3
28	8. 58. 5	,062370	11. 55. 31	,051010	1. 4. 52	,043359	2
29	9. 13. 15	,062066	11. 48. 17	,050615	0. 32. 28	,043332	1
30	9. 27. 57	,061754	11. 40. 0	,050224	0. 0. 0	,043323	0
	Sig. V. XI. −		Sig. IV. X. −		Sig. III. IX. −		

961. When EH becomes a tangent to the circle, the equation then becomes a maximum, and this is found by saying, $EF : FH ::$ rad. : sin. HEF; after that, as the point H approaches K the equation grows less, and the apogee, so far as its motion arises from this equation, goes backward, and therefore when this velocity becomes equal to the mean progressive velocity, they will destroy each other, and the apogee becomes stationary. This position of the apogee might be determined by making the fluxions of these two angular motions equal, but such a method would be very troublesome in practice, and it can be done very readily from inspection in the Table, by the method of trial and error. The mean motion of the sun in a day is 59′. 8″, and the mean motion of the apogee in a day is 6′. 41″; hence, in a day the sun recedes 52′. 27″ from the mean apogee, considering the sun as moving with its mean motion, which will be sufficiently accurate for the purpose for which it is here wanted; in the time therefore that the sun recedes 1° from the apogee, the apogee has moved 7′. 43″. Now from 1ˢ. 28° to 1ˢ. 29° of argument, the motion of the apogee (from the equation) has been retrograde 7′. 14″, and from 1ˢ. 29° to 2ˢ the motion has been 8′. 17″; hence, the regressive motion must have been equal to the progressive motion somewhere between 1ˢ. 29° and 2ˢ of argument. By trial and error therefore it is readily found, that with the argument 1ˢ. 29°. 23′ the motions become equal; and this will appear by computing its motion for a very small increase of argument, and comparing it with the contemporary mean motion. Now the equation to the argument 1ˢ. 29°. 23′ is 11°. 42′; hence, 1ˢ. 29°. 23′ − 11°. 42′ = 47°. 41′ the distance of the sun from the *true* place of the apogee when it becomes stationary from the effect of its greatest equation, which must be very near the true distance, as the other equations which affect its motion are very small. In the passage therefore of the sun from about 47°. 41′ before it comes to the apside, to 47°. 41′ beyond the apside, the apogee is *progressive*; and from 42°. 19′ before the sun comes into quadratures, to 42°. 19′ beyond quadratures, the apogee is *regressive.*

962. In half a day, the sun, moving with its mean motion, recedes 26′. 13″,5 from the mean apogee, and with that argument (the sun being so far from the mean apogee) the equation is found to be 9′. 12″; hence, in one day whilst the sun passes through the apogee, its progressive motion has been 18′. 24″ from the equation, to which add 6′. 41″ the mean motion of the apogee in the same time, and we have 25′. 5″ for the *progressive* motion of the apogee in a day, whilst the sun passes through the apogee. The same is true when the sun passes through the perigee. When the sun's distance from the apogee is 90°, the equation in half a day from thence is 14′. 11″, therefore in a day whilst the sun passes through quadratures, the equation is 28′. 22″, giving that regressive

motion of the apogee; hence, 28′. 22″ − 6′. 41″ = 21′. 41″ * the *regressive* motion of the apogee in a day when the sun passes its quadratures. It appears then from these two Articles, that the progressive motion of the apogee in syzygies is greater than the regressive motion in quadratures; and that in a whole revolution of the sun in respect to the apogee, it is progressive for a longer time than it is regressive. This is the meaning of what Sir I. NEWTON has delivered in Cor. 8. Pr. 66. Lib. I. He says, Manifestum est quod apsides in syzygiis suis, per vim ablatitiam, progredientur velocius, inque quadraturis suis tardius recedent per vim addititiam. Ob diuturnitatem temporis, quo velocitas progressus vel tarditas regressus continuatur, fit hæc inequalitas longè maxima. It must be acknowledged, however, that what he has here advanced is no immediate consequence of any thing which he has before investigated.

963. Some of the principal equations of the moon's motion (349 . . . . 354) were discovered by observation; but Sir I. NEWTON having found that the moon was retained in its orbit by a force, which, at different distances from the earth, varied inversely as the square of the distance†; and concluding from analogy that the same law of attraction might take place between all the bodies in the system, applied this theory (called the *Theory of Gravity*) to compute the effect of the sun's attraction upon the earth and moon, so far as it might affect the relative situation of the latter as seen from the former; and hence he discovered, besides the irregularities before observed, other smaller inequalities of the moon's motion, which also were found to agree with observations. From these, and other applications of his theory, he was confirmed in his conjectures concerning the principle of universal gravitation; and the same principle having since been further applied, and found to produce conclusions conformable

* Sir I. NEWTON, in the first edition of his *Principia*, page 462, has given the following Scholium. Hactenus de motibus lunæ quatenus excentricitas orbis non consideratur. Similibus computationibus inveni, quod apogæum, ubi in conjunctione vel oppositione solis versatur, progreditur singulis diebus 23′ respectu fixarum; ubi verò in quadraturis est, regreditur singulis diebus 16⅓′ circiter: quodque ipsius motus medius annus sit quasi 40°. Per Tabulas Astronomicas à CL. FLAMSTEDIO ad hypothesin HORROXII accommodatas, apogæum in ipsius syzygiis progreditur cum motu diurno 24′. 28″, in quadraturis autem regreditur cum motu diurno 20′. 12″, et motu medio annuo 40°. 41′ fertur in consequentia. Quod differentia inter motum diurnum progressivum apogæi in ipsius syzygiis, et motum diurnum regressivum in ipsius quadraturis, per Tabulas sit 4′. 16″, per computationem verò nostram 6⅔′, vitio Tabularum tribuendum esse suspicamur. Sed neque computationem nostram satis accuratam esse putamus. Nam rationem quandam ineundo prodiere apogæi motus diurnus progressivus in ipsius syzygiis, et motus diurnus regressivus in ipsius quadraturis, paulo majores. Computationes autem, ut nimis perplexas et approximationibus impeditas, neque satis accuratas, apponere non lubet.

† Sir I. NEWTON found, that if the force with which bodies fall upon the earth's surface were extended to the moon, and to vary inversely as the square of the distance from the center of the earth, it would in one minute draw the moon through a space which is equal to the versed sine of the arc which the moon describes in a minute. He concluded therefore that the moon was retained in its orbit by the same force as that by which bodies are attracted upon the earth.

to observation, his theory of gravity is now firmly established. In the year 1747, M. CLAIRAUT, in a Memoir read before the Academy of Sciences at Paris, made an objection to this law, upon this ground, that it would not account for the motion of the moon's apogee, it giving, according to his calculations, that motion only one half of what it was found to be by observations; and he concluded, that it was necessary to change this law, by adding something to correct it. He however soon afterwards discovered his mistake, and was the first who gave a complete theory of the moon, in which he showed that Sir I. NEWTON'S law of gravity would not only account for the motion of the moon's apogee, but also for all the other irregularities of the moon. M. EULER has done great justice to M. CLAIRAUT upon his solution of this important problem, in a letter to the Rev. Mr. CASPAR WETSTEIN; in which he observes, that "this question is of the last importance; and I must own, that, till now, I always believed, that this theory did not agree with the motion of the apogee of the moon. M. CLAIRAUT was of the same opinion; but he has publicly retracted it, by declaring that the motion of the apogee is not contrary to the Newtonian theory. Upon this occasion I have renewed my enquiries on this affair; and, after most tedious calculations, I have at length found to my satisfaction, that M. CLAIRAUT was in the right, and that this theory is entirely sufficient to explain the motion of the apogee of the moon. As this enquiry is of the greatest difficulty, and those, who hitherto pretended to have proved this nice agreement of the theory with the truth, have been much deceived, it is to M. CLAIRAUT that we are obliged for this important discovery, which gives quite a new lustre to the theory of the GREAT NEWTON; and it is but now, that we can expect good Astronomical Tables of the moon." Sir I. NEWTON in his *Principia*, Lib. I. Prop. 45. Cor. 2. by assuming the mean force $\left(\frac{1}{EM^2} - \frac{1}{2}m \times EM\right)$ of the moon to the earth in the direction of the radius ME, and considering that force as acting for a whole revolution of the moon, finds (855) the motion of the apsides to be only one half of the real motion; for this force $\frac{1}{EM^2} - \frac{1}{2}m \times EM = \frac{EM - \frac{1}{2}m \times EM^4}{EM^3}$ gives the distance of the apsides $= 180° \times \sqrt{\frac{1 - \frac{1}{2}m}{1 - 2m}} = 180° \times \sqrt{1 + \frac{3}{2}m} = 180° \times \overline{1 + \frac{3}{4}m}$, and consequently the motion is $180° \times \frac{3}{4}m$; whereas (903) the true motion is $180° \times \frac{3}{2}m$. But in this he has neglected that part of the force in a direction perpendicular to EM, and which is found to produce the other half of the motion; and Sir I. NEWTON, in this place, intended only to show what part of the motion the mean force in the direction of the radius would produce. In Lib. III. Pr. 3. he says, that "the action of the sun, so far as it draws the moon from the earth, is twice as great as he has assumed it above;" by which he does not mean that he has assumed the mean force $(-\frac{1}{2}m \times EM)$ of the

sun too little by one half, but that, as it would require twice such a force of the sun to give the true motion of the apsides, the force which acts in a direction perpendicular to the radius EM, must, in its effects upon this occasion, be equivalent to the force $-\frac{1}{2}m \times EM$ in the direction of the radius. In the first edition of the *Principia*, in the Scholium to the Theory of the moon, he said that he had found, by a very complex calculation, that the mean annual motion of the apogee was about 40°; and that the diurnal motion of the apogee, when in syzygies, was progressive about 23′, and when it was in quadratures, it was regressive about $16\frac{1}{3}$′; this he omitted in his future editions, as he was not perfectly satisfied with his computations. Mr. MACHIN, in his *Laws of the Moon's Motion according to Gravity*, makes the mean annual motion of the apogee to be 40°. 40′. $40\frac{1}{2}$″, upon a principle which he first suggested, and upon which we have in this Work computed that motion, according to the method given by FRISI. M. WALMSLEY, in his *Theory of the Motion of the Apsides*, has, upon the same principle, computed the mean motion of the moon's apogee, and his conclusion agrees very well with observation; but his principles are altogether wrong; for he has entirely omitted that part of the force which acts in a direction perpendicular to the radius, which, as we have shown, produces just one half of the motion; he also assumes the mean disturbing force in the direction of the radius as acting constantly, instead of the real disturbing force; and he has also wrongly computed the periodic time of the moon; it was by accident therefore that he obtained the mean motion; in respect to the true motion, his conclusions are erroneous. Mr. MACHIN has not given us his process; we cannot therefore say how far it was just. In the *Phil. Trans.* 1751, Mr. P. MURDOCK has given a method of computing the mean motion of the moon's apogee, by first considering only that part of the disturbing force, which acts in the direction of the radius; and then, instead of supposing the earth to be at rest, by conceiving the earth and moon to revolve about their common center of gravity, he imputes about one half the motion to that cause, and thence deduces a conclusion agreeing nearly with observation. What we have already observed in Art. 861. is sufficient to show, that no part of the effect can arise from the latter circumstance; and he has also (as we have already shown) omitted a cause which produces about one half of the motion; by two mistakes he has therefore fallen upon a true conclusion.

964. Now to bring into one point of view, the sources of all the equations which we have here found, we may consider, *first*, the equation of the center, which arises from the elliptic form of the orbit. *Secondly*, we may consider that the disturbing forces would change a circular orbit into that of an ellipse, having the earth in the center, with the minor axis lying in syzygies, and therefore the moon's orbit must suffer very nearly a similar effect; the areas (849) are also accelerated and retarded; and from these causes, the mean place differs

from the true; thus there is produced an equation, called the *Variation*. But at different distances of the earth from the sun, the disturbing forces vary; this equation therefore being first calculated for the mean distance of the earth from the sun, will be subject to a variation, from the variation of that distance; and hence arise some new equations. *Thirdly*, the moon's orbit being dilated and contracted as the earth approaches to, or recedes from, the sun, its motion will accordingly be diminished or increased, and hence arises an annual equation, assigning the difference between the mean motion at the mean distance of the earth from the sun, and the mean motion at any other distance of the sun. *Fourthly*, the variation depending on the true distance of the sun from the moon, if the mean distance at first used be corrected by the first term of the equation of the center, and by the annual equation, new equations of the variation will arise. And if the second term of the equation of the center be also taken, and applied, there will thence arise further equations of the variation. Also, if the distance of the moon from the sun be further corrected by the evection, new equations will be introduced into the variation. Again, another correction of the variation may be introduced, by considering the difference between the disturbing forces arising from the situation of the nodes of the lunar orbit. Thus, by correcting the moon's distance from the sun, from what was first assumed, we get new equations of the variation. *Fifthly*, the orbit of the moon being inclined to the ecliptic, the disturbing force in the orbit is different from what it would be, if the orbit coincided with the ecliptic, and that difference of forces will produce another equation, depending on the situation of the nodes in respect to the sun. *Sixthly*, the disturbing forces cause a motion of the apogee; but that motion is not uniform, it being regressive when the sun is in its quadratures, and progressive, when in syzygies, but upon the whole it is progressive; there arises therefore an equation of the motion of the apogee, which depends upon its distance from the sun; there is also a smaller annual equation, arising from the disturbing forces being different at different times of the year. *Seventhly*, the excentricity of the orbit is subject to a change, and that change causes a change of the equation of the center, called the evection; hence arise new equations to be applied, depending on the situation of the apogee, the change of excentricity depending on that circumstance. *Eighthly*, the evection at the mean distance of the earth from the sun being found, the disturbing force of the sun at different distances from the earth being different, equations of the evection will hence arise. Also, the disturbing force varies as the situation of the nodes vary, and hence also arise new equations of the evection. *Lastly*, if the distance of the moon from the sun be corrected by the annual equation, and the motion of the moon from the apogee be corrected by the same equation of the mean motion, and by the annual equation of the apogee, further equations of the evection are found. Thus, by correcting the first assumed values of the quantities representing the

several sources of the equations, new equations arise, which may be considered as corrections of the original equations; and hence we derive the equations of the moon's motion, which applied to the mean place, will give the true place. Sir I. NEWTON calls the equations *Menstruæ*, *Semestres*, *Annuæ*, *&c.* according to the periods in which they return. The mean motion of the nodes, and the mean inclination of the orbit, are also corrected by equations found upon similar principles, which applied to the mean place of the node, and mean inclination, give the true. The conclusions thus deduced will be found generally to agree very well with those which are derived from a direct solution; the few cases where they were not sufficiently accurate for a ground of computation, we have pointed out, and corrected; but in whatever manner this subject is treated, some corrections are applied from observations, in order to render the equations more perfect; not that the principle of attraction is insufficient to furnish conclusions which shall agree with observations, but that the method of deducing those conclusions being only by approximation, small errors are introduced, which are easier to be corrected by observation, than by continuing such laborious calculations in order to a further correction*. MAYER's Tables†, founded upon a very elegant theory which he corrected by observations, do not err more than half a minute in longitude; but we have now more correct Tables; see Vol. III. The method however by which we have here treated the subject, has this advantage, that it shows the causes of all the equations, and thereby gives a very clear and comprehensive view of the whole business. Thus we have given the reader all the satisfaction we are able upon this difficult subject, without entering into a direct solution of the problem, which requires the integration of a fluxional equation of the second order, and this can be done only by an approximation of a very intricate nature, and of great labour; see Chapter XXXVII. By so treating the subject, we obtain the following very important conclusion, which was first observed by Mr. SIMPSON: That as no terms enter into the equation of the orbit, but what consist of the cosine of an arc, or of its multiples, all the terms, by a regular increase and decrease, do after a certain time return again to their former values, and therefore the mean motion of the moon, and the greatest quantities of the several equations, undergo no change from gravity.

* MAYER in his *Theoria Lunæ,* page 50, makes this observation. Præterea in eadem hac formula plures termini occurrunt, quos theoria, licet summo studio tractata, accurate præbere non potest, ob rationes nulli non cognitas, qui in hac re vires suas ac patientiam exercuit. Hos igitur terminos malui ex observationibus definire, quam illorum gratia calculum tædiosissimum, à me jamdudum longe accuratius institutum, quam à quoque hactenus factum, ulterius adhuc persequi, novisque ac forte insuperabilibus augere difficultatibus. Denique etiam eos terminos, quos theoria satis manifeste ostendit, per observationes ita correxi, ut paucis secundis adjectis vel demtis cum cœlo magis consentirent.

† As corrected by Dr. MASKELYNE. See the Preface to the *Nautical Almanac*, 1797, *&c.*

CHAP. XXXIII.

ON THE FIGURE OF THE EARTH.

Art. 965. IF a fluid body had no motion about its axis, and all its parts were at rest and kept together by the mutual attraction of its constituent particles, the body would form itself into a sphere; for the pressures of all the columns upon the central particle could not be equal, unless they were all of the same length. Now Sir I. NEWTON having proved that the bodies in our system attract each other with forces varying inversely as the squares of their distances, concluded it to be an universal principle for every particle of matter, and that the parts of each body were kept together by the same power (835). On this supposition therefore, if the earth were a fluid body, and perfectly at rest, its form must be that of a sphere. But as the earth revolves about an axis, each particle, besides its gravity, will be urged by a centrifugal force, by which the particles will all have a tendency to recede from the axis; and hence, M. HUYGENS and Sir I. NEWTON concluded that the earth must put on a spheroidical form, whose polar diameter is the shortest. For let POp be the axis, EQ the equator, draw abm perpendicular to Pp, and let bm represent the centrifugal force of the point b, and resolve it into two, one bn in the direction of the tangent, and the other mn perpendicular to it. Then the force bn draws the fluid from p towards E, and consequently it will diminish the radius pO and increase the radius EO. We shall therefore first consider, what will be the form of the curve PEp, and then determine the ratio of pO to EO, according to the principles of M. CLAIRAUT, in his Treatise, entitled, *Théorie de la Figure de la Terre*, who has followed MAC LAURIN in his investigation of the attraction of a corpuscle upon the surface of a spheroid, in directions parallel to each axis. FIG 214.

966. Let Pp, Qa be two concentric circles, O the center, draw $PQOap$, and mQM, ar, perpendicular to it, and join Mr, and make the angles aQb, aQc, rMs, rMw equal, then $Qb+Qc=Ms+Mw$. For from the construction, Mr is parallel to Qa, and hence $Mr=Qa$; draw rt perpendicular to Ms, and take $rv=rs$. Then as the angle $rMt=aQb$, $rtM=abQ$, and $Mr=Qa$, we have $Mt=Qb$. Now as the angle $sMr=wMr$, the chord $rw=$ chord $rs=rv$; therefore the triangles Mrv, Mrw are similar and equal; hence, $Mw=Mv$; consequently $Mw+Ms=Mv+Ms=2Mt=2Qb=Qb+Qc$. FIG. 215.

967. Now let us suppose this figure to be orthographically projected upon a plane passing through Mm, and the two circles will be projected into similar

ellipses, and the lines Qb, Qc, Ms, Mw being all diminished in the same ratio, we shall have in the ellipses, $Qb + Qc = Ms + Mw$. When by increasing the angle vMw it becomes greater than the angle rMx of the segment, Mw will lie on the other side of M, or w falls in the arc MPm, in which case, $Ms - Mw = Qb + Qc$.

FIG. 216. 968. The attraction of a corpuscle at A towards any pyramid $Avwzx$, the area $vwzx$ of whose base is indefinitely small, varies as the length, the angle A being given, and the attraction to each particle varying inversely as the square of the distance. For put $a =$ the area $xzvw$, $m = Ax$, $x = Aa$, and let ab be a section parallel and therefore similar to zv; then, $m^2 : x^2 :: a : \frac{ax^2}{m^2} =$ the section ab; hence, $\frac{ax^2\dot{x}}{m^2} =$ the fluxion of Aab, and the fluxion of the attraction of $Aab = \frac{a\dot{x}}{m^2}$, whose fluent is $\frac{ax}{m^2}$, the attraction of any length x, which therefore varies as x. Hence, the attractions of corpuscles at the vertices of similar pyramids are in proportion to their lengths. If $x = m$, we get the attraction to the whole pyramid $= \frac{a}{m}$.

969. Hence, if two corpuscles be similarly situated in respect to two similar solids, the attractions to the solids will be as their lengths. For if the two solids be divided into similar pyramids, having the corpuscles in the vertices, the attractions to all the corresponding pyramids will be as their lengths, or as the lengths of the solids; for the pyramids being similarly situated in the two similar solids, their lengths must be in proportion to the lengths of the solids, and therefore the whole attractions will be in proportion to the lengths of the solids; or in proportion to any two lines similarly situated in them.

FIG. 217. 970. Let FIG. 217. be the projection described in Art. 967. and make the angles bQv, cQg, sMx, wMz indefinitely small and equal, and conceive the whole figure to revolve about Mm through an indefinitely small angle, then the pyramids generated by bQv, cQg, sMx, wMz being similar, the attractions of the corpuscles at Q and M towards them will (968) be as their lengths. But $Qb + Qc = Ms + Mw$, or $Ms - Mw$ when w falls on the other side of M; hence, the attraction of Q to the pyramids generated by bQv, cQg is equal to the attraction of M towards the pyramids generated by sMx, wMz, the attraction to wMz being reckoned negative when w lies on the other side of M. Hence, if each attraction be divided into two, one perpendicular to Mm, as bn, cq, sr, wy, and the other parallel to Mm, then from similar triangles, $bn + cq = sr + wy$, consequently the attractions of the corpuscles at Q and M in a direction perpendicular to Mm are equal. And as this reasoning holds for

every such corresponding positions of pyramids about Q and M, it follows that the attraction of the corpuscles at Q and M to the whole solids generated by $Qead$, $Pupt$ about Mm through an indefinitely small angle, in directions perpendicular to Mm, will be equal.

971. Now let $Qead$, $Pupt$ be two similar spheroids, and conceive the line Pp to pass through the center, and Mm to be perpendicular to it. Then every plane which passes through Mm, cutting the two spheroids, will be similar ellipses; hence, by the last Article, if we conceive any two of these planes to be inclined to each other at an indefinitely small angle, the attraction of the corpuscles at Q and M in a direction which is perpendicular to Mm, towards the solids between these planes in the two respective spheroids, will be equal. Hence, by the resolution of forces, the whole attraction of a corpuscle at Q in the direction QO towards the spheroid $Qead$ is equal to the attraction of a corpuscle at M in a direction parallel to QO towards the spheroid $Pupt$. In like manner it appears, that if Tef be drawn parallel to the axis Pp, the attraction of a corpuscle at e in the direction eO towards the spheroid $eadQ$ is equal to the attraction of a corpuscle at T in a direction parallel to eO towards the spheroid $uptP$.

972. The attraction of a corpuscle at P towards the spheroid $Pupt$ is to the attraction of a corpuscle at Q towards the spheroid $Qead$ :: PO : QO. For conceive two similar pyramids, whose bases are indefinitely small, to be similarly situated in the two spheroids, having their vertices in P and Q; then (968) the attractions of the corpuscles at P and Q will be as their lengths, or as PO to QO from their similar situations. Hence, if we resolve the attraction of each into the direction PO, QO, the attractions in this direction will still be as PO to QO, from the pyramids being equally inclined to PO and QO. Therefore by dividing the whole of each spheroid into similar pyramids, it follows that the attraction of the corpuscles at P and Q to the centers of their respective spheroids will be as PQ : QO.

973. Let $PEpU$ be an ellipse, Pp the minor and EU the major axis, draw MG perpendicular to the curve, MT a tangent to it, and MQ, MR perpendicular to OP and OE. Now $QT : QM :: QM : QG = \frac{QM^2}{QT}$, and by the property of the ellipse, $OT : OP :: OP : OQ = \frac{OP^2}{OT}$; hence, $QG : OQ :: \frac{QM^2}{QT} : \frac{OP^2}{OT} :: QM^2 \times \frac{OT}{QT} : OP^2$; but from the second proportion, $OT : OQ :: OP^2 : OQ^2$; hence, $OT : TQ :: OP^2 : OP^2 - OQ^2 = \overline{OP + OQ} \times \overline{OP - OQ} = pQ \times PQ :: OE^2 : QM^2$, therefore $OE^2 = QM^2 \times \frac{OT}{QT}$; hence, $QG : OQ :: OE^2 : OP^2$, FIG. 218.

consequently $QG = \frac{OE^2}{OP^2} \times OQ$.

974. A fluid body will preserve its figure, if the direction of its gravity at every point be perpendicular to its surface; for then gravity cannot put its surface in motion.

975. If the particles of an homogeneous fluid body attract each other with forces varying inversely as the squares of their distances, and it revolve about an axis, it will put on the form of a spheroid. For if $PEpU$ be a fluid, Pp its axis about which it revolves, then may the spheroid revolve in such a time, that the centrifugal force of any particle M combined with its attraction to the spheroid may make this whole force act perpendicular to the surface. For let $E =$ the attraction at the equator E, $P =$ that at the pole P, $F =$ the centrifugal force at the equator. Now (971) the attraction of M in the direction $MR =$ the attraction of Q to a spheroid similar to $PUpE$ whose semi-axis is QO, and (972) the attractions at P and Q upon these similar spheroids are as $QO : PO$; hence, the attraction of M in the direction MR is $P \times \frac{QO}{PO}$; take $Mr =$ this quantity. For the same reason, the attraction of M in the direction MQ is $E \times \frac{OR}{OE}$. But the centrifugal forces of bodies revolving in equal times being as their radii*, we have $OE : QM$, or $OR, :: F : F \times \frac{OR}{OE}$ the centrifugal force of M; hence, $\overline{E - F} \times \frac{OR}{OE} =$ the whole force of M in the direction MQ; take $Mg =$ this quantity. Complete this parallelogram $Mrqg$, and Mq will be the direction of the whole force acting on the particle at M. Produce Mq to G. It remains therefore only to be proved, that OE, OP may have such a ratio to each other, that MG shall be every where perpendicular to the curve. Now by similar triangles, gq, or Mr, $: Mg :: QG : QM$, that is, $P \times \frac{QO}{PO} : \overline{E - F} \times \frac{OR}{OE} :: \frac{OE^2}{OP^2} \times OQ : QM$, or $P : E - F :: OE : OP$, in which there is no line concerned but the two axes; therefore to a spheroid having two axes in such a ratio, the whole attractive force will at every point be perpendicular to its surface; hence (974), the fluid will be at rest. And F may always have such a value as will satisfy this proportion, by adjusting the time of revolution. Or having given F together with P and E, the spheroid whose axes are as $P : E - F$ must be that into which the fluid will form itself.

* For (826) the centrifugal force varies as $\frac{\mathfrak{Q}r^2}{Sr}$, and in this case $\mathfrak{Q}r$ varies as Sr; hence, the centrifugal force varies as Sr, or as the radius.

976. The attraction at any point M in the direction MR is as $P \times \frac{QO}{PO} = P \times \frac{MR}{PO}$; let therefore P (the attraction at P) be represented by PO, and MR will represent the attraction at M in the direction MR, consequently Mv will represent the whole attraction acting in the direction perpendicular to the surface. Draw ve perpendicular to MO; then $MO : Ma :: Mv : Me ::$ the attraction in the direction Mv ($= Mv$) : the attraction in the direction $MO = \frac{Ma \times Mv}{MO}$ which varies as $\frac{1}{MO}$, because $Ma \times Mv$ is constant, by the property of the ellipse.

977. To determine the attraction of a corpuscle at P the pole of a spheroid, to the spheroid. Draw Pm, PM, indefinitely near each other, and MQ, Mr, perpendicular to Pp, Pm, and conceive the plane PEp to revolve about Pp through an indefinitely small angle whose arc is equal a, radius being unity; put $PO = 1$, $OE = m$, $PQ = z$, $QM = u$, the cosine of $MPQ = s$, and $\sqrt{m^2 - 1} = n$. Now $ua =$ the indefinitely small arc described by M about Pp, consequently $ua \times Mr =$ the base of the pyramid generated by PMr; hence (968), the attraction of P to this pyramid $= \frac{ua \times Mr}{PM}$; but considering the angle MPm as the increment of MPQ, and Mr the increment of the arc to the radius PM, we have $Mr : \dot{s} :: PM : \sqrt{1 - s^2}$; hence, the attraction of the pyramid in the direction $PO = \frac{uas\dot{s}}{\sqrt{1-s^2}}$. But $u^2 = 2m^2z - m^2z^2$, also $s = \frac{PQ}{PM}$ (radius being unity) $= \frac{z}{\sqrt{z^2 + u^2}}$; hence, $\frac{us}{\sqrt{1-s^2}} = z = 1 + \sqrt{1 - \frac{u^2}{m^2}}$ from the first equation, from which $u = \frac{2m^2\sqrt{1-s^2} \times s}{1 + n^2s^2}$; hence, the attraction of the pyramid in the direction $PO = \frac{2am^2s^2\dot{s}}{1 + n^2s^2} = \frac{2am^2}{n^3} \times \overline{n\dot{s} - \frac{n\dot{s}}{1 + n^2s^2}}$, whose fluent is $\frac{2am^2}{n^3} \times \overline{ns - z}$ where z is a circular arc whose radius $= 1$, tangent $= ns$; and when $s = 1$, we have $\frac{2am^2}{n^3} \times \overline{n - z}$ for the attraction of P towards the solid generated by the revolution of PEp through an indefinitely small angle, where z is the arc whose tangent is n. Hence, as this is the attraction for every indefinitely small solid thus cut, if we put $c =$ the circumference of a circle whose radius is unity, and substitute c instead of a, we shall get $P = \frac{2cm^2}{n^3} \times \overline{n - z}$ for the attraction to the whole spheroid. But $z = n - \frac{1}{3}n^3 + \frac{1}{5}n^5 - \frac{1}{7}n^7 + \&c.$ put also $m = 1 + d$, then $n^2 = m^2 -$ FIG. 219.

$1=2d+d^2$; hence, by substitution, $P=\frac{2c}{3}\times\overline{1+\frac{4}{5}d-\frac{4}{7}d^2+\frac{8}{105}d^3}$ &c. If $d=0$, we get $\frac{2}{3}c$ for the attraction to a sphere whose radius $=1$.

FIG. 220. 978. To find the attraction of a corpuscle at the equator E of a spheroid $EPDp$. Draw EK parallel to Pp, and suppose a sphere EH to be inscribed in the spheroid, whose radius is equal to unity. Now if we conceive any plane to pass through EK cutting the spheroid and sphere, the section of the spheroid will be similar to $EPDp$, and the section of the sphere a circle; hence, if we find the attraction to the two solids between any two planes passing through EK and forming an indefinitely small angle e with each other, we shall get the ratio of the attractions to the two whole solids. Draw EN, En indefinitely near, and NL perpendicular to En, and NK to EK. Put $EQ=m$, $OR=1$, $\overline{m^2-1}|^{\frac{1}{2}}=n$, $EK=u$, $NK=z$, the arc of the angle e to radius unity $=a$, and the sine of $NEQ=s$, then $az=$ the arc described by N; hence, the base of the pyramid described by $ENL=az\times LN$, and (968) the attraction of the pyramid $=\frac{az\times LN}{EN}$, and the attraction in the direction $EQ=\frac{az\times LN}{EN}\times\sqrt{1-s^2}=az\dot s$, because, considering NL as a circular arc whose radius is EN, and the angle NEL as the increment of NEO, $LN:\dot s::EN:\sqrt{1-s^2}$. But by the property of the ellipse, $u^2m^2=2mz-z^2$; also, $s=\frac{u}{\sqrt{u^2+z^2}}$; make the two values of u^2 from these two equations equal to each other, and thence we get $z=\frac{2m\times\sqrt{1-s^2}}{1+n^2s^2}$; hence, the attraction $=\frac{2ma\times\sqrt{1-s^2}\times\dot s}{1+n^2s^2}=2a\times\frac{m^3}{n^3}\times\frac{n\dot s}{1+n^2s^2}-\frac{m\dot s}{n^2}$, whose fluent is $2a\times\frac{m^3}{n^3}\times z-\frac{ms}{n^2}$, where z is a circular arc whose radius $=1$, tangent $=ns$; and when $s=1$, we get $2a\times\overline{\frac{m^3}{n^3}\times z-\frac{m}{n^2}}$ for the whole attraction to the part cut off from the spheroid. If we make $m=1$, and consequently $n=0$, we have $2as-2as^2\dot s$ for the attraction to the pyramid of the sphere generated by Ewv, whose fluent, when $s=1$, is $\frac{4a}{3}$ for the attraction to the corresponding part of the sphere cut off by the two planes. Hence, the ratio of these attractions is as $1:\frac{3}{2}\times\overline{\frac{m^3}{n^3}\times z-\frac{m}{n^2}}$. But (977) the attraction to a sphere whose radius $=1$, is $\frac{2}{3}c$; hence, $E=c\times\overline{\frac{m^3}{n^3}\times z-\frac{m}{n^2}}$ is the attraction to the spheroid at E. Put $m=1+d$, then $n^2=2d+d^2$; also, $z=n-\frac{1}{3}n^3+\frac{1}{5}n^5-\frac{1}{7}n^7+$&c. hence, by substitution, $E=\frac{2c}{3}\times\overline{1+\frac{2}{5}d-\frac{3}{35}d^2+\frac{4}{105}d^3}$ &c. Now (975) when the fluid is

in equilibrio, $P : E - F :: 1 + d : 1$; hence, $F = E - \frac{P}{1+d}$; substitute for E and P their values, and we get $F = \frac{4}{3} c \times \frac{2}{5} d - \frac{2}{35} d^2 - \frac{2}{35} d^3$ &c.

979. Let v express the centrifugal force of a body at the equator compared with its weight, then $v = \frac{F}{E-F}$, the weight being unity; substitute for E and F their values, and $v = \frac{2}{5} d - \frac{4}{175} d^2 + \frac{1}{875} d^3$ &c. hence, by reverting the series, $d = \frac{5}{4} v + \frac{25}{56} v^2 - \frac{125}{392} v^3$ &c. consequently $1 + d = 1 + \frac{5}{4} v + \frac{25}{56} v^2 - \frac{125}{392} v^3$ &c. is the equatorial radius, the polar radius being unity.

980. To determine from hence the actual ratio of the diameters of the earth. By Art. 965. mn is that part of the centrifugal force bm which acts in opposition to gravity; now $Ob : ba :: bm : mn = \frac{ba \times bm}{Ob}$; but bm (975) varies as ba, and Ob is constant, therefore mn varies as ba^2 the square of the cosine of latitude. Now according to Sir I. Newton, if the earth be a sphere, its radius is 19615800 Paris feet. In the latitude of Paris, a body falls 2174 lines the first second. The versed sine of the arc described by the equator in $1''$ is 7,54064 lines, which therefore represents the centrifugal force at the equator. Hence, the force of gravity at Paris : centrifugal force at the equator :: 2174 : 7,54064. But $\text{rad.}^2 : \text{cos. lat. Paris}^2 :: 7,54064 : 3,267$ that part of the centrifugal force at Paris which is opposite to gravity; hence, $2174 + 3,267 = 2177,267$ is the force of gravity at Paris; therefore the force of gravity at Paris : centrifugal force at the equator :: 289 : 1; and this ratio may be taken for the gravity at the equator to the centrifugal force. Hence, $v = \frac{1}{288}$, consequently $d = \frac{5}{1152} = \frac{1}{230{\cdot}4}$ neglecting all the terms after the first on account of their smallness. Hence, the ratio of the diameters is as $1 : 1 + \frac{1}{230{,}4} :: 230{,}4 : 231{,}4$, or as 230 : 231 without any sensible difference. Sir I. Newton makes it 229 : 230, from which ratio the above does not sensibly differ.

981. If the whole density of the body should vary, then the gravity E at the equator will vary as the density, but the centrifugal force F will not be altered; now $v = \frac{F}{E-F} = \frac{F}{E}$ nearly; hence, v varies inversely as the density nearly; consequently d will vary nearly in the inverse ratio of the density; that is, by increasing the whole density, the body approaches nearer to a sphere.

982. To find the ratio of the diameters of *Jupiter*. Let t = the time of its rotation, T = the time of revolution of one of its satellites, h = the distance of that satellite from the center of Jupiter, the radius of Jupiter (here supposed a sphere) being = 1. Then (977) as $\frac{2c}{3}$ expresses the attraction at the surface of Jupiter, $\frac{2c}{3h^2}$ = the attraction of the satellite. Now the centrifugal forces being

FIG. 214.

as the radii directly and squares of the periodic times inversely*, and the centrifugal force of the satellite being equal to its centripetal force (these forces in a circle being always equal), we have, $\frac{h}{T^2} : \frac{1}{t^2} :: \frac{2c}{3h^2} : F = \frac{2cT^2}{3h^3t^2}$. Hence (978), $\frac{4}{5}d - \frac{6}{35}d^2 - \frac{2}{75}d^3$ &c. $= \frac{T^2}{h^3t^2} = w$, and by the reversion of series, $d = \frac{5}{4}w + \frac{75}{224}w^2 + \frac{125}{392}w^3$ &c. consequently $1 + d = 1 + \frac{5}{4}w + \frac{75}{224}w^2 + \frac{125}{392}w^3$ &c. Now according to CASSINI, the time of the rotation of Jupiter is 596′, and the distance of the fourth satellite, according to Mr. POUND, is $26{,}63 = h$, and the mean time of its revolution $= 24032' = T$. Hence, the ratio of the diameters becomes 100,5 : 90,5. By observation, Mr. POUND found the ratio of the diameters to be 12 : 13; and Dr. BRADLEY as 12,5 : 13,5.

983. Hence, the difference of the diameters being $\frac{5}{4}w$, taking the first term only for the value of d, it appears that that difference, or w, varies as $\frac{1}{t^2}$ or directly as the square of the velocity of the planet about its axis. Hence, and by Art. 981. if the density and time of rotation should vary, the difference of the diameters will vary as the square of the velocity directly and density inversely.

984. Sir I. NEWTON determines the ratio of the diameters of the earth in the following manner†. He assumes the figure to be a spheroid, and finds the

* For by Art. 826. the centrifugal force varies as $\frac{\mathfrak{Q}x^2}{Sx}$. Let $P =$ the periodic time in the circle $\mathfrak{Q}x$, $v =$ the velocity in the circle, then v varies as $\mathfrak{Q}x$, because the time is given; but P varies as $\frac{\text{circum.}}{v}$, or as $\frac{Sx}{v}$, therefore $\frac{Sx}{P^2}$ varies as $\frac{v^2}{Sx}$ which varies as $\frac{\mathfrak{Q}x^2}{Sx}$, or as the centrifugal force.

FIG. 221. † Sir I. NEWTON in his *Principia*, Lib. I. Pr. 91. Cor. 3. proves, that if a corpuscle P be placed within a spheroid, it is attracted to the center O by a force proportional to PO. For conceive the spheroid $Pp\mathfrak{Q}$ to be similar to the given spheroid MTR, and draw the radii OpV, OPM; and draw $mpwv$, $npst$, making an indefinitely small angle npm; then considering pmn, ptv as similar pyramids, the attraction of p to them (968) will be as their lengths; but by the property of the ellipses, $wv = pm$; therefore the attraction of p to the part $wstv =$ the attraction to pmn; hence, p is attracted only by the pyramid pws. Thus it appears, that the attraction of p is only to the spheroid $P\mathfrak{Q}$; hence, the attraction (969) at p : the attraction at $V :: pO : VO$. But this is not true for corpuscles in different radii. For $P\mathfrak{Q}$ is a spheroid similar to MTR; and (as above proved) the corpuscles P, p, will not be disturbed by the attraction of the matter exterior to $P\mathfrak{Q}$, consequently (976) the attraction of p to O : the attraction of P to $O :: PO : pO$; but (969) considering rz similar to $P\mathfrak{Q}$, the attraction of a corpuscle P : the attraction of a corpuscle $r :: PO : rO$; therefore the attraction of p to O : the attraction of r to $O :: PO^2 : pO \times rO :: pO \times PO^2 : pO^2 \times rO :: pO : rO \times \frac{pO^2}{PO^2} :: pO : rO \times \frac{VO^2}{MO^2}$. Therefore for corpuscles situated in different radii, this last proportion must be applied, and not that of Sir I. NEWTON. From hence it also appears, that if VO, MO be two canals meeting at O and filled with a fluid, they will balance; for the fluxions of their pressures will be as the forces of attraction multiplied

centrifugal force at the equator : gravity :: 1 : 289 as in Art. 980. He then assumes the ratio of the diameters to be as 100 : 101, and finds the gravity at the pole : gravity at the equator :: 501 : 500. Now if we conceive two canals to be cut from the pole and equator down to the center of the earth, and filled with a fluid, they must balance each other. But the force of gravity at different parts of the same canal varies directly as the distance from the center, and the centrifugal force of every point of the equatorial canal varying (975) in the same ratio, therefore the whole force to the center varies as the distance from the center; and if we take any two indefinitely small parts of each canal, similarly situated, their weights must be as the weights of the whole; and as the weights are as the magnitudes and gravities, they will be as 101×500 : 100×501, or as 505 : 501. Hence, that there may be an equilibrium, the centrifugal force must take off $\frac{4}{505}$ from the equatorial canal, and then the weights of each will be equal, and they will balance. But the centrifugal force of the earth at the equator takes off $\frac{1}{289}$ part of gravity; hence (266), $\frac{4}{505}$: $\frac{1}{289} :: \frac{1}{100} : \frac{1}{229}$ the excess of the equatorial above the polar radius; therefore the ratio of the radii is $1 : 1 + \frac{1}{229}$ or 229 : 230.

985. To find the ratio of the diameters of any other body, Sir ISAAC proceeds thus. If the body be greater or less than the earth, the density and time of rotation being the same, the ratio of the centrifugal force to gravity, and therefore the ratio of the diameters, will remain the same. But if the time of rotation and the density vary, the difference of the diameters will (963) vary very nearly in a duplicate ratio of the velocity directly and the density inversely. Now the earth revolves in 23*h.* 56′, and *Jupiter* in 9*h.* 56′, the squares of these are as 29 : 5 very nearly, and densities (as will afterwards appear) are as 400 : 94,5; hence, the difference of the diameters of Jupiter : its least diameter :: $\frac{29}{5} \times \frac{400}{94,5} \times \frac{1}{229} : 1$, or as $1 : 9\frac{1}{3}$ very nearly; hence, the equatorial : the polar diameter of Jupiter :: $10\frac{1}{3} : 9\frac{1}{3}$, agreeing very nearly with Art. 982.

986. The ratio of the diameters of the earth and planets here determined from the principle of attraction, supposes the earth to be of an uniform density; but as it appears that this does not give accurately the ratio of the diameters of Jupiter, it will be proper to examine, how this determination agrees with the figure of the earth deduced from an actual mensuration. The method of performing this operation, we shall explain from the measurement of a degree of

into the fluxions of their lengths, or as $pO \times \dot{\overline{pO}} : rO \times \dot{\overline{rO}} \times \frac{VO^2}{MO^2}$, whose fluents are as $pO^2 : rO^2 \times \frac{VO^2}{MO^2}$, and when Op and Or become OV and OM, we get the whole pressures as 1 : 1, or they balance each other. It is from assuming this principle, that Sir I. NEWTON proves the ratio of the diameters of the earth, and that the attraction on the surface varies inversely as the radius. Hence, also, any similar parts Op, OP will balance each other.

the meridian at the polar circle in Lapland *, by CLAIRAUT, CAMUS, LE MONNIER, MAUPERTUIS, the Abbé OUTHIER, and M. CELSIUS of Upsal. They set out together from Stockholm, and went to Tornea; from thence they departed
FIG. 222. on July 6, 1736, to survey the country, and fix the proper stations. FIG. 222. represents the triangles, upon which the calculations of the degree of the meridian were founded. T represents Tornea; n, Niwa; K, Kakama; C, Cuitaperi; A, Avasaxa; P, Pullingi; Q, Kittis; N, Njemi; and H, Horrilakero. These are the stations from which they measured the angles, which they found to be as follows.

$CTK =$	24°.	22'.	54",5	$HAP =$	53°.	45'.	56",7
$KTn =$	19.	38.	17,8	$HAC =$	112.	21.	48,6
$TnK =$	87.	44.	19,4	$APH =$	31.	19.	55,5
$HnK =$	73.	58.	5,7	$HPN =$	37.	22.	2,1
$AnH =$	21.	32.	2,5	$NPQ =$	87.	52.	24,3
$HnC =$	31.	57.	3,6	$NQP =$	40.	14.	52,7
$TKn =$	72.	37.	27,8	$QNP =$	51.	53.	4,3
$CKn =$	45.	50.	44,2	$PNH =$	93.	25.	7,5
$HKn =$	89.	36.	2,4	$HNK =$	27.	11.	53,3
$HKN =$	9.	41.	47,7	$CHA =$	36.	42.	3,1
$KCn =$	28.	14.	54,7	$CHn =$	19.	38.	21
$KCT =$	37.	9.	12	$nHK =$	16.	26.	6,3
$KCH =$	100.	9.	56,8	$AHP =$	94.	53.	49,7
$HCA =$	30.	56.	53,4	$PHN =$	49.	13.	9,3

These are the angles as measured with a quadrant of two feet radius, furnished with a micrometer, and reduced to the horizon.

987. Let QM be a meridian line, and from the several stations draw the dotted lines perpendicular to it; and QM is the difference of the latitudes of Tornea and Kittis, which we want to determine in measure of toises. For this purpose, it was necessary to measure some base line, and connect it with the above triangles. A line Bb lying on the ice was therefore accurately measured, and found from the mean of the two measurements, which differed only four inches, to be 7406,86 toises. And the following angles were found by mensuration.

* From the measurement of the degrees of the meridian in France, the longest degree appeared to be that which lay most to the south, from which CASSINI concluded that the earth was an oblong spheroid, or the polar diameter the greatest. To settle therefore the figure of the earth, the measurement of a degree of the meridian at Lapland was undertaken.

$$ABb = 9.\ 22.\ 0$$
$$AbB = 77.\ 31.\ 50$$
$$ACB = 54.\ 40.\ 28$$
$$BAC = 22.\ 37.\ 20$$

988. Now in any triangle, the angles and one side being known, the other sides may be computed; hence, in the triangle ABb, we find $BA = 7242{,}76$; therefore in the triangle ABC, we find $AC = 8659{,}94$ toises. By proceeding thus with the triangles ACH, CHK, CKT, AHP, HNP, NPQ, we find $AP = 14277{,}43$, $PQ = 10676{,}9$, $CT = 24302{,}64$ toises.

989. To determine the position of these triangles in respect to the meridian QM, the passages of the sun at Q through a vertical circle to P and N was observed for many days, which gave the angle $PQM = 28°.\ 51'.\ 52''$, and $NQM = 11°.\ 22'.\ 54''$. Hence, $PQD = 61°.\ 8'.\ 8''$, $APE = 84°.\ 33'.\ 54''$, $ACF = 81°.\ 33'.\ 26''$, $CTG = 69°.\ 49'.\ 9''$; therefore by the resolution of the right angled triangles PDQ, AEP, AFC, CGT, we get

$$\begin{aligned} PD &= 9350{,}45 \text{ toises.} \\ AE &= 14213{,}24 \\ AF &= 8566{,}08 \\ CG &= 22810{,}62 \\ \hline QM &= 54940{,}39 \end{aligned}$$

Which is the arc of the meridian passing through Kittis, and terminated by a perpendicular from Tornea.

990. The same may be computed from the triangles ACH, CHK, CKT, HKN, HNP, NPQ, by the resolution of which, we find $QN = 13564{,}64$, $NK = 25053{,}25$, $KT = 16695{,}84$ toises; also, $QNd = 11°.\ 22'.\ 54''$, $KNL = 86°.\ 7'.\ 12''$, $KTg = 85°.\ 48'.\ 7''$; hence, by the resolution of the right angled triangles, NQd, KNL, Kgt, we get,

$$\begin{aligned} Nd &= 13297{,}88 \text{ toises.} \\ KL &= 24995{,}83 \\ Kg &= 16651{,}05 \\ \hline QM &= 54944{,}76 \end{aligned}$$

The mean of these two values of QM, gives $QM = 54942{,}57$ toises.

991. If we take the observations from n, we may compute the value of QM, from a great variety of triangles. Accordingly, ten other values of QM were computed, the mean of which gave $QM=54922,1$. The observers however (for reasons not assigned in their account of the mensuration) preferred the determinations of QM from the mean of the two first values of it.

992. The next thing to be determined was to find the degrees in the arc QM. This was done by observing the difference of the zenith distances of the same star at Kittis and Tornea. The instrument they used for this purpose was made by Mr. GRAHAM; and the divisions being verified by a micrometer adapted to it, the error never amounted to above 2″ or 3″.

993. The star δ *Draconis* was observed at Kittis and Tornea, and the difference of the zenith distances, after applying the proper corrections, was found to be 57′. 26″,9. By α *Draconis*, it was found to be 57′. 30″,4; the mean of these is 57′. 28″,7; but this is not the arc QM, because the point at Kittis where the observation was made, was 3 toises 4 feet 8 inches more north than the point Q, and the point at Tornea where the observation was made, was 73 toises 4 feet $5\frac{1}{2}$ inches more south than the point T; there must also be added 3,38 toises, because the points T and Q are not in the same meridian; the sum of these is 80,9; this added to 54942,57 gives 55023,47 the length of an arc of 57′. 28″,65; hence, 57′. 28″,65 : 1°:: 55023,47 : 57438 the length of a degree of the meridian at the place of measurement. But we must subtract 16 toises from this, on account of refraction, which MAUPERTUIS neglected; hence, the length of the degree becomes 57422 toises. The latitude of the middle of QM was 66°. 20′. And by comparing the length of this degree with the length 57183 toises of a degree measured by M. PICARD between Paris and Amiens, the latitude of the middle of which was 49°. 22′, the earth was found (995) to be an *oblate* spheroid, and the ratio of the diameters 178 : 179.

994. In taking the angular distance of two objects upon the earth with a quadrant*, if they be at any altitude above or below the surface, that angular distance must be reduced to the horizon, if we want it for the purpose of carrying on a measurement upon the earth's surface; that reduction may be thus made. Let Z be the zenith, MN the horizon, A and B the two objects. Find
FIG. 223. their altitudes MA, NB, and also their distance AB; then in the triangle ZAB, we know all the sides, to find the angle Z, or the arc MN, which is the angular distance of the two objects reduced to the horizon. We avoid this reduction

* For this purpose, the quadrant must be fixed upon a center, so that it may be put into any

however, when we observe with a theodolite having two telescopes which move vertically on an horizontal axis.

995. To find the ratio of the diameters, from the lengths of two degrees in two known latitudes, let Pp be the earth's axis, $EPDp$ any meridian, E the equator, M any place, draw the tangent Mt meeting OE in t, and DOK parallel to it, also Mva perpendicular to Mt, and Mr to OE, and let Mx be the radius of curvature to M. Now by Conics, $Mx=\frac{OK^2}{Ma}$; but $Ma=\frac{OP^2}{Mv}$, and $OK^2=\frac{PO^2\times OE^2}{Ma^2}$; hence, $Mx=\frac{OE^2\times Mv^3}{OP^2}$; but a degree of latitude must vary as Mx, and consequently as Mv^3. Now $Rt=\frac{OE^2}{OP^2}\times\frac{MR^2}{OR}$, and $Rt=\frac{MR^2}{vR}$; hence, $vR=\frac{OP^2}{OE^2}\times OR$. But $OR^2=\frac{OE^2}{OP^2}\times\overline{PO^2-MR^2}$; hence, $vR^2=\frac{OP^4}{OE^2}-\frac{OP^2\times MR^2}{OE^2}$, consequently $Mv^2=\frac{OP^4}{OE^2}-\frac{OP^2\times MR^2}{OE^2}+MR^2$; but if $T=$ the sine of tMR, or of latitude, $MR^2=Mv^2\times T^2$; by substitution therefore, we get the value of $Mv^2=\frac{OP^4}{OE^2\times\overline{1-T^2}+OP^2\times T^2}$; hence, a degree of latitude, or Mv^3, varies inversely as $\overline{OE^2\times\overline{1-T^2}+OP^2\times T^2}^{\frac{3}{2}}$, or $\overline{OE^2-OE^2-OP^2\times T^2}^{\frac{3}{2}}$. But if the spheroid be very nearly a sphere, and $OP : OE :: 1 : 1+d$, then Mv^3 varies inversely as $\overline{1-2dT^2}^{\frac{3}{2}}$, or inversely as $1-3dT^2$ very nearly, d being very small. Hence, if T, t be the sines of any two latitudes, and m, and n represent the lengths of a degree of each respectively, we have $m : n :: 1-3dt^2 : 1-3dT^2$, consequently $d=\frac{m-n}{3mT^2-3nt^2}$. Hence, the ratio of the diameters is as $\frac{3mT^2-3nt^2}{m-n} : \frac{3mT^2-3nt^2}{m-n}+1$. FIG. 218.

996. Now the length of a degree in different latitudes is, according to

		Toises.
MAUPERTUIS - - -	in latitude 66°. 20′ - - - -	57422
CASSINI and de la CAILLE - -	49. 23 - - - -	57069
	45. 0 - - - -	57028
BOSCOVICH - - - - - - - - - -	43. 0 - - - -	56979
JUAN and ULLOA	at the equator - - -	56768
BOUGUER	at the equator - - -	56753
De la CONDAMINE	at the equator - - -	56750
MASON and DIXON - - - - - -	39. 12 - - - -	56888

Hence, we get the following ratios of the diameters of the earth;

Lat. 66°. 20′ and 49°. 23′ - -	give 130 : 131, or 200 : 201,54.
66. 20 and 45. 0 - -	give 150 : 151, or 200 : 201,33.
66. 20 and 43. 0 - -	give 147 : 148, or 200 : 201,36.
66. 20 and 39. 12 - -	give 140 : 141, or 200 : 201,43.
66. 20 and at equat. by BOUGUER	216 : 217, or 200 : 200,92.
49. 23 and at equat. by BOUGUER	312 : 313, or 200 : 200,64.
39. 12 and at equat. by BOUGUER	370 : 371, or 200 : 200,54.
66. 20 and mean at the equator -	217 : 218, or 200 : 200,92.
45. 0 and 49°. 23′ -	give 321 : 322, or 200 : 200,62.
45. 0 and 39. 12 - -	give 112 : 113, or 200 : 201,78.
45. 0 and at equat. by BOUGUER	311 : 312, or 200 : 200,64.
43. 0 and 39°. 12′ -	give 109 : 110, or 200 : 201,83.

997. The mean of all the consequents of the last ratio, is 201,13; hence, the ratio of the diameters from the mean of these twelve comparisons is 200 : 201,13, or reduced to that ratio, the difference of whose terms is unity, it is 177 : 178, which is (993) extremely near to the ratio deduced from the measurement at the polar circle and in France. But the great difference of the results from the different comparisons, show that we cannot depend upon the accuracy of the mean ratio. Indeed other authors have deduced a mean ratio from mensuration, agreeing very nearly with Sir I. NEWTON.

998. In the year 1738, when M. BOUGUER was at Peru, measuring a degree of longitude, it occurred to him to put the Newtonian theory of gravity to the test, by examining the attraction of mountains. This he communicated to his colleague M. de la CONDAMINE, and they made the trial upon the mountain Chimboraco, the attraction of which they judged would be about the 2000th part of the attraction of the whole earth, and therefore they concluded that a plumb line would be drawn out of its vertical situation through an angle of 1′. 43″ towards the mountain; whereas it amounted only to $7\frac{1}{2}''$. But the experiments were made under so many disadvantages, that no great dependance can be placed upon the accuracy of the result. This satisfied them however that the mountain had an attraction, although it was much less than what was expected from its bulk. But it appeared, that this mountain had once been a volcano, and therefore was probably hollow in many places. M. BOUGUER concludes his account thus: "that as in France or in England, a hill may be found of sufficient height for the purpose, and especially if the observer would double the action, by making a station on each side, he should be happy to hear on his return to Europe, that the experiment had been repeated, whether the result

tended to confirm his observations, or to throw some better light upon that enquiry." Accordingly, the Royal Society requested Dr. MASKELYNE to undertake the business, who repeated the experiments upon Schehallien in Scotland, with an excellent zenith sector, made by Mr. SISSON; his MAJESTY very liberally undertaking to defray the expences. From observations of ten stars near the zenith, he found the difference of latitudes of the two stations on the opposite sides of the mountain, to be 54″,6; and by a measurement by triangles, he found the distance of the two parallels to be 4364,4 feet, answering, in that latitude, to an arc of the meridian of 42″,94, which is 11″,6 less than by observation; its half therefore, 5″,8, is the effect of the attraction of the mountain; and from its magnitude, compared with the bulk of the whole earth, Dr. MASKELYNE discovered the mean density of the earth to be about double that of the mountain. Thus, the doctrine of *Universal Gravitation** is firmly established.

* Dr. MASKELYNE deduced the following consequences:

1. It appears from this experiment, that the mountain Schehallien exerts a sensible attraction; therefore, from the rules of philosophising, we are to conclude, that every mountain, and indeed every particle of the earth, is endued with the same property, in proportion to its quantity of matter.

2. The law of the variation of this force, in the inverse *ratio* of the squares of the distances, as laid down by Sir I. NEWTON, is also confirmed by this experiment. For, if the force of attraction of the hill had been only to that of the earth, as the matter in the hill to that of the earth, and had not been greatly increased by the near approach to its center, the attraction thereof must have been wholly insensible. But now, by only supposing the mean density of the earth to be double to that of the hill, which seems very probable from other considerations, the attraction of the hill will be reconciled to the general law of the variation of attraction in the inverse duplicate *ratio* of the distances, as deduced by Sir I. NEWTON from the comparison of the motion of the heavenly bodies with the force of gravity at the surface of the earth; and the analogy of nature will be preserved.

3. We may now, therefore, be allowed to admit this law; and to acknowledge, that the mean density of the earth is at least double of that at the surface, and consequently, that the density of the internal parts of the earth is much greater than near the surface. Hence also, the whole quantity of matter in the earth will be at least as great again as if it had been all composed of matter of the same density with that at the surface; or will be about four or five times as great as if it were all composed of water. The idea thus afforded us, from this experiment, of the great density of the internal parts of the earth, is totally contrary to the hypothesis of some naturalists, who suppose the earth to be only a great hollow shell of matter; supporting itself from the property of an arch, with an immense vacuity in the midst of it. But, were that the case, the attraction of mountains, and even smaller inequalities in the earth's surface, would be very great, contrary to experiment, and would affect the measures of the degrees of the meridian much more than we find they do; and the variation of gravity in different latitudes in going from the equator to the poles, as found by pendulums, would not be near so regular as it has been found by experiment to be.

4. The density of the superficial parts of the earth, being, however, sufficient to produce sensible deflections in the plumb-lines of astronomical instruments, will thereby cause apparent inequalities in the mensurations of degrees in the meridian; and therefore it becomes a matter of great importance to chuse those places for measuring degrees, where the irregular attractions of the elevated parts may be small, or in some measure compensate one another; or else it will be necessary to make allowance for their effects, which cannot but be a work of great difficulty, and perhaps liable to great uncertainty.

Dr. HUTTON in the *Phil. Trans.* for 1778, has calculated the attraction of this mountain from the observations of Dr. MASKELYNE, and found that the density of the mountain was to the mean density of the earth as 5 : 9; now the density of the mountain was found to be to the density of rain water as $2\frac{1}{2} : 1$; hence, the mean density of the earth is to the density of rain water as $4\frac{1}{2} : 1$. The internal parts of the earth are therefore much denser than those at the surface, but in what manner the dense parts are disposed must be uncertain. If we suppose the earth at first to have been in a fluid state, and the different parts to have taken their places according to their gravity, the central parts must be the most dense, the effects of which upon the ratio of the diameters of the earth we shall afterwards state.

999. The vibration of pendulums upon different parts of the earth have been used as a means to determine the ratio of its diameters; for this purpose, we must find the force of gravity upon different parts of its surface. To investigate this, let Pp be the polar, and EQ the equatorial diameter. By Art. 976. the attraction at M perpendicular to the surface varies as $\frac{1}{MO}$. But if $1 =$ the mean radius of the earth, and $1+e=EO$, then (882) $MO=1+e\times\cos. 2MOE$; hence, $\frac{1}{MO}=\frac{1}{1+e\times\cos. 2MOE}=1-e\times\cos. 2MOE=1-e+e\times$ ver. sin. $2MOE$; therefore the increase of attraction from the equator to the poles varies as the versed sine of double the latitude very nearly, or as the square of the sine of the latitude; which is the same ratio as that by which the degrees of latitude increase.

1000. If the time of vibration of a pendulum be given, the length varies as the gravity, and consequently (976) inversely as MO. Hence, the length of a pendulum vibrating seconds, increases as it is carried towards the poles. If therefore the length of a pendulum vibrating seconds in two latitudes could be accurately ascertained, we might ascertain the ratio of the diameters of the earth, the density of the earth being supposed to be uniform. Now it is found by observations, that the length of a pendulum vibrating seconds increases from the equator towards the poles, agreeable to what ought to take place according to our theory; but if we deduce the ratio of the diameters of the earth from the lengths of two pendulums in two latitudes, the conclusions from different experiments are considerably different. This probably arises from the irregularity of the density of the interior parts of the earth; for in that case, the above rule for the variation of the length of the pendulum cannot hold. M. CLAIRAUT observes (*Figure de la Terre*, Sect. 69.) that the variations of the lengths of the pendulum make the ratio of the diameters nearer to a ratio of ility than 229 : 230, indicating a greater density towards the center. On nt of the length of the investigations, we shall refer the reader to the and also the *Phil. Trans.* Vol. XL. and only give the results.

1001. There are two principles upon which we may find the figure of the earth—either by supposing an equilibrium between two canals from the surface meeting at the center—or by supposing the whole force at every point of the surface to act in a direction perpendicular to the surface. There are some spheroids which unite both principles. If the earth was at first in a state of fluidity, it must have acquired such a form as results from the equilibrium of the columns, and from the gravitation acting perpendicularly to the surface. The second principle indeed is absolutely necessary, when the surface is covered with a fluid, as the fluid on the surface could not possibly rest, unless gravity acted perpendicularly to it.

1002. If a fluid spheroid contain a nucleus of uniform density, and the part exterior to the nucleus be also of uniform density; and if $1=$ the semidiameter of the whole solid, $a=$ that of the nucleus, $n=$ the elliptic form of the nucleus (measured by dividing the difference of the diameters by the greater diameter), the elliptic form of the spheroid $=m$, the density of the fluid $=1$, and that of the nucleus $=1+f$, and the centrifugal force at the equator : gravity :: $d : 1$; then M. CLAIRAUT has proved that $m=\frac{6a^5fn+5a^3fd+5d}{10a^3f+4}$ (*Figure de la Terre*, p. 219). Hence, he deduces these conclusions.

"If the spheroid contain a solid nucleus of uniform density, but different from that of the fluid which covers it, and if the figure of the nucleus be similar to that of the spheroid, but of less density, the spheroid will be more elliptical than an homogeneous body would have been.

"If the density of the nucleus be greater than that of the fluid, the spheroid may still be more flat than an homogeneous one would have been, provided the elliptic form of the nucleus be within certain limits.

"If n be negative and greater than $\frac{5a^3fd+5d}{6a^5f}$, m will be negative; that is, the spheroid will be oblong, or the equatorial diameter will be shorter than the polar. It is therefore possible that the earth might have been in the form of an oblong spheroid, under a certain limit of the elliptic form of the nucleus, which in this case would also be an oblong spheroid."

1003. If a fluid cover a solid body composed of an infinite number of elliptic strata of different densities, of which the elliptic form and density vary as any function of the distance from the center, the surface will be a spheroid very nearly, the centrifugal force being supposed to be very small. Also, the variation of weight from the equator to the poles is as the square of the sine of latitude. M. CLAIRAUT also deduces the following conclusions.

"If p be the weight of a body at the equator, P its weight at the pole, E the elliptic form of the body if it had been homogeneous (denoted by the difference of the axes divided by the greater axis), e the real elliptic form; then $\frac{P-p}{p}=2E-e$. Or as the weights of the same body are as the forces of gravity, or as the lengths of pendulums vibrating in the same time, if the lengths of two pendulums vibrating seconds at the pole and equator be L and l; then $\frac{L-l}{l}=2E-e$. If we apply this to the earth, $E=\frac{1}{230}$; hence, $\frac{L-l}{l}=\frac{1}{115}-e$. Now gravity is found to increase from the equator to the pole in a greater ratio than it would if the earth were homogeneous; hence, $\frac{L-l}{l}$ is increased, and consequently the elliptic form is diminished. If gravity had decreased in a less ratio, the flatness of the earth would have increased. This is contrary to the opinion of Sir I. NEWTON. M. CLAIRAUT however observes, that he does not mean to decide against Sir ISAAC's determination, because he cannot be assured of his meaning, when he says, that the density of the earth diminishes from the center towards the circumference; as the parts, instead of being composed of parallel beds, may be conceived to be otherwise arranged, so that the opinion of Sir ISAAC may be true. M. CLAIRAUT shows also that his conclusions differ from those of MAC LAURIN, in consequence of their having solved the problem upon different suppositions. From the whole view of the subject, M. CLAIRAUT observes, that if we make the variation of the density from the center to the surface, and the variation of the axes of the strata, as general as possible, the difference of the axes (from the vibration of pendulums) is found to be less than $\frac{1}{230}$."

1004. The length L of a pendulum vibrating seconds at the equator, and the length l at the pole, may thus be found, from knowing the lengths L', l' in two latitudes whose sines are S, s; for $L-L' : L-l' :: S^2 : s^2$; hence, $L=\frac{s^2\times L'-S^2\times l'}{s^2-S^2}$; and $L-L' : L-l :: S^2 : 1^2$, therefore $l=\frac{L\times S^2-\overline{L-L'}}{S^2}$.

1005. The method of finding the ratio of the diameters of the earth by the vibration of pendulums upon different parts of its surface, is, either by observing how much a pendulum of the same length will gain in a day, as it is carried from the equator towards the pole; or by observing how much it must be lengthened, in order to continue to vibrate in a second.

The following Table shows the seconds gained in one day by a pendulum vibrating seconds in different latitudes, when it remains of the same length.

Lat. Place	Seconds gained.	Lat. Place	Seconds gained.
5°	1″,7	50°	134″,0
10	6, 9	55	153, 2
15	15, 3	60	171, 2
20	26, 7	65	187, 5
25	40, 8	70	201, 6
30	57, 1	75	213, 0
35	75, 1	80	221, 4
40	94, 3	85	226, 5
45	114, 1	90	228, 3

1006. The following Table exhibits the actual length of pendulums vibrating seconds in different latitudes, in French feet, lines, and decimals of a line.

			F.	L.
Under the equator		(Bouguer)	36.	7,07
At Portobello	lat. 9°. 34′	(Bouguer)		7,16
At Pondicherry	lat. 11. 56	(Gentil)		7,26
At Manilla	lat. 14. 34	(Gentil)		7,43
At Madagascar	lat. 17. 40	(Gentil)		7,39
At St. Domingo	lat. 18. 27	(Bouguer)		7,33
At the Isle of France,	lat. 20. 10			7,66
Cape of Good Hope,	lat. 33. 55			8,07
At Malta	lat. 35. 54	(d'Angos)		8,22
At Toulouse	lat. 43. 36	(d'Arquier)		8,40
At Geneva	lat. 46. 12	(M. Mallet)		8,17
At Paris	lat. 48. 50	(De la Caille)		8,55
At Leyden	lat. 52. 9	(M. Lulofs)		8,71
At Petersburg	lat. 59. 56	(M. Mallet)		9,25
At Archangel	lat. 64. 33			9,15
At Pello	lat. 66. 48	(Maupertuis)		9,17
At Lapland	lat. 67. 4	(M. Mallet)		9,17
At Kola	lat. 68. 52			9,31
At Spitzbergen	lat. 79. 50	(Lyons)		9,40

A Table of the Lengths of a Pendulum, in French Lines, vibrating Seconds upon the Surface of the Earth, from the Observations made at Peru, Paris, and Spitsbergen.

Latitude.	Length of the Pendulum.	Latitude.	Length of the Pendulum.	Latitude.	Length of the Pendulum.
Degrees	Lines.	Degrees	Lines.	Degrees	Lines.
0	439,07	30	439,72	60	440,92
1	439,07	31	439,76	61	440,95
2	439,08	32	439,80	62	440,97
3	439,08	33	439,84	63	441,01
4	439,09	[redacted]	439,87	64	441,04
5	439,09	35	439,91	65	441,07
6	439,11	36	439,95	66	441,09
7	439,12	37	440,00	67	441,12
8	439,13	[redacted]	440,04	[redacted]	441,15
9	439,14	39	440,08	69	441,18
10	439,15	40	440,13	70	441,20
11	439,16	41	440,17	71	441,22
12	439,18	[redacted]	440,22	72	441,24
[redacted]	439,20	43	440,27	73	441,26
14	439,22	44	440,31	74	441,29
15	439,24	45	440,35	75	441,31
16	439,27	46	440,40	76	441,33
17	439,30	47	440,45	77	441,35
18	439,32	48	440,49	78	441,36
19	439,35	49	440,54	79	441,37
20	439,38	50	440,58	80	441,38
21	439,41	51	440,62	81	441,39
22	439,44	52	440,65	82	441,40
23	439,47	53	440,68	83	441,41
24	439,50	54	440,71	84	441,42
25	439,53	55	440,75	85	441,43
26	439,56	56	440,79	86	441,43
27	439,59	57	440,82	87	441,44
28	439,63	58	440,85	88	441,44
29	439,67	59	440,88	89	441,44
30	439,72	60	440,92	90	441,45

This Table is computed upon supposition that the increase of the length of the pendulum is as the square of the sine of the latitude; for when the time of vibration is the same, the lengths vary as the forces; therefore the variation of the lengths vary as the variation of the forces, or (999) as the square of the sine of latitude.

1007. The ratio of the diameters of *Jupiter* is less (982) than that given by theory, upon supposition that it is homogeneous, which M. CLAIRAUT shows may happen, if Jupiter have a nucleus denser than the other part of the planet, with a certain elliptic form (1002). It is therefore unnecessary to suppose, with Sir I. NEWTON, that Jupiter is more dense towards its equator, and which (he thinks) may arise from the heat of the sun at those parts. But this is much more likely to be the case with our earth than with Jupiter, and yet Sir ISAAC thinks that the earth is denser towards the center. He appears to have been led into his conjectures, from thinking, that an increase of flatness of the body must be attended with a greater increase of weight in going from the equator to the poles, than if the body had been homogeneous, which (1003) is not necessarily the case. The greater density of the internal parts of the earth, and the variation of gravity upon its surface seem to favour the supposition that the difference of the diameters is less than that which Sir I. NEWTON has determined. M. de la LANDE assumes the difference to be $\frac{1}{300}$ part of the whole.

1008. The horizontal parallax of the moon is as the radius of the earth directly, and the distance of the moon from the center of the earth inversely. The distance of the moon therefore being known, if we know also its horizontal parallax, the radius of the earth will be known. If therefore two radii of the earth in two known latitudes be thus determined, the figure of the earth may be found. But observations of sufficient accuracy to settle this matter, have never been made. It has also been proposed to find the ratio of the diameters of the earth from solar eclipses, as the computation of the parallax of the moon, and consequently the times of the beginning and end of such an eclipse, will vary according as the ratio of the diameters of the earth vary. M. de la LANDE thinks that a difference of $\frac{1}{300}$ of the diameters will make such computations best agree with observations. From a consideration of all the circumstances, it is probable that the difference of the polar and equatorial diameters of the earth, is less than that which is determined by Sir I. NEWTON.

1009. The length of a degree of latitude at the equator, taking the mean of three measures, is $56756\frac{2}{3}$ toises, which multiplied by 6 gives 340540 French feet; and as a Paris foot : an English foot :: 4,263 : 4, a degree at the equator in English feet is 362930,5; hence, the circumference, corresponding to this arc of 1°, is 130654980, the radius of which is 3938,334 miles, which we may consider as the radius of curvature at the equator. Assume $230x$ and $231x$ for

the polar and equatorial radii; then, by Conics, $\overline{\frac{230x^2}{231x}}=3938,334$; hence, $230x=\frac{3938,334\times 231}{230}=3955,4$ miles the polar radius, and $231x=3972,3$ the equatorial radius; and the difference of the two radii $=17,1$ miles, from this ratio of the diameters; also, the mean radius $=3963,95$ which we may take 3964. Now the circumference corresponding to this mean radius is 24907; consequently the length of a degree corresponding to this mean radius, or a mean degree, will be 69,2 miles.

1010. By Art. 995. the length of a degree varies inversely as $1-3dT^2$; now at the equator $T=0$; hence, the length of a degree at the equator : length in any other latitude :: $1-3dT^2 : 1$; thus the length of a degree for any latitude may be computed. Hence also, the increment of a degree from the equator to the pole will increase as $3dT^2$, or as T^2 the square of the sine of latitude. And as the length of a degree must be in proportion to the radius of curvature, the variation of the radius of curvature of an ellipse which is very nearly a circle, must be as the square of the sine of latitude.

FIG. 218. 1011. To find the angle OMv between a perpendicular Mv to the surface and MO drawn to the center. Let $PO=1$, $OE=1+d$, then $OE^2=1+2d$; put $OR=x$, and draw vc perpendicular to MO. Now $Rv=x\times\frac{1}{1+2d}=x\times\overline{1-2d}$, therefore $Ov=2dx=2d\times$ cos. lat. nearly; but $cv=Ov\times$ sin. $vOc=Ov\times$ sin. lat. nearly; therefore $cv=2d\times$ cos. lat. $\times$ sin. lat. nearly $=d\times$ sin. 2 lat. nearly, and this is the sine of vMO to radius Mv which we may consider here as unity. This angle OMv is the reduction of the elevation of the pole in Art. 173.

1012. The earth being supposed to be a sphere, the length of a degree of longitude, as you go from the equator to the poles, decreases as the arcs of the circles parallel to the equator intercepted between any two meridians decrease, which arcs are as their radii, or as the cosines of latitude; therefore, radius : cos. of latitude :: the length of a degree of longitude at the equator : the length of a degree of longitude at that latitude. But as the earth is a spheroid, this rule will want a little correction. Let POp be the axis of the earth, $EPUp$ a meridian, EOU a diameter of the equator; then the latitude of M is MvE, and the angle vMO is given in Art. 1011. therefore $MvE-vMO=MOE$ is known; hence, we get QOM. But (882) if unity represent the mean radius of the earth, and $e=$ the difference between the mean and the greatest or least radii, then will $1+e$ cos. $2MOE=MO$; hence, knowing QOM and MO, we find QM, and this we must use instead of the cosine of the latitude, in order to find the length of a degree of longitude upon the surface of the earth. The length of a degree therefore being known for one latitude, the length for every

2

other latitude may be found. Hence, the calculations for the following Table may be made by this Article and Article 1010.

M. de la PLACE (*Mec. Col. Tom.* ii. *L.* iii. *Ch.* 8.) has shewn, that with the *same* time of revolution of the earth, there are two different spheroids which will preserve their equilibrium; one having the ratio of the polar to the equatorial diameter, as 229 : 230, and the other in the ratio of 1 : 680; which latter is nearly a flat, circular body, having a convex edge.

A Table of the Lengths of a Degree of Latitude and Longitude upon the Earth in French Toises, on supposition that the Difference of its Diameters is the 300th part of the whole.

Height of pole.	Degrees of Latitude.	Differ.	Degrees of Longitude.	Differ.	Height of pole.	Degrees of Latitude.	Differ.	Degrees of Longitude.	Differ.
0°	56747	0	57127	9	45°	57031	10	40462	710
1	56747	1	57118	25	46	57041	10	39732	722
2	56748	1	57093	44	47	57051	10	39030	734
3	56749	1	57049	60	[illegible]	57061	10	38296	746
4	56750	1	56989	78	49	57071	10	37550	757
5	56751	2	56911	95	50	57081	10	36793	769
6	56753	2	56816	112	51	57091	10	36024	780
7	56755	3	56704	129	52	57101	9	35244	791
8	56758	3	56575	147	53	57110	10	34453	801
9	56761	3	56428	163	54	57120	[illegible]	33652	812
10	56764	4	56265	181	55	57129	10	32840	822
11	56768	4	56084	197	56	57139	9	32018	831
12	56772	4	55887	215	57	57148	9	31187	841
13	56776	4	55672	231	58	57157	9	30346	851
14	56780	5	55441	248	59	57166	8	29495	860
15	56785	5	55193	265	60	57174	9	28635	868
16	56790	5	54928	281	61	57183	9	27767	878
17	56795	6	54647	299	62	57192	8	26889	885
18	56801	6	54348	314	63	57200	8	26004	893
19	56807	6	54034	331	64	57208	7	25111	902
20	56813	7	53703	347	65	57215	8	24209	908
21	56820	7	53356	363	66	57223	7	23301	916
22	56827	7	52993	380	67	57230	7	22385	923
23	56834	7	52613	396	68	57237	7	21462	930
24	56841	7	52217	411	69	57244	7	20532	936
25	56848	8	51806	428	70	57251	6	19596	941
26	56856	[illegible]	51378	442	71	57257	[illegible]	18655	948
27	56864	8	50936	459	72	57263	6	17707	954
28	56872	8	50477	473	73	57269	5	16753	958
29	56880	9	50004	489	74	57274	[illegible]	15795	963
30	56889	8	49515	504	75	57280	5	14832	[illegible]
31	56897	9	49011	519	76	57285	4	13864	972
32	56906	9	48492	534	77	57289	4	12892	977
33	56915	10	47958	548	78	57293	4	11915	979
34	56925	9	47410	563	79	57297	[illegible]	10936	984
35	56934	9	46847	577	80	57301	3	9952	986
36	56943	10	46270	591	81	57304	3	8966	989
37	56953	9	45679	605	82	57307	3	7977	992
38	56962	10	45074	619	83	57310	2	6985	994
39	56972	10	44455	633	84	57312	2	5991	995
40	56982	10	43822	646	85	57314	1	4996	998
41	56992	10	43176	659	86	57315	2	3998	[illegible]
42	57002	10	42517	672	87	57317	0	3000	1000
43	57012	9	41845	685	88	57317	1	2000	1000
44	57021	10	41160	698	89	57318	0	1000	1000
45	57031		40462		90	57318		0000	

A TABLE OF FRENCH MEASURES,

Taken from M. de la LANDE's *Astronomy*.

	IN.	LINES.
The foot of the ancient Romans	10.	10,9
The Roman foot of VESPASIAN	11.	1,105
The Greek foot according to AUZOUT	11.	3,8
———— M. le ROY	11.	4,56
The Arabic foot	9.	10,72
The foot of Alexandria	13.	2,9
The English foot	11.	3,1154
The foot of the Rhine, Leyden, and Denmark	11.	7,183
The foot of Bologne	14.	0,6
The foot of Turin	18.	11,7
The foot of Venice	12.	10
The foot of Padua	15.	9,9
The foot of Vienna	11.	8,117
The foot of Sweden	10.	11,75
The foot royal of China	11.	9,9
The *Braccio da Panno* of Florence	21.	6,454
The modern Roman palm	8.	3,033
The palm of Naples	9.	8,15
The vare of Castile	30.	11
The archine of Russia	26.	6,3

	TOISES.
The stade of the ancient Romans	94,693
The stade of the Egyptians	114,13
The verst of Russia	547
The li of China	295
The English mile	830
The Italian mile	958
The modern Roman mile	764
The Roman mile from STRABO	766
———— PLINY	757,5

An English fathom is to a French toise as 1000 to 1065,75, according to General ROY. The toise contains six feet, the foot contains twelve inches, and the inch contains twelve lines.

1013. The figure of the earth is so near that of a sphere, that in estimating its magnitude, we may consider it as a sphere whose radius is the mean radius of the earth. Now this mean radius is 3964 miles; hence, $3964 \times 6{,}28318 = 24907$ miles the circumference; also, $\overline{3964}^2 \times 3{,}14159265 \times 4 = 197459101$ the number of square miles upon the earth's surface; lastly, $\overline{3964}^3 \times 4{,}188790204785 = 260909292265$ the number of cubic miles contained in the earth. Dr. Long estimated the proportion of the land and water upon the surface of the earth, so far as discoveries had then been made, in the following manner. He took the paper off a terrestrial globe, and then cut out the land from the sea, and weighed the two parts; by this means he found the proportion of the water to the land as 349 : 124. The conclusion would be more accurate, if the land were cut out from the sea, *before* the paper was put upon the globe. After all the modern discoveries, this method would probably give the proportion of land to water to a considerable degree of accuracy.

CHAP. XXXIV.

ON THE PRECESSION OF THE EQUINOXES, AND THE NUTATION OF THE EARTH'S AXIS.

Art. 1014. IT has already been observed (148), that the equinoctial points have a retrograde motion of about $50\frac{1}{4}''$ in a year. Sir I. NEWTON was the first who accounted for this motion. Having proved that, from the centrifugal force of the parts of the earth arising from its rotation, the equatorial diameter must be greater than the polar, he proceeded to show, that if we conceive a sphere to be inscribed in the earth, the attraction of the sun and moon upon the excess of the quantity of matter in the earth above that of the sphere will cause a motion in the plane of the equator, and make the points where it intersects the ecliptic go backwards upon it. But although he assigned the true cause of the precession, it is acknowledged that he fell into an error in his investigation of the effect. Without, however, any inquiry relative to the circumstances in which he erred, we shall show how to obtain a true solution from the common principles of motion.

1015. Let S be the sun, $ABDC$ the earth, T its center, EQ the equator, P, p the poles; draw CTB perpendicular to SAD, and join SE, which produce to meet CB in K. Call the radius TE unity, and let the force of the sun on a particle at T be $\frac{1}{ST^2}$; then the force on a particle at $E = \frac{1}{SE^2}$; hence, if we resolve this latter force into two others, one in the direction ET and the other in a direction parallel to TS, we have $SE : ST :: \frac{1}{SE^2}$: the force in the direction parallel to $TS = \frac{ST}{SE^3} = \frac{ST}{ST-EK^3} = \frac{1}{ST^2} + \frac{3EK}{ST^3}$, omitting the other terms of the series on account of their smallness. Hence, the force with which a particle at E is drawn *from* $CB = \frac{3EK}{ST^3}$; consequently the effect of this force in a direction perpendicular to ET will be $\frac{3EK \times KT}{ST^3}$; hence, this force : the force of the sun on a particle at $T :: \frac{3EK \times KT}{ST^3} : \frac{1}{ST^2} :: 3EK \times KT : ST$. Now if $P =$ the periodic time of the earth, $p =$ the periodic time of a body revolving at the earth's surface, then (858) the force of the earth to the sun : force of the body to the earth, or the force of gravity, $:: \frac{ST}{P^2} : \frac{1}{p^2}$; hence, FIG. 224.

the force on a particle at E perpendicular to ET : force of gravity :: $\frac{3EK\times KT\times p^2}{P^3}$: 1.

1016. Let v be the center of gyration, and put M = the quantity of matter in the earth; then the effect of the inertia of M placed at v to oppose the communication of motion is the same as the effect of the inertia of the earth; and hence, $TE^2 : Tv^2$ $(=\frac{2}{5}TE^2) :: M : \frac{2}{5}M$, which is the quantity of matter to be placed at E to have the same effect.

FIG. 225. 1017. Let $PEpQ$ represent the earth, P, p its poles, EQ the equator, $Pepq$ a sphere whose diameter is the axis PTp, TA a radius directed to the sun, and CTB a plane perpendicular to it, PXp a great circle perpendicular to $PEApQ$, and let IR represent a small circle parallel to the equator; take the arc $XL = Xl$, and draw LM, lm, XY perpendicular to the plane CTB. Now (847) the disturbing force of the sun at L, l, in the directions ML, ml are as ML, ml, consequently these forces are the same as they would be if the corpuscles at L, l were orthographically projected upon the plane $CABD$; let us therefore conceive the whole matter in the earth to be thus projected. Draw XN, ln parallel to BC; put $p = 3{,}14159$, &c. a = the mean radius of the earth, $Ee = m$, $r = IX$ on the projection, $TX = v$, s = sin. ATE, c = cos. ATE, the arc XL, or $Xl = z$, and y = sin. XL; then (in the projection) $ln = NX = sy$, $Xn = NL = cy$, $XY = sv$ and $TY = cv$; therefore $LM = sv + cy$, $lm = sv - cy$, $TM = cv - sy$, and $Tm = cv + sy$. Now the forces at L and l in the directions ML, ml, being as ML, ml, their effects to turn the earth about T in the direction BAC are as $ML \times MT$ and $ml \times mT$, or the whole effects are as $\overline{sv+cy} \times \overline{cv-sy} + \overline{sv-cy} \times \overline{cv+sy} = 2cs \times \overline{v^2-y^2}$; therefore the fluxion of the force of all the matter in the circumference IR is as $2cs\dot{z} \times \overline{v^2-y^2}$; hence, the fluxion of the force to turn the earth in the direction CAB is as $2cs\dot{z} \times \overline{y^2-v^2} = 2cs \times \frac{ry^2\dot{y}}{\sqrt{r^2-y^2}} - 2csv^2\dot{z}$, whose fluent, when $y = r$, is $\frac{1}{2}pr \times cs \times \overline{r^2-2v^2}$ the force upon the semicircumference IR; hence, the force on that whole circumference $= pr \times cs \times \overline{r^2-2v^2}$ $=$ (as $r^2 = a^2 - v^2$) $pr \times cs \times \overline{a^2-3v^2}$. Now $a : r :: m : Ii = \frac{mr}{a}$; hence, the fluxion of the force of the annulus $IieE$ is as $pr \times \frac{mr}{a} \times cs\dot{v} \times \overline{a^2-3v^2} = \frac{pmcs}{a} \times \dot{v} \times \overline{a^2-v^2} \times \overline{a^2-3v^2} = \frac{pmcs}{a} \times \overline{a^4\dot{v} - 4av^2\dot{v} + 3v^4\dot{v}}$, whose fluent, when $v = a$, is $\frac{4pmcs}{15} \times a^4$, which is as the whole force on the matter exterior to the sphere $Pepq$, on one side of EQ; hence, $\frac{8pmcs}{15} \times a^4$ is as the whole force of the sun

upon the earth to turn it about in the direction CAB. Now $a+\frac{1}{2}m=TE$, $a-\frac{1}{2}m=Te$; hence, the solid content of the spheroid $=\frac{4}{3}p\times\overline{a-\frac{1}{2}m}\times\overline{a+\frac{1}{2}m}^2=\frac{4}{3}pa^3+\frac{2}{3}pma^2$ very nearly; and the content of the sphere $=\frac{4}{3}p\times\overline{a-\frac{1}{2}m}^3=\frac{4}{3}pa^3-2pma^2$ very nearly; the difference of these is $\frac{8}{3}pma^2$ the content of the part exterior to the sphere; place one fifth of this matter, that is, $\frac{8}{15}pma^2$, at E; then as $EK=ca$, and $KT=sa$, the effect of the sun to turn the matter $\frac{8}{15}pma^2$ at E about $T=\frac{8}{15}pmcsa^4$, which is equal to the effect of the sun upon the whole earth to turn it about its center. Hence, the effect of the sun upon the matter of the earth exterior to the sphere to turn it about its center, is equal to the effect which would be produced if one fifth part of that matter were placed at E.

1018. Put $q=$ the quantity of matter in the earth above that of its inscribed sphere; now (1017) the attraction upon the matter exterior to the sphere would generate an angular velocity about an axis perpendicular to $CABD$, equal to the angular velocity which would be generated in a quantity of matter $=\frac{1}{5}q$ placed at E. Let us therefore suppose the sun's attraction perpendicular to ET to be exerted upon a quantity of matter at $E=$ to $\frac{1}{5}q$, and at the same time to have a quantity of matter to move $=\frac{2}{5}M$, and then (1016, 1017) it appears, that the effect will be the same as the accelerative force of the sun to turn about the earth. Hence, that accelerative force is (1015) equal to $\frac{3EK\times KT\times p^2\times\frac{1}{5}q}{\frac{2}{5}M\times P^3}=\frac{3EK\times KT\times p^2\times q}{2M\times P^3}$. Now if $TE:TP::1:1-r$, then $M:M-q::1:1-2r$, therefore $M:q::1:2r$; hence, $\frac{q}{2M}=r$; consequently the accelerative force $=\frac{3EK\times KT\times p^2\times r}{P^3}$, the force of gravity on the earth being unity.

1019. Let $\dot{z}=$ the arc described by a point of the equator about its axis in an indefinitely small given time, which may therefore represent its velocity; and let $a\dot{z}$ represent the arc described in the same time by a body revolving about the earth at its surface; then $\frac{a^2\dot{z}^2}{2}=$ the sagitta of the arc described by the body in the same time, and consequently $a^2\dot{z}^2=$ the velocity generated by gravity whilst a point of the equator describes $\dot{z}$. Hence, (1018), $1:\frac{3EK\times KT\times p^2\times r}{P^3}::a^2\dot{z}^2:\frac{3EK\times KT\times p^2\times r\times a^2\dot{z}^2}{P^3}$ the velocity of the point E perpendicular to ET, generated by the action of the sun whilst the equator describes $\dot{z}$ about its axis; consequently the ratio of these velocities is as $\frac{3EK\times KT\times p^2\times r\times a^2\dot{z}}{P^3}:1$.

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1020. Let y be an arc described by the sun in the ecliptic to a radius equal to unity, whilst a point of the equator describes $\dot{z}$ about its axis, then (as ap = the time of the earth's rotation, and the arcs described in equal times to equal radii are inversely as the periodic times,) $\frac{1}{P} : \frac{1}{ap} :: \dot{y} : \dot{z} = \frac{P\dot{y}}{ap}$; hence, if v and w be put for the sine and cosine of the sun's declination, the ratio of the velocities in the last Article becomes $\frac{3aprvw\dot{y}}{P} : 1$.

FIG. 226. 1021. Hence, if SAL be the ecliptic to the radius unity, P the place of the sun, SBL the equator, PE the sun's declination, and we take $Ec : dc$ (dc being perpendicular to Ec) $:: 1 : \frac{3aprvw\dot{y}}{P}$, and through d, E, describe the great circle TEM, then will ST be the precession of the equinox, during the time the sun describes $\dot{y}$ in the ecliptic (861); hence, Ed or Ec, or $1 : dc$, or $\frac{3aprvw\dot{y}}{P}$, $::$ sin. $SE : SV = \frac{3aprvw \times \text{sin. } SE \times \dot{y}}{P}$, therefore the sin. STV, or ESP, $: 1 :: SV : ST = \frac{3aprvw \times \text{sin. } SE \times \dot{y}}{P \times \text{sin. } ESP}$.

1022. Now $\frac{v}{\text{sin.} ESP} = \text{sin. } SP$, and $w = \frac{\text{cos. } SP}{\text{cos. } ES}$; hence, $\frac{vw}{\text{sin. } ESP} = \frac{\text{sin. } SP \times \text{cos. } SP}{\text{cos. } ES}$; but $\frac{\text{cos. } ESP}{\text{tan. } ES \times \text{cot. } SP} = 1$; hence, $\frac{vw}{\text{sin. } ESP} = \frac{\text{sin. } SP \times \text{cos. } SP \times \text{cos. } ESP}{\text{cos. } ES \times \text{tan. } ES \times \text{cot. } SP} = \frac{\text{sin. } SP^2 \times \text{cos. } ESP}{\text{sin. } ES}$; consequently $ST = \frac{3apr \times \text{sin. } SP^2 \times \text{cos. } ESP \times \dot{y}}{P} =$ (if x = sine of SP) $\frac{3apr \times \text{cos. } ESP \times x^2\dot{x}}{P \times \sqrt{1-x^2}}$, whose fluent is $\frac{3apr \times \text{cos. } ESP}{2P} \times \overline{y - x\sqrt{1-x^2}}$, and when $x = 1$, it becomes $\frac{3apr \times \text{cos. } ESP \times m}{2P}$ (y being now $= m$ a quadrant) the arc of precession whilst the sun describes 90° from the equinox; and to find the degrees, say $4m : 360° :: \frac{3apr \times \text{cos. } ESP \times m}{2P} : 360° \times \frac{3apr \times \text{cos. } ESP}{8P}$; consequently the precession in a year $= 360° \times \frac{3apr \times \text{cos. } ESP}{2P} = 21''. 6'''$. This would be the precession of the equinox arising from the attraction of the sun, if the earth were solid of an uniform density, and the ratio of the diameters as 229 : 230; but, from what follows, if the greatest nutation of the earth's axis be rightly ascertained, the precession is only about $14\frac{1}{2}''$; which difference between the theory and what is deduced from observation, must arise, either from the fluidity of the earth's surface, an increase of density towards the center, or the ratio of

the diameters being different from that which is here assumed; or probably from all the causes conjointly. This regression of the equinoxes (caused by the plane of the equator moving backwards upon the ecliptic) must necessarily cause the poles of the earth to describe circles about the poles of the ecliptic, in a direction contrary to the order of the signs, setting aside the effect of nutation.

1023. Having ascertained the precession, the corresponding nutation may be immediately found thus. Take $SB = SA = 90°$, and draw the great circle BbA, then BA is the measure of the angle BSA (12); and as we may consider Tb and TA to be each equal to $90°$ without any sensible error, Ab will be the measure of the angle bTA; hence, Bb will measure the difference of the angles BSA, bTA, or the variation of the inclination of the equator to the ecliptic, or the nutation of the axis of the equator. Now $SV : Bb ::$ sin. SE : sin. BE, or cos. SE, :: tan. SE : rad.; also, $ST : SV ::$ rad. : sin. T; hence, $ST : Bb ::$ tan. SE : sin. T; but tan. $SE =$ cos. $T \times$ tan. TP; therefore $Bb = \frac{ST \times \text{sin. } T}{\text{cos. } T \times \text{tan. } TP}$. But $ST = \frac{3apr \times \text{cos. } ESP \times x^2\dot{x}}{P \times \sqrt{1-x^2}}$, and x being the sine of TP, its tangent $= \frac{x}{\sqrt{1-x^2}}$; hence, (considering the angles ESP and T as equal) $Bb = \frac{3apr \times \text{sin. } ESP \times x\dot{x}}{P}$, whose fluent is $\frac{3apr \times \text{sin. } ESP \times x^2}{2P}$ the arc of nutation whilst the sun describes SP; and if $m =$ an arc of $90°$ of the ecliptic from Aries, we have, $4m : 360° :: \frac{3apr \times \text{sin. } ESP \times x^2}{2P} : 360° \times \frac{3apr \times \text{sin. } ESP \times x^2}{8Pm}$ the angle of nutation; and when $x = 1$, we have, $360° \times \frac{3apr \times \text{sin. } ESP}{8Pm}$ for the whole nutation whilst the sun moves from the equinox to the tropic. Whilst the sun moves from the equinox to the tropic, BA is greater than bA, and therefore the inclination of the equator to the ecliptic decreases; but from the tropic to the equinox, BA is less than bA, and therefore the inclination increases. Now the nutation varies as the square of the sine of the sun's longitude; and as x increases till the sun comes to the tropic, and then decreases again until it comes to the equinox, when it is $= 0$, the inclination, from this cause, is least when the sun is at the tropic, and greatest when the sun comes to the equinox, and is then the same as at the preceding equinox.

1024. Hence, the precession during the sun's motion from the equinox to the tropic : the nutation at any time :: cos. ESP : $\frac{\text{sin. } ESP \times x^2}{m}$ :: m : tan. $ESP \times x^2$, and at the tropic, this ratio becomes m : tan. ESP, or as 1,5708

: ,4341. Hence, if we take the whole precession for the time the sun is moving from the equinox to the tropic to be one fourth of $14\frac{1}{2}''$, the nutation at the tropic $= 1''$, which is the greatest nutation arising from the force of the sun, x at the tropic being the greatest. Therefore the nutation from the time the sun leaves the equinox $= 1'' \times x^2 = \frac{1}{2}'' - \frac{1}{2}'' \times \cos. 2y$; and at 45° from the equinoxes, the inclination is the mean, it varying half a second each way from thence. Hence, at the equinoxes, the nutation $= \frac{1}{2}''$, and at the tropic, the nutation $= -\frac{1}{2}''$ from the mean inclination; and the nutation at any time from the mean inclination $= \frac{1}{2}'' \cos. 2y$.

1025. To find the equation of the precession, we have $m : y :: \frac{1}{4}$ of $14\frac{1}{2}'' : 14\frac{1}{2}'' \times \frac{y}{4m}$ the *mean* precession corresponding to the longitude y, upon supposition that the annual precession arising from the force of the sun is $14\frac{1}{2}''$. Also (1022), the *mean* precession corresponding to the longitude m is $\frac{3apr \times \cos. ESP \times m}{2P}$; hence, $m : y :: \frac{3apr \times \cos. ESP \times m}{2P} : \frac{3apr \times \cos. ESP \times y}{2P}$ the *mean* precession corresponding to the longitude y; but the *true* precession in the same time is (1022) $\frac{3apr \times \cos. ESP}{2P} \times \overline{y - x\sqrt{1-x^2}}$, from which take the mean precession, and we have $\frac{3apr \times \cos. ESP}{2P} \times -x\sqrt{1-x^2}$ for the equation of precession, which therefore is to the mean precession as $-x\sqrt{1-x^2} : y :: -2x\sqrt{1-x^2} : 2y :: -\sin. 2y : 2y$. But the mean precession at the same time is $14\frac{1}{2}'' \times \frac{y}{4m}$; hence, $2y : -\sin. 2y :: 14\frac{1}{2}'' \times \frac{y}{4m} : \frac{-14\frac{1}{2}'' \times \sin. 2y}{8m} = -1''.\ 9''' \times \sin. 2y$ the equation of the precession. Hence, this equation is greatest in the middle point between the equinox and tropic, and is there $= 1''.\ 9'''$; and it is to be subtracted in the first and third quadrants of the ecliptic, and added in the second and fourth.

FIG. 227. 1026. Let E be the pole of the ecliptic ♈ L, P the mean pole of the equator, about which as a center describe the circle $abcd$ with a radius $= \frac{1}{2}''$; draw $EcPaC$, and make the angle $aPp =$ twice the motion of the sun in longitude from the equinox, and p will represent the true place of the pole, very nearly.

$$\text{For } Pp = \tfrac{1}{2}'' : Pr :: \text{rad.} : \sin. 2y$$
$$Pr : Cm :: \sin. 23°.\ 28' : \text{rad.}$$
$$\therefore Cm = \frac{\frac{1}{2}'' \times \sin. 2y}{\sin. 23°.\ 28'} = 1'.\ 15'' \times \sin. 2y$$

Which is (1025) very nearly the equation of precession; if we therefore suppose the true pole at p, it throws the tropic from C to m, and the equinox as

much from ♈ to ♈′, and this is the case till $y = 90°$; and as y increases from 90° to 180°, p lies on the other side of EC, and throws the true from the mean equinox on the other side of ♈ to ♈″. This agrees with Art. 1025, where the equation was shown to be negative in the *former* case, denoting thereby the true to lie at ♈′ in respect to ♈ the mean equinox, and positive in the *latter*, denoting the true to lie at ♈″ in respect to ♈ the mean equinox. Also, $Pp = \frac{1}{2}'' : pr ::$ rad. $= 1 : \cos. 2y$, therefore $pr = \frac{1}{2}'' \times \cos. 2y$ the nutation, which diminishes the mean inclination whilst the sun passes from 45° before the tropic to 45° after, and increases it for the other part, agreeable to Art. 1024. Hence, the inequality of the precession, and the nutation, may be represented, by supposing the pole of the equator to describe a circle of 1″ diameter about the mean pole every half year, making the true pole p to set off from a at the equinoxes, and to move *in consequentia* with an angular motion about P which is equal to double the sun's motion in longitude. The apparent distance therefore of every star from the pole of the equator, will be subject to a variation of 1″ twice in a year from this cause. This motion of the pole of the equator, Dr. Maskelyne has mentioned in the Preface to his Tables, published in the first Volume of his excellent Observations.

On the Precession and Nutation arising from the Action of the Moon.

1027. The inequality of the precession of the equinoxes, and the nutation of the earth's axis, arising from the attraction of the moon in different situations of its nodes, was discovered by Dr. Bradley. With his zenith sector fixed at Wanstead, he informs us, that as soon as he discovered the cause, and settled the laws of the aberration of the fixed stars arising from the progressive motion of light, his attention was excited by another *new phænomenon*, that is, an apparent change of declination in some of the fixed stars; which seemed to be sensibly greater about that time, than a precession of 50″ in a year would have occasioned. In consequence of this he continued his observations, and from 1727 to 1732 he found that some of the stars near the solstitial colure had changed their declinations 9″ or 10″ *less* than a precession of 50″ would have produced; and at the same time, that others near the equinoctial colure had altered theirs about the same quantity *more* than such a precession would have occasioned; the north pole of the equator seeming to have approached the stars which come to the meridian with the sun about the vernal equinox and the winter solstice; and to have receded from those which come to the meridian with the sun about the autumnal equinox and summer solstice. Considering these circumstances, and the situation of the ascending node of the moon's orbit at the time when he first began his observations, he suspected that the moon's action upon the equatorial parts of the earth might produce these effects: for

the plane of the moon's orbit being at one time, above 10° more inclined to the plane of the equator, than at another, it was reasonable to conclude, that the part of the whole annual precession, which arises from the moon, would in different years be varied in its quantity; whereas, the plane of the ecliptic, wherein the sun appears, keeping always very nearly the same inclination to the equator, *that* part of the precession which is owing to the sun, must be the same every year. Hence it would follow, that although the *mean* annual precession, proceeding from the joint actions of the sun and moon, was 50″, yet the *true* annual precession might sometimes exceed, and sometimes fall short, of that mean quantity, according to the various situations of the nodes of the moon's orbit. In the year 1727, the moon's ascending node was near the beginning of *Aries*, and consequently its orbit was as much inclined to the equator as it can at any time be; and then the *true* annual precession was found, by his first year's observations, to be *greater* than the *mean*; and the stars near the equinoctial colure, whose declinations are most affected by precession, had changed *theirs* above a tenth part more than a precession of 50″ would have caused. The succeeding years' observations proved the same thing; and in three or four years' time the difference became so considerable, as to leave no room to suspect, that it was owing to any imperfections either in the instrument or observations.

But some of the stars which he observed, that were near the solstitial colure, having appeared to move, during the same time, in a manner contrary to what they ought to have done, by an increase in the precession; and the deviations in them being as remarkable as in the others, he perceived that something more than a mere change in the quantity of the precession, would be requisite to solve the *phænomenon*. Upon comparing his observations of stars near the solstitial colure, that were nearly opposite in right ascension, he found that they were equally affected by this cause; for whilst γ *Draconis* appeared to have moved northward, the small star, which is the thirty-fifth *Camelopardali Hevel.* in the *British Catalogue*, seemed to have gone as much towards the south; which showed, that this apparent motion, in both these stars, might proceed from a nutation of the earth's axis. Upon making the like comparison between the observations of other stars that lie nearly opposite in right ascension, whatever their situations were with respect to the cardinal points of the equator, it appeared that their change of declination was nearly equal, but contrary, and such as a nutation of the earth's axis would effect.

The moon's ascending node being got back towards the beginning of *Capricorn* in the year 1732, the stars near the equinoctial colure appeared, about that time, to change their declinations no more than a precession of 50″ required; whilst some of those near the solstitial colure altered theirs above 2″ in a year less than they ought. Soon after, he perceived the annual change of declination of the former to be diminished, so as to become *less* than 50″ of pre-

cession would cause; and it continued to diminish till the year 1736, when the moon's ascending node was about the beginning of *Libra*, and its orbit had the *least* inclination to the equator. By this time, some of the stars near the solstitial colure had altered their declinations 18″ *less*, since the year 1727, than they ought to have done from a precession of 50″. For γ *Draconis*, which in those nine years should have gone about 8″ more *southerly*, was observed in 1736 to appear 10″ more *northerly* than it did in 1727.

As this appearance of γ *Draconis* indicated a diminution of the inclination of the earth's axis to the ecliptic; and as it had been observed that that inclination was regularly diminished, if this phænomenon depended upon such a cause, and amounted to 18″ in nine years, the obliquity of the ecliptic would, at that rate, alter a minute in 30 years, which was faster than had been found from observations. He therefore thought that some part of this motion, if not the whole, might arise from the action of the moon upon the equatorial parts of the earth, which, he conceived, might cause a libratory motion of the earth's axis. But as he was unable to judge, from only nine years observations, whether the axis would entirely recover the same position that it had in 1727, he found it necessary to continue his observations through a whole period of the moon's nodes; at the end of which he found that the stars returned into the same positions again, as if there had been no alteration in the inclination of the earth's axis, which convinced him that he had rightly assigned the cause of the *phænomenon*; and the very near agreement of his observations upon different stars with this theory, through a revolution of the moon's node, indisputably confirmed it. The following Table contains his observations upon γ *Draconis* for 20 years. The first column contains the times of the observations; the second shows the number of seconds the star was south of 38°. 25′, that being the point of the limb of the sector with which this star was compared; the third contains the alteration of the polar distance, which the *mean* precession, at the rate of one degree in 71½ years, would cause in this star, from March 27, 1727, to the day on which the observation was taken; the fourth shows the aberration of light; the fifth, the equations arising from the aforementioned hypothesis; and the sixth gives the *mean* distance of the star from the point with which it was compared, found, by collecting the several numbers, according to their signs, in the third, fourth and fifth columns, and applying them to the *observed distances* contained in the second.

If the observations had been perfectly exact, and the several equations of their *due* quantity, then all the numbers in the last column would have been equal; but since they differ a little from one another, if the *mean* of all be taken, and the extremes are compared with it, we shall find no greater difference, than what may be supposed to arise from the uncertainty of the observations themselves; it no where amounting to more than 1½″. The hypothesis

therefore seems, in this star, to agree extremely well with the observations here set down; but as I had made 300 of it, I took the trouble of comparing each of them with the hypothesis: and although it might have been expected that, in so large a number, some great errors would have occurred, yet these are very few; viz. only eleven, that differ from the mean of these so much as 2″; and not one that differs so much as 3″. This surprising agreement, therefore, in so long a series of observations, taken in all the various seasons of the year, as well as in the different positions of the moon's nodes, seems to be a sufficient proof of the truth, both of *this* hypothesis, and also of *that* which I formerly advanced, relating to the aberration of light; since the polar distance of this star may differ, in certain circumstances, almost a minute, viz. 56½″, if the corrections resulting from both these hypotheses are neglected; whereas, when those equations are rightly applied, the mean place of the star comes out the same, as nearly as can be reasonably expected. Thus far Dr. Bradley.

γ *DRACONIS.*	South of 38°. 25′	Precession.	Aberration	Nutation.	Mean Distance.
1727 September 3	70″,5	− 0″,4	+19″,2	−8″,9	80″,4
1728 March 18	108,7	− 0,8	−19,0	−8,6	80,3
—— September 6	70,2	− 1,2	+19,3	−8,1	80,2
1729 March 6	108,3	− 1,6	−19,3	−7,4	80,0
1729 September 8	69,4	− 2,1	+19,3	−6,9	80,2
1730 September 8	68,0	− 2,9	+19,3	−3,4	80,5
1731 September 8	66,0	− 3,8	+19,3	−1,0	80,5
1732 September 6	64,3	− 4,6	+19,3	+2,0	81,0
1733 August 29	60,8	− 5,4	+19,0	+4,8	79,2
1734 August 11	62,3	− 6,2	+16,9	+6,9	79,9
1735 September 10	60,0	− 7,1	+19,3	+7,9	80,1
1736 September 9	59,3	− 8,0	+19,3	+9,0	79,6
1737 September 6	60,8	− 8,8	+19,3	+8,5	79,8
1738 September 13	62,0	− 9,6	+19,3	+7,0	78,7
1739 September 2	66,6	−10,5	+19,2	+4,7	80,0
1740 September 5	70,8	−11,3	+19,3	+1,9	80,7
1741 September 2	75,4	−12,1	+19,2	−1,1	81,4
1742 September 5	76,7	−12,9	+19,3	−4,0	79,1
1743 September 2	81,6	−13,7	+19,1	−6,4	80,6
1745 September 3	86,3	−15,4	+19,2	−8,9	81,2
1746 September 17	86,5	−16,2	+19,2	−8,7	80,8
1747 September 2	86,1	−17,0	+19,2	−7,6	80,7

The conclusion derived from these observations is, that the gradual diminution of the obliquity of the ecliptic to the equator does not arise from an alteration in the position of the earth's axis, but from some alteration in the ecliptic itself; because the stars at the end of the period of the moon's node, appeared in the same places with respect to the equator, as they ought to have done, if the earth's axis had retained the same inclination to an invariable plane.

Dr. BRADLEY, in his observation upon *η Ursæ Majoris*, in the years 1740 and 1741, found that they gave the polar distance 3″ greater than the *mean* of the other years; and observes that, had there been only a single observation in each of these years, part of this difference might have been supposed to have arisen from their uncertainty; but as there were eight observations taken within a week in 1740, which agree well with each other; and three were made within twenty days in 1741 which likewise correspond with each other, he was inclined to think that the aforementioned difference must be owing to some other cause; and he suspected that the position of the moon's apogee, as well as of its nodes, had some relation to this apparent motion.

1028. Dr. BRADLEY communicated his observations to Mr. MACHIN, who soon after sent him a Table of the annual precession and the corresponding nutation, in the various situations of the moon's nodes. These were calculated upon supposition that the pole of the equator, during a period of the moon's nodes, moved round in the periphery of a circle of 18″ diameter, having the center 23°. 29′ from the pole of the ecliptic, that circle having an angular motion of 50″ about the same pole. The north pole of the equator was conceived to be in that part of the small circle which is furthest from the north pole of the ecliptic, when the moon's ascending node is in the beginning of Aries; and the opposite point of it, when the same node is in the beginning of Libra. But Dr. BRADLEY afterwards observed, that the calculations would agree better with observations, if the true pole of the equator described an ellipse instead of a circle, whose minor axis is about 16″, and lying parallel to the ecliptic. This is confirmed by theory. From all the observations of Dr. BRADLEY, Dr. MASKELYNE fixed the whole nutation at 19″,1. In the Table we have computed, we have assumed it 19″.

1029. Sir I. NEWTON had taken notice of the nutation of the earth's axis; and Mr. FLAMSTEAD in his *Hist. Cœl.* Vol. III. p. 113, informs us that he attempted to discover its quantity, but found his instruments not sufficiently accurate for the purpose. M. de la LANDE also observes, that the following passage was found in the manuscripts of ROMER: *Sed de altitudinibus non perinde certus reddebar, tam ob refractionum varietatem quam ob aliam non dum liquido perspectam causam; scilicet per hos duos annos, quemadmodum et alias, expertus sum esse quandam in declinationibus varietatem quæ nec refractionibus nec*

parallaxibus tribui potest, sine dubio ad vacillationem aliquam poli terrestris referendam, cujus me verisimilem dare posse theoriam, observationibus munitam, spero. Notwithstanding however the nutation of the earth's axis had been so long suspected, the discovery of the cause and quantity thereof was reserved for Dr. BRADLEY. We proceed now to consider, how all this agrees with the conclusions deduced from the theory of gravity.

FIG. 224. 1030. The effect (853) of the body at S upon a particle at E varies as the cube of the apparent diameter of S seen from T and the density of S conjointly; therefore as the apparent diameters of the sun and moon seen from the earth may be considered as equal, the effects of the sun and moon upon a particle E of the earth, given in position, will be as their densities. Hence, if $m:1::$ density of the moon : density of the sun, and $A=$the whole precession generated by the sun in one year, mA would represent the mean precession by the moon in the same time, if its orbit had the same inclination to the equator as the ecliptic has. But this is not the case. To find therefore the effect produced by the moon, let $fFNeE$ represent the orbit of the moon intersecting the

FIG. 228. ecliptic $♈CL$ in N, and let $♈ELF$ be the position of the equator when the moon passed it at F, and $aebf$ when it passed it again at e; bisect $♈L$ in C, and draw the great circle CrR perpendicular to $♈L$. Now (1022) the precession, *cæteris paribus*, varies as the cosine of the angle which the orbit of the body makes with the equator; hence, $\cos.♈ : \cos. E :: mA : mA \times \frac{\cos. E}{\cos. ♈}$ the mean precession arising from the moon in a year; hence, if $t=$the time the moon is moving in its orbit from F to e, 1 year : $t :: mA \times \frac{\cos. E}{\cos. ♈} : tmA \times \frac{\cos. E}{\cos. ♈}$ the mean precession Ee caused by the moon in respect to its own orbit. Now to reduce this to the ecliptic, and find the precession during a revolution of the moon's node, we shall follow and explain the method given by Mr. T. SIMPSON in his *Miscellaneous Tracts*, it appearing to be as simple as the nature of the subject will admit of.

1031. As the inclination of the earth's axis at the end of every half revolution, on the return of the sun or moon again to the equator, is (1023) restored to its former quantity on the respective orbits, the angles E, e, F, f are equal, and the triangles DEe, DFf are similar and equal in all respects; therefore $DE+De$ being $=DE+DF=$a semicircle, both DE and De may be taken as quadrantal arcs. Now sin. ED ($=$rad.) : sin. e, or E, :: sin. Ee, or Ee, : sin. eDE, or eDE $=$ sin. $E \times Ee$; also, sin. a, or $♈$: sin. $♈D$, or cos. $♈E$, :: sin. eDE, or eDE, : sin. $a♈$, or $a♈ = \frac{\cos. ♈E}{\sin. ♈} \times eDE = \frac{\cos. ♈E}{\sin. ♈} \times \sin. E \times Ee$ $= tmA \times \frac{\sin. E \times \cos. E \times \cos. ♈E}{\sin. ♈ \times \cos. ♈}$ the precession upon the ecliptic in the

time t. Again, sin. Υ, or rad. : sin. DR, or ΥE :: sin. eDE, or eDE : sin. Re, or $Rr = \text{sin.}\ \Upsilon E \times eDE = \text{sin.}\ \Upsilon E \times \text{sin.}\ E \times Ee = tmA \times \frac{\text{sin.}\ \Upsilon E \times \text{sin.}\ E \times \text{cos.}\ E}{\text{cos.}\ \Upsilon E}$ the corresponding nutation. But sin. E : sin. ΥN :: sin. N : sin. ΥE; hence, sin. $\Upsilon E \times \text{sin.}\ E = \text{sin.}\ \Upsilon N \times \text{sin.}\ N$, consequently the nutation $Rr = tmA \times \frac{\text{sin.}\ \Upsilon N \times \text{sin.}\ N \times \text{cos.}\ E}{\text{cos.}\ \Upsilon E}$. Hence, (from the former expression for the nutation), the nutation : the corresponding precession :: sin. Υ : $\frac{\text{cos.}\ \Upsilon E}{\text{sin.}\ \Upsilon E} = \text{cotan.}\ \Upsilon E$. Having determined the precession and nutation, for any given position of the moon's node, we proceed next to determine the same, during a revolution of the node.

1032. Let $\Upsilon N = z$, its sine $= x$, cosine $= y$, the sine of $N \Upsilon E = a$, cosine $= b$, sine of $N = c$, its cosine $= d$, $\Upsilon CL = e$, and the time of half a revolution of the node $= R$, and let ΥQ be perpendicular to NE. Then, cos. $N\Upsilon$, or y : rad. $= 1$ :: cotan. N, or $\frac{d}{c}$: tan. $N \Upsilon Q = \frac{d}{cy}$; let this be denoted by h; then the secant of the same angle $= \sqrt{1+h^2}$, its sine $= \frac{h}{\sqrt{1+h^2}}$, and cosine $= \frac{1}{\sqrt{1+h^2}}$, hence, the sine of the difference of the two angles $N \Upsilon Q$ and $N \Upsilon E$ will be $\frac{hb-a}{\sqrt{1+h^2}}$, and the cosine $= \frac{ha+b}{\sqrt{1+h^2}}$. Also, sin. $N \Upsilon Q$: sin. $E \Upsilon Q$ :: cos. N : cos. $E = \frac{\overline{hb-a} \times d}{h} = bd - \frac{ad}{h} = bd - acy$, and cos. $N \Upsilon Q$: cos. $E \Upsilon Q$ :: cot. $N\Upsilon$, or $\frac{y}{x}$, : cotan. $\Upsilon E = \overline{b+ha} \times \frac{y}{x} = \frac{ad+bcy}{cx}$, because $h = \frac{d}{cy}$. But (1031) the sine of nutation for the time $t = tmA \times \frac{\text{sin.}\ \Upsilon N \times \text{sin.}\ N \times \text{cos.}\ E}{\text{cos.}\ N \Upsilon E} = tmA \times \frac{cx \times \overline{bd-acy}}{b}$, and the time t, (whilst the node describes $\dot{z}$) : R :: $\dot{z}$: e, therefore $t = R \times \frac{\dot{z}}{e} = \frac{R\dot{x}}{e\sqrt{1-x^2}}$; hence, the nutation whilst the node describes $\dot{z}$ is (writing $\sqrt{1-x^2}$ for y) $= mAR \times \frac{c}{eb} \times \overline{\frac{bdx\dot{x}}{\sqrt{1-x^2}} - acx\dot{x}}$, whose correct fluent $mAR \times \frac{c}{cb} \times \overline{bd - bd\sqrt{1-x^2} - \frac{1}{2}acx^2}$ is the nutation, or decrease of the inclination of the equator to the ecliptic, caused by the moon, for the time the node has moved from Υ to N. Next, with regard to the corresponding precession, the increase thereof being (1031) in proportion to the decrement of

nutation as the cotangent of ♈ E to the sine of N♈E, or as $\frac{ad+bcy}{cx}=\frac{ad+bc\sqrt{1-x^2}}{cx}$ to a, therefore the fluxion of the precession $= mAR \times \frac{1}{abe} \cdot \times \overline{\frac{abd^2\dot{x}}{\sqrt{1-x^2}} + \overline{b^2-a^2} \times cd\dot{x} - abc^2\dot{x}\sqrt{1-x^2}}$, whose fluent $mAR \times \frac{1}{abe} \times \overline{abd^2z + \overline{b^2-a^2} \times cdx - \frac{1}{2}abc^2z - \frac{1}{2}abc^2x\sqrt{1-x^2}}$ is the true precession.

1033. Hence, at the end of half a revolution of the node, or when it arrives at Libra L, $x=0$, and $z=e$; therefore the nutation $= mAR \times \frac{2cd}{e}$; and the precession $= mAR \times \overline{1-\frac{3}{2}c^2}$, because $c^2+d^2=1$. Hence, the whole quantity of nutation during half a revolution of the node from ♈ : the corresponding quantity of precession :: $\frac{2cd}{e}$: $1-\frac{3}{2}c^2$ :: 10 : 174 very nearly.

1034. Hence, the mean precession of the equinox from the action of the moon is to what it would be if the moon's orbit coincided with the ecliptic as $1-\frac{3}{2}c^2$: 1. But if the moon's orbit coincided with the ecliptic, the effect of the moon would (1030) be to that of the sun in the ratio of their densities, therefore the mean precession of the moon : that from the sun, in a compound ratio of $1-\frac{3}{2}c^2 = 0{,}988$: 1, and of the density of the moon : the density of the sun; consequently the density of the moon : the density of the sun :: precession from the moon : precession from the sun $\times$ 0,988.

1035. As the precession in half a revolution of the node $= mAR \times \overline{d^2-\frac{1}{2}c^2}$, we have, $e : z :: mAR \times \overline{d^2-\frac{1}{2}c^2} : mAR \times \frac{z}{e} \times \overline{d^2-\frac{1}{2}c^2}$ the *mean* precession whilst the node moves over the arc z, which subtracted from the *true* precession (found in Article 1032), we get $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cdx$ for the *equation of the equinoxes*, neglecting $-\frac{1}{2}abc^2x\sqrt{1-x^2}$ as never amounting to $\frac{1}{4}''$. Hence, the equation, when the node has made one fourth of a revolution, will be $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cd$, which is to the greatest nutation $mAR \times \frac{2cd}{e}$, during half a revolution of the node, as b^2-a^2 : $2ab$, or as 1 : $\frac{2ab}{b^2-a^2}$, that is, as radius : the tangent of double the inclination of the equator to the ecliptic.

1036. As c is but very small, we may neglect the last term in the expression for the nutation, as well as in the equation of precession, without any considerable error; hence, the nutation becomes $mAR \times \frac{cd}{e} \times \overline{1-\sqrt{1-x^2}}$, which varies as $1-\sqrt{1-x^2}$ the versed sine of the nodes' true longitude; and the equation of precession $mAR \times \frac{1}{abe} \times \overline{b^2-a^2} \times cdx$ varies as x the sine of the node's true longitude.

1037. If the annual precession arising from the sun be taken $= 21''. 6'''$ as in Art. 1022. and the whole precession $= 50''$, then the part arising from the action of the moon will be $28''. 54'''$; hence (1034), the density of the moon : density of the sun :: $28''. 54''' : 21''. 6''' \times 0,988 = 20''. 8'''$, which ratio does not agree, either with the proportion deduced from the tides, or with the accurate observations of Dr. Bradley. The best method of settling this point, is from the greatest nutation.

1038. The nutation, during half a revolution of the moon's node from Aries, Dr. Maskelyne fixed at $19'',1$, and which we shall here assume $19''$; hence (1033), $10 : 174 :: 19'' : \frac{174 \times 19''}{10}$ the precession from the moon during that time, which we may take equal 9,31 years; hence, the mean precession from the moon in one year $= \frac{174 \times 19''}{10 \times 9,31}$; therefore if we take the whole precession in a year to be $50\frac{1}{4}''$, we have $50\frac{1}{4}'' - \frac{174 \times 19''}{10 \times 9,31} = \frac{10 \times 9,31 \times 50\frac{1}{4}'' - 174 \times 19''}{10 \times 9,31}$ for the part of the precession arising from the sun. Hence (1034), the density of the moon : the density of the sun :: $174 \times 19'' : \overline{10 \times 9,31 \times 50\frac{1}{4}'' - 174 \times 19''} \times 0,988 :: 2,44 : 1$. The part therefore of the precession arising from the action of the moon $= 35''. 39'''$, and that of the sun $= 14''. 36'''$; and the greatest equation (1035) of the precession arising from the moon $= 17'',7$.

1039. The equation of the precession (1036) varies as x the sine of the node's distance from ♈ measured contrary to the order of the signs; but if $L =$ the longitude of the node, sin. $x = -$sin. L; therefore *rad.* : $-$*sin.* L :: $17'',7$: *the equation of precession* $= -$ *sin.* $L \times 17'',7$; hence, the equation is to be *subtracted* from the mean precession when L is *less* than six signs, and *added*, when *greater*.

1040. As (1036) the decrease of the inclination of the equator to the ecliptic, from the time the node coincides with ♈, is as the versed sine of the node's distance from that point, the inclination must be at its mean value when the node is in the solstice; hence, the difference between the mean and true values will be as the difference between the versed sine and radius, or as the

cosine of the node's distance from ♈; therefore to find the nutation at any time, say, *rad.* : *cos.* z :: 9",5 : *the nutation* = 9",5 × *cos.* z; which must be *added*, when the node is in the *ascending* signs ♑, ♒, ♓, ♈, ♉, ♊, but *subtracted*, when in the *descending* signs, ♋, ♌, ♍, ♎, ♏, ♐, to get the true obliquity of the equator to the ecliptic; hence, *the equation of the obliquity of the ecliptic* = − 9",5 × cos. z.

The following Table shows the Equation of the Precession, and the Equation of the Obliquity of the Ecliptic, arising from the Action of the Moon, both computed by the Rules here given.

The Equation of the Precession of the Equinox.				
☽'s ☊ from ♈	Sig. O. – Sig. VI. +	Sig. I. – Sig. VII. +	Sig. II. – Sig. VIII. +	
0°	0″,0	8″,8	15″,3	30°
5	1, 5	10, 1	16, 1	25
10	3, 0	11, 4	16, 7	20
15	4, 5	12, 5	17, 2	15
20	6, 0	13, 6	17, 5	10
25	7, 4	14, 5	17, 7	5
30	8, 8	15, 3	17, 7	0
	Sig. V. – Sig. XI. +	Sig. IV. – Sig. X. +	Sig. III. – Sig. IX. +	☽'s ☊ from ♈

The Equation of the Obliquity of the Ecliptic.				
☽'s ☊ from ♈	Sig. O. + Sig. VI. –	Sig. I. + Sig. VII. –	Sig. II. + Sig. VIII. –	
0	9″,5	8″,2	4″,7	30°
5	9, 4	7, 8	4, 0	25
10	9, 3	7, 3	3, 3	20
15	9, 2	6, 7	2, 4	15
20	9, 0	6, 1	1, 7	10
25	8, 7	5, 5	0, 9	5
30	8, 2	4, 7	0, 0	0
	Sig. V. – Sig. XI. +	Sig. IV. – Sig. X. +	Sig. III. – Sig. IX. +	☽'s ☊ from ♈

Ex. Let the distance of the ascending node of the moon from the firs point of Aries be $4^s. 18°. 40'$; to find the equations of the precession and ob liquity.

The equation of the precession for $4^s. 15°$. is $-12'',5$, and for $4^s. 20°$ it i $-11'',4$; hence, $5° : 3°. 40' :: 1'',1 : 0'',8$, which taken from $-12'',5$ leave $-11'',7$ the equation of precession. Also, the equation of the obliquity fo $4^s. 15°$ is $-6'',7$, and for $4^s. 20'$ it is $-7'',3$; hence, $5° : 3°. 40' :: 0'',6 : 0'',4$ which added to $-6'',7$ gives $-7'',1$ the equation of the obliquity. Thes equations, with those arising from the sun (1024, 1025), applied to the *mea* precession and obliquity, give the *true*, so far as regards the displacement of th equator.

1041. The *nutation* arising from the moon, and the *equation of the precession* may be both together represented thus. Let P be the mean place of the pol
FIG. 229. of the equator ♈ V, E the pole of the ecliptic ♈ Y; in EP♋ take $PA = PB = 9'',5$ half the greatest nutation, and describe the circle $BDAG$, an draw DPG perpendicular to AB, and take $PC : PD ::$ cos. $2EP$: cos. $EP ::$ 6828 : 9173; describe the ellipse $BCAF$, and the true place of the pole o the equator will always be found in the circumference of this ellipse; an make $APS =$ the distance of the moon's ascending node from ♈, draw Sp perpendicular to AB, and p is the true place of the pole. For draw the grea circles EC, EpT; then it is manifest, that EA and EB will be the greates and least distances of the two poles, AB being $= 19''$ the greatest nutation; and rad. : cos. APS, or cos. y, :: $PS = 9'',5 : PR = 9'',5 \times$ cos. $z =$ (1040) the nutation, therefore ER, or Ep very nearly, is the true distance of the poles. Also, by *Construction*, $CP : AB :: \frac{1}{2}$ cos. $2EP$: cos. $EP :: \frac{b^2-a^2}{2} : b$; and by *Spherics*, sin. PEC : sin. PC, or (on account of their smallness) $PEC : PC$, :: rad. : sin. EC, or EP, or a; hence, by compounding these two proportions, $PEC : AB :: \frac{b^2-a^2}{2} : ab :: b^2 - a^2 : 2ab$, which proportion to find PEC is the same as that in Art. 1035. to determine the greatest difference of the true and mean longitude, consequently PEC represents that difference. Hence it follows, that the angle REp will express the difference of the mean and true longitudes, at the given position of the node; for rad. : sin. $APS :: PD : RS :: PC : Rp ::$ the angle $PEC : REp$, as it ought to be by Art. 1036. As therefore Ep is the true distance of the poles, and REp expresses the difference between the mean and true longitudes, p must be the true place of the pole. Now as the inclination of the equator to the ecliptic decreases from the time the ascending node of the moon's orbit leaves *Aries* till it gets back to

Libra; therefore the pole of the equator during that time has moved from A to B; and as the equation of the precession for that time is to be added to the mean precession by Art. 1039. it makes the true precession greater than the mean, and therefore the true place of the pole must have been behind the mean place, consequently the pole has moved from A in the direction ACB. This matter therefore may be simply explained thus. If the precession were uniform, and there was no nutation, then the true place of the pole P would (1021) describe a circle about E contrary to the order of the signs, with an angular velocity equal to that of the precession; but the precession is not uniform, and there is also a nutation, in consequence of which, the true motion of the pole of the equator is in the ellipse $ACBF$, whilst the center P is carried about E as above-mentioned. The motion of the pole in the ellipse, therefore, takes into consideration the effect of the equation of precession and the nutation.

1042. Let s be the place of a star, ΥV the equator to the mean pole P, $\Upsilon' W$ the equator to the true pole p; draw the great circles $Psma$, $psnbd$, and let Υc be perpendicular to $\Upsilon' W$, and pr to Ps. Put $v = APp$, $z = APS$, $r = APs$, $d =$ the declination of the star, $a =$ its right ascension; then $sPp = r - v$; also, by the property of the ellipse, $PD = 9'',5 : PC = 7'',07 :: RS : Rp ::$ tan. z : tan. v, therefore tan. $v = \frac{7'',07}{9'',5} \times$ tan. z; and $PD = 9'',5 : PC = 7'',07 :: RS : Rp = \frac{7'',07}{9'',5} \times RS$; but rad. $= 1 :$ sin. $z :: PS = 9'',5 : RS = 9'',5 \times$ sin. z; hence, $Rp = 7'',07 \times$ sin. z; also, cos. v : rad. $= 1 :: PR = 9'',5 \times$ cos. z (1040) : $Pp = 9'',5 \times \frac{\text{cos. } z}{\text{cos. } v}$.

1043. The *mean* right ascension is $\Upsilon a = \Upsilon b + ba$*, and the *true* right ascension is $\Upsilon' d = \Upsilon' c + cd = \Upsilon' c + \Upsilon b$; hence, the variation of the right ascension $= \Upsilon' c - ba = \Upsilon\Upsilon' \times$ cos. $23°. 28' - ba = T♋ \times$ cos. $23°. 28' - ba$. Now $T♋ = \frac{Rp}{\text{sin. } 23°. 28'} = \frac{7'',07 \times \text{sin. } z}{\text{sin. } 23°. 28'}$; hence, $T♋ \times$ cos. $23°. 28' = 7'',07 \times$ sin. z $\times$ cot. $23°. 28' = 16'',29 \times$ sin. z. Also, sin. sPp : sin. sp :: sin. Psp : sin. Pp, and as Psp and Pp are very small, we may put the quantities themselves for their sines; hence, $Pp \times \frac{\text{sin. } \overline{r - v}}{\text{sin. } sp} = Psp = bsa$; therefore $ab = Pp \times \frac{\text{sin. } \overline{r - v} \times \text{cos. } sp}{\text{sin. } sp} = Pp \times \text{sin. } \overline{r - v} \times \text{tan. } d$. Hence, the variation of right

* The arc ab is called the *deviation* in right ascension, mn the *deviation* in longitude, and Pr the *deviation* in north polar distance; $\Upsilon\Upsilon'$ is called the *equation of the equinoxes* in longitude, and $\Upsilon' c$ the *equation of the equinoxes* in right ascension.

ascension $= 16'',29 \times \sin. z - Pp \times \sin. \overline{r-v} \times \tan. d = 16'',29 \times \sin. z - 9'',5 \times \frac{\cos. z}{\cos. v} \times \sin. \overline{r-v} \times \tan. d = 16'',29 \times \sin. z - 9'',5 \times \frac{\cos. z}{\cos. v} \times \sin. \overline{r-v} \times \tan. d =$ $16'',29 \times \sin. z - 9'',5 \times \frac{\cos. z}{\cos. v} \times \tan. d \times \overline{\sin. r \times \cos. v - \sin. v \times \cos. r} =$ (as the tangent $v = \frac{7'',07}{9'',5} \times$ the tangent z) $16'',29 \times \sin. z -$ the tangent $d \times \overline{9'',5 \times \sin. r \times \cos. z - 7'',07 \times \cos. r \times \sin. z}$.

$$= - \left\{ \begin{array}{l} 4'',75 \times \sin. \overline{r-z} + 4'',75 \times \sin. \overline{r+z} \\ 3'',53 \times \sin. \overline{r-z} - 3'',53 \times \sin. \overline{r+z} \end{array} \right\} \times \tan. d$$

$$+ \ 16'',29 \times \sin. z.$$

But (in this figure) $r = 90° - a$, therefore $r \pm z = 90° - a \pm z =$ (so far as regards the angle) $a \pm z - 90°$; and the same is true if a be greater than $90°$, and $r = a + 90°$. Hence, the variation of *Right Ascension*

$$= - \left\{ \begin{array}{l} 4'',75 \times \sin. \overline{a-z-90°} + 4'',75 \times \sin. \overline{a+z-90°} \\ 3'',53 \times \sin. \overline{a-z-90°} - 3'',53 \times \sin. \overline{a+z-90°} \end{array} \right\} \times \tan. d$$

$$+ \ 16'',29 \times \sin. z.$$

$= \tan. d \times (-8'',28 \times \sin. \overline{a-z-90°} - 1'',22 \times \sin. \overline{a+z-90°}) + 16'',29 \times \sin. z.$

For south declination d is negative.

1044. The variation in *Declination* is $sP - sp = Pr$ very nearly, $= Pp \times \cos. \overline{r-v} = 9'',5 \times \frac{\cos. z}{\cos. v} \times \cos. \overline{r-v} = 9'',5 \times \frac{\cos. z}{\cos. v} \times \overline{\cos. r \times \cos. v + \sin. r \times \sin. v}$

$= 9'',5 \times \cos. r \times \cos. z + 9'',5 \times \sin. r \times \cos. z \times \tan. v = 9'',5 \times \cos. r \times \cos. z + 7'',07 \times \sin. r \times \sin. z$

$$= \left\{ \begin{array}{l} 4'',75 \times \cos. \overline{r-z} + 4'',75 \times \cos. \overline{r+z} \\ 3'',53 \times \cos. \overline{r-z} - 3'',53 \times \cos. \overline{r+z} \end{array} \right\}$$

$= 8'',28 \times \cos. \overline{r-z} - 1'',22 \times \cos. \overline{r+z} = 8'',28 \times \sin. \overline{a-z} - 1'',22 \times \sin. \overline{a+z}.$

1045. From the expressions for the variation in right ascension and declination, we calculated the following Tables for more readily finding those quantities. "They were computed by M. LAMBERT, upon supposition that the nutation was 18″; but a more correct value being 19″ (1028), we have here calculated the Tables for that quantity.

	TABLE I.			TABLE II.			TABLE III.			
	Signs.			Signs.			Signs.			
Degrees	O. VI. + —	I. VII. + —	II. VIII. + —	O. VI. + —	I. VII. + —	II. VIII. + —	O. VI. — +	I. VII. — +	II. VIII. — +	Degrees
0	0″,00	4″,14	7″,17	0″,00	0″,61	1″,06	0″,00	8″,14	14″,11	30
1	0, 14	4, 26	7, 24	0, 02	0, 63	1, 07	0, 28	8, 39	14, 25	29
2	0, 29	4, 39	7, 31	0, 04	0, 65	1, 08	0, 57	8, 63	14, 38	28
3	0, 43	4, 51	7, 38	0, 06	0, 66	1, 09	0, 85	8, 87	14, 51	27
4	0, 58	4, 63	7, 44	0, 09	0, 68	1, 10	1, 14	9, 11	14, 64	26
5	0, 72	4, 75	7, 50	0, 11	0, 70	1, 11	1, 42	9, 34	14, 76	25
6	0, 86	4, 87	7, 56	0, 13	0, 72	1, 11	1, 70	9, 58	14, 88	24
7	1, 01	4, 98	7, 62	0, 15	0, 74	1, 12	1, 99	9, 80	15, 00	23
8	1, 15	5, 10	7, 68	0, 17	0, 75	1, 13	2, 27	10, 03	15, 10	22
9	1, 30	5, 21	7, 73	0, 19	0, 77	1, 14	2, 55	10, 25	15, 21	21
10	1, 44	5, 32	7, 78	0, 21	0, 78	1, 15	2, 83	10, 47	15, 31	20
11	1, 58	5, 43	7, 83	0, 23	0, 80	1, 15	3, 11	10, 69	15, 40	19
12	1, 72	5, 54	7, 87	0, 25	0, 82	1, 16	3, 39	10, 90	15, 49	18
13	1, 86	5, 65	7, 92	0, 27	0, 83	1, 17	3, 66	11, 11	15, 58	17
14	2, 00	5, 75	7, 96	0, 30	0, 85	1, 17	3, 94	11, 32	15, 66	16
15	2, 14	5, 85	8, 00	0, 32	0, 86	1, 18	4, 22	11, 52	15, 73	15
16	2, 28	5, 96	8, 03	0, 34	0, 88	1, 18	4, 49	11, 72	15, 81	14
17	2, 42	6, 06	8, 07	0, 36	0, 89	1, 19	4, 76	11, 91	15, 87	13
18	2, 56	6, 15	8, 10	0, 38	0, 91	1, 19	5, 03	12, 11	15, 93	12
19	2, 70	6, 24	8, 13	0, 40	0, 92	1, 20	5, 30	12, 29	15, 99	11
20	2, 83	6, 34	8, 15	0, 42	0, 93	1, 20	5, 57	12, 48	16, 04	10
21	2, 97	6, 43	8, 18	0, 44	0, 95	1, 20	5, 84	12, 66	16, 09	9
22	3, 10	6, 52	8, 20	0, 46	0, 96	1, 21	6, 10	12, 84	16, 13	8
23	3, 23	6, 61	8, 22	0, 48	0, 97	1, 21	6, 36	13, 01	16, 17	7
24	3, 37	6, 70	8, 23	0, 50	0, 99	1, 21	6, 63	13, 18	16, 20	6
25	3, 50	6, 78	8, 25	0, 52	1, 00	1, 22	6, 88	13, 34	16, 23	5
26	3, 63	6, 86	8, 26	0, 53	1, 01	1, 22	7, 14	13, 50	16, 25	4
27	3, 76	6, 94	8, 27	0, 55	1, 02	1, 22	7, 40	13, 66	16, 27	3
28	3, 89	7, 02	8, 27	0, 57	1, 03	1, 22	7, 65	13, 81	16, 28	2
29	4, 01	7, 10	8, 28	0, 59	1, 05	1, 22	7, 90	13, 96	16, 29	1
30	4, 14	7, 17	8, 28	0, 61	1, 06	1, 22	8, 14	14, 11	16, 29	0
	V. XI. + —	IV. X. + —	III. IX. + —	V. XI. + —	IV. X. + —	III. IX. + —	V. XI. — +	IV. X. — +	III. IX. — +	

Ex. On January 1, 1790, the right ascension of α *Lyræ* was $9^s. 7°. 27'$, and its declination was $38°. 35'. 44''$; to find its nutation.

The longitude of the moon's ascending node was $7^s. 16°. 36'$; hence,

$$
\begin{array}{lll}
a = 9^s.\ 7°.\ 27'. & & \\
z = 7.\ 16.\ 36 & - - - - - & +11'',64 \text{ Tab. III.} \\
\hline
a - z = 1.\ 20.\ 51 & - - - - - & +\ 6,42 \text{ Tab. I.} \\
a + z = 4.\ 24.\ \ 3 & - - - - - & +\ 0,72 \text{ Tab. II.} \\
\hline
\text{Nutation in } \textit{Declination} & & = +\ 7,14
\end{array}
$$

$$
\begin{array}{lll}
a - z - 90° = 10^s.\ 20°.\ 51'. & -5'',23 & \text{Tab. I.} \\
a + z - 90° = 1^s.\ 24°.\ 3'. & +0,99 & \text{Tab. II.} \\
\hline
 & -4,24 & \\
\text{Tan. dec.} & = ,798 & \\
\hline
 & -\ 3,38 & \text{Product.} \\
 & +11,64 & \\
\hline
\text{Nutation in } \textit{Right Ascen.} = & +\ 8,26 &
\end{array}
$$

When the declination is south, its tangent becomes negative.

To find the Variation in Right Ascension and Declination of a Star, from the Precession of the Equinoxes.

1046. Let $♈L$ be the ecliptic, $♈Q$ the equator, rQ its next position at the end of any given time, s a star, sx, svw two circles of declination to the two positions of the equator, and $♈a$ perpendicular to rQ. As we here consider the effect arising only from the regression of the equinoctial points, the inclination of the equator to the ecliptic remaining the same, we have (1031) $Q♈ = 90°$; hence (13), rad. $= 1$: cos. $♈w :: a♈ (= ♈r \times \sin. ♈) : vw = ♈r \times \sin. ♈ \times \cos. ♈w$, the *variation of declination*. FIG. 230.

1047. Hence, we can find the variation of right ascension. From the variation of the right angled spherical triangle Qsw, whose side Qs is constant,

we have* $\Upsilon w \times$ tan. $ws = vx \times$ tan. Qx, therefore $vx = \frac{\Upsilon w \times \text{tan. } ws}{\text{cot. } \Upsilon w}$ (1046) $= \frac{\Upsilon r \times \text{sin. } \Upsilon \times \text{cos. } \Upsilon w \times \text{tan. } ws}{\text{cot. } \Upsilon w} = \Upsilon r \times \text{sin. } \Upsilon \times \text{sin. right ascension} \times \text{tan. dec.}$ This is one part of the variation of right ascension. But as we now reckon from r and not from Υ, it is manifest that there is another part ra which is common to all the stars; now $ra = \Upsilon r \times \text{cos. } \Upsilon$; hence, the *variation of right ascension* $= \Upsilon r \times \text{cos. } \Upsilon - \Upsilon r \times \text{sin. } \Upsilon \times \text{sin. right ascen.} \times \text{tan. dec.} = \text{prec. in long.} \times \overline{\text{cos. } 23°. 28'. - \text{sin. } 23°. 28' \times \text{sin. right ascen.} \times \text{tan. dec.}}$ In the last six signs, the sin. right ascen. becomes negative; also, if the declination becomes south, the tang. of declination becomes negative; the second term therefore is sometimes additive, and sometimes subtractive.

On the preceding theory, Mr. SIMPSON has made the following remarks, in order to explain certain difficulties and objections that may thence arise.

1048. It may be observed, in the first place, that we have, all along, considered the effects of the sun and moon separately; and, consequently, have supposed them to be no-ways influenced or disturbed by each other. This may seem too bold an assumption; especially, as it is known that the tides, which are produced by the very same forces, depend upon, and are greatly varied by, the different positions of the two luminaries.

FIG. 231. 1049. To remove this objection, let $\Upsilon SM \simeq$ represent the plane of the earth's equator, $\Upsilon O \simeq$ its intersection with the plane of the ecliptic, ΥS the right ascension of the sun, ΥM the right ascension of the moon; and let the forces of the two bodies to turn the earth about its center, in those positions, be represented by f and F respectively.

1050. These forces may be considered as acting perpendicular to the plane of the equator in the points S and M, and will be equivalent to, and have the same effect with, one single force, equal to them both, acting in their center of gravity N. But, by Mechanics, the force $f + F$, acting at N, will (if the radius OP be drawn through N) be equivalent to another force, acting at P, expressed by $\overline{f+F} \times \frac{ON}{OP}$, or $\overline{f+F} \times \frac{NQ}{PR}$ (supposing NQ, PR, as also SB, MC, to be perpendicular to $\Upsilon O \simeq$).

* If S and s represent the sines of the two legs A, a of a right angled triangle, C and c their cosines, T and t their tangents, and the hypothenuse be constant, then $C \times c = \text{rad.} \times \text{cos. hyp.}$ a constant quantity; hence, $C\times\dot{c} + c \times \dot{C} = 0$; but $\dot{c} = s \times \dot{a}$, and $\dot{C} = S \times \dot{A}$; therefore $C \times s \times \dot{a} + c \times S \times \dot{A} = 0$, and $\frac{s}{c} \times \dot{a} = -\frac{S}{C} \times \dot{A}$, or $t \times \dot{a} = -T \times \dot{A}$; the signs of the quantities thus deduced we have omitted above, as we only wanted the values of the quantities.

4

1051. But the quantity of precession, during a given moment of time, is known (1018 to 1021) to be as the force, and as the sine of the right ascension, conjointly; from whence the two quantities arising from the sun and moon, considered separately, are expounded by $f \times SB$, and $F \times MC$, respectively. But, supposing both bodies to act together, or, which is the same, supposing one single force, expressed by $\overline{f+F} \times \frac{NQ}{PR}$, to act at P, the quantity of the precession will then (by the very same rule) be truly defined by $\overline{f+F} \times \frac{NQ}{PR} \times PR$, or its equal $\overline{f+F} \times NQ$; which quantity, by the property of the center of gravity, is known to be equal to $f \times SB + F \times MC$. Hence, it is manifest that, whether the forces of the luminaries be joined together, or treated apart, the result will be the same.

1052. The next difficulty relates to the excentricity of the lunar orbit, and the inequality of the motion in that orbit; which may be thought sufficient to occasion a sensible deviation from rules founded on a supposition that pays no regard to them.

1053. In order to clear up this point also, imagine $ADBE$ to be an ellipse, in which the moon is supposed to revolve, about the center of the earth placed in the lower focus F of the ellipse; let AB be the transverse axis of the ellipse, perpendicular to which, through F, draw the ordinate IH; moreover, let there be drawn any two other lines DE, de, through the focus F, to make a very small (given) angle DFd with each other. FIG. 232.

1054. The perturbating force of the moon, at the distance DF will (856) be *inversely* as the cube of that distance; and the time of describing the given angle DFd will * be *directly* as the square of the same distance. Therefore, by composition, the quantity of the moon's action, during the time of describing this angle, will be in the simple ratio of the said distance, *inversely*. Hence, it appears, that the sum of the forces employed, during the times of describing the opposite angles DFd, EFe, will be truly defined by $\frac{1}{FD} + \frac{1}{FE}$; or its equal $\frac{FE+FD}{FE \times FD}$.

1055. Upon AB let fall the perpendiculars DN and EM; so shall $FE - FH : FI\ (FH) - FD :: FM : FN$ (by the property of the ellipse) $:: FE : FD$ by similar triangles; consequently $FE \times FD - FH \times FD = FH \times FE - FD \times FE$, or $2FE \times FD = FH \times \overline{FE+FD}$; therefore, as it appears from hence that $\frac{FE+FD}{FE \times FD}$, the measure of the said forces, is, every where, equal to the con-

* For the angle DFd being given, the area DFd varies as $DF \times dF$, or as DF^2; therefore (805), the time of describing the angle varies as DF^2.

stant quantity $\frac{2}{FH}$, it is evident that the excentricity of the orbit and the position of the apogee have no effect on the motion of the earth's axis.

1056. An objection may, perhaps, arise, with regard to the addition of the forces employed by the moon in opposite parts of its orbit; which step may be looked upon as arbitrary; but the reason upon which it is founded will be clear, by considering that the moon's inclination to the plane of the equator, in opposite points of its orbit, is always the same; and that, therefore, the very same effect in the alteration of the position of the equator will be produced, whether the whole force employed during the description of the corresponding opposite angles, be equally, or unequally, divided, with respect to the said angles; since the said force acts with the same advantage, or under the same circumstance of declination, in both cases.

1057. Another difficulty, that may arise, is in relation to our having made the effect of the sun's force to be about one third part less than the quantity resulting from calculations founded on hydrostatical principles and the hypothesis of an uniform density of all the parts of the earth. But, that the *phænomenon* cannot be truly accounted for, upon this hypothesis, appears from the concurrence of all experiments in general: for, whether we regard the mensuration of the degrees of the earth, the accurate observations of Dr. BRADLEY, or the proportions and times of the tides, the case is the same, and requires a much less effect from the action of the sun than results from, or can consist with, the said hypothesis.

1058. But if the density of the earth, instead of being uniform, is supposed to increase from the surface to the center (as there is the greatest reason to imagine it does), then the *phænomenon* may be easily made to quadrate with *the principles of gravitation;* and that according to innumerable suppositions, respecting the law whereby the density may be conceived to increase.

CHAP. XXXV.

ON THE DENSITIES, QUANTITIES OF MATTER, LIGHT AND HEAT OF THE PLANETS.

Art. 1059. TO measure the quantity of matter in distant bodies, appears at first sight, to be a problem of insuperable difficulty, and such it was before the discovery of the laws of gravitation; but those principles led Sir I. NEWTON to a very easy solution of this important problem, in all those planets which have satellites revolving about them; and in the other planets, they also furnish a method by which their quantities of matter may be assigned, to a considerable degree of accuracy, by the effects which such planets produce upon the others. To understand the principle upon which this determination rests, we may observe, that the effect of attraction at equal distances will be in proportion to the quantity of matter in the attracting body; and at different distances, as the quantity of matter and the inverse square of the distance conjointly. The quantity of matter is also in proportion to the magnitude of the body and its density conjointly. If therefore we know the effects of the attraction of different bodies, together with their magnitudes, we can find their densities, and thence their quantities of matter.

1060. To find their densities, put

d = the density of the central body.
m = its diameter.
a = its quantity of matter.
P = the periodic time of the revolving body.
D = the mean distance of the revolving body from its central body.
s = the sine of the angle under which m appears at the distance D, to radius unity.

Then a varies as dm^3; but (818) P^2 varies as $\frac{D^3}{a}$ which varies as $\frac{D^3}{dm^3}$; hence, d varies as $=\frac{D^3}{m^3 P^2}$. But $s = \frac{m}{D}$; hence, d varies as $\frac{1}{s^3 P^2}$; we will therefore assume $d = \frac{1}{s^3 P^2}$.

For the *Sun*. If we take the earth as the revolving body, $P = 365,25639$

days, according to M. de la CAILLE, $s = 0{,}0093155 = \sin. 32'. 1'',5$ the mean apparent diameter of the sun; hence, $d = \frac{1}{0{,}0093155^3 \times 365{,}25639^2}$ $= 9{,}2722$.

For the *Earth*. Here we must take the moon for the revolving body; therefore $P = 27{,}32167$ days, according to MAYER, $s = 0{,}033155 = \sin. 1°. 54'$, the mean angle under which the earth's mean diameter appears at the moon; hence, $d = \frac{1}{0{,}033155^3 \times 27{,}32167^2} = 36{,}7569$.

For *Jupiter*. Mr. POUND observed the greatest elongation of its fourth satellite to be $8'. 16''$, and the corresponding diameter of Jupiter to be $39''$ *; hence, the sine s of the angle under which the diameter of Jupiter appeared at that satellite at that time was $0{,}078629$; also, $P = 16{,}68898$ days, according to M. WARGENTIN; hence, $d = \frac{1}{0{,}078629^3 \times 16{,}68898^2} = 7{,}3857$.

For *Saturn*. According to Mr. POUND, the greatest elongation of its fourth satellite is $2'. 58''$, and the corresponding diameter of Saturn $= 18''$; hence, $s = 0{,}10112$; also, $P = 15{,}9454$ days, according to Dr. HALLEY; hence, $d = \frac{1}{0{,}10112^3 \times 15{,}9454^2} = 3{,}8038$.

For the *Georgian*. If we take the second satellite, we have, according to Dr. HERSCHEL, its greatest elongation $= 44'',23$, and the corresponding diameter of the planet $= 3'',90554$; hence, $s = 0{,}0883$; also, $P = 13{,}462$ days; hence, $d = \frac{1}{0{,}0883^3 \times 13{,}462^2} = 8{,}0149$.

1061. These densities of the Sun, Earth, Jupiter, Saturn and the Georgian, are as 0,25226, 1, 0,20093, 0,10349 and 0,21805. The other planets not having any satellites revolving about them, their densities cannot be thus determined; but they may be found, by observing the effects which those planets

* From the times in which the first and third satellites are in passing over the body of Jupiter, Sir I. NEWTON computes the diameter of that planet at its mean distance to be $37'',25$, which he uses. But M. de la GRANGE thinks it is safer to trust to the diameter directly measured by a telescope; he accordingly supposes the diameter to be $39''$. Sir I. NEWTON reduces the observed diameter $18''$ of Saturn to $16''$, on account of the irradiation of light (1063). From all the observations, we have judged $18''$ to be the most correct value.

produce upon the other planets in disturbing their motion. EULER, however, in his *Recherches sur les Perturbations des Planetes*, observing that the densities of the *Earth*, *Jupiter*, and *Saturn*, were very nearly as the square roots of their mean motions, or inversely, as $d^{\frac{3}{4}}$, d being their mean distance from the sun, supposed that the same law might hold for all the planets, from whence he estimated the densities of *Mercury*, *Venus* and *Mars*. But the density of the *Georgian*, thus determined, does by no means agree with that found from Dr. HERSCHEL's observations. M. de la GRANGE, in his *Théorie des Var. Sec. des Planets*, in the *Hist. de l'Acad. Roy. des Scien.* 1782, assumes the densities to be inversely as their distances from the sun, as being the most simple law, and which, by his calculations, answers very nearly for the *Earth*, *Jupiter* and *Saturn*. He computes the disturbing forces of all the planets upon that supposition, and from the agreement of the results with observation he sees no reason for changing his hypothesis. This gives the densities of Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian as 2,5833, 1,3825, 1, 0,6563, 0,20155, 0,11215, and 0,052077. As however the density of the Georgian, deduced from its second satellite, by no means agrees with the above law, we may conclude that that law for the densities is not generally true; as it cannot be supposed that Dr. HERSCHEL should have erred so much in his observations. M. de la LANDE makes the density of Venus 1,0379, as best answering to the motion of the sun's apogee, of the aphelion of the orbit of Mercury, and of the nodes of Mercury. From a diminution of the inclination of the equator to the ecliptic of 50″ in 100 years, Dr. MASKELYNE has determined the density of Venus to be 1,024, that of the earth being unity. Dr. HERSCHEL makes the time of the rotation of Mars to be 24,656 hours, and the ratio of the diameters as 16 : 15. Hence (983), the density of Mars is about 0,07, that of the earth being unity. But from some observations of Dr. MASKELYNE upon this planet, he has reason to think that the ratio of its diameters is much nearer to a ratio of equality than that given above, which renders the density, thus deduced, subject to great uncertainty. We will therefore assume the densities of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian, as 0,25226, 2,5833, 1,024, 1, 0,6563, 0,20093, 0,10349 and 0,21805.

1062. By Art, 1088, the density of the moon : density of the sun as 2,44 : 1; and the density of the sun being to that of the earth as 0,252 : 1; it follows that the density of the moon : density of the earth :: 0,6149 : 1. Hence, in proportion to the above densities of the planets, the moon's density would be 0,6149.

To find the Ratio of the Diameters of the Sun and Planets.

[illegible]. Dr. Herschel found the diameter of the *Georgian*, at its mean distance, to be [illegible]; if therefore it were seen at the mean distance of the earth from the sun, it would be 74",52. Mr. Pound found the diameter of Saturn, at its mean distance from the earth, to be 18"; and therefore if it were seen at the mean distance of the earth from the sun, the diameter would be [illegible]. He also found the diameter of *Jupiter*, at its mean distance, to be 39"; hence, at the mean distance of the earth from the sun, it would be [illegible]. M. Picard and Flamstead found the diameter of *Mars* to be 30" when its distance from the earth was 0,3815, the mean distance of the earth from the sun being unity; hence, if Mars were seen at the earth's mean distance from the sun, its diameter would be 11",4. Dr. Herschel makes it 8",94, which is probably the most accurate. The apparent diameter of the *Earth* seen from the sun is twice the sun's horizontal parallax, or 17",5 (629). In the transit of *Venus* over the sun in 1761, M. de la Lande, from his own observations, and those of Mr. Short, found its diameter to be 57",8; hence, the diameter of Venus, seen at the mean distance of the earth from the sun, would be 16",7. The diameter of *Mercury*, measured by Dr. Bradley in 1723, in its transit over the sun's disc, with a micrometer to Huygens's telescope of 120 feet long, was found to be 10", 75; hence, its diameter, at the mean distance of the earth, will be 7",27. M. de la Lande, from the transit in 1753, found it to be 6",5; we will therefore take it 7". The diameter of the sun in its apogee seen from the earth is 31'. 29",2, according to Dr. Maskelyne; according to Mr. Short, 31'. 28"; according to M. de la Lande, 31'. 30",5; Dr. Maskelyne's is a mean between the other two; we will therefore state it at 31'. 29"; and as the difference of the diameters in its apogee and perigee is 1'. 5", if we add its half, 32",5, to 31'. 29" we have 32'. 1",5 for its mean diameter. Sir I. Newton supposes that there is a sensible aberration in all telescopes, which makes the image of the object in the focus of a telescope greater than it ought to be. He observes, that this aberration has a less ratio to the diameter of *Jupiter* in long than in short telescopes; the latter therefore will give the greater diameter. What reduction must be made from the measured diameter by any telescope in order to get the real diameter, it is not easy to say. M. de la Lande thinks you may allow 5" for a small telescope not very perfect. This will make the diameter nearly the same as that used by M. Cassini in his Tables, who probably had not a very good telescope for that purpose. On the contrary, when a planet appears *on* the sun, its diameter, measured by a telescope, appears less than it is, owing to the irradiation of the sun. The diameter of the moon in 1748, measured upon the

disc of the sun, appeared 6″ less than when measured off the sun. Hence, the diameters of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian are as 109,8, 0,4, 0,9543, 1, 0,5109, 11,593, 9,812 and 4,258.

1064. We have found the densities (1060), and (1063) apparent diameters of all the planets seen at the mean distance of the earth from the sun, which must represent the ratio of their real diameters; and as the quantities of matter in spherical bodies are as the cubes of their diameters and densities conjointly, we find the ratio of the quantities of matter in the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian as 333928, 0,16536, 0,88993, 1, 0,08752, 312,101, 97,762 and 16,837.

1065. The diameter of the earth is to that of the moon as 11 : 3, or as 1 : 0,2727; therefore the magnitude of the earth : that of the moon :: 1 : ,02028, or very nearly as 49 : 1; and their densities (1062) are as 1 : 0,6149; therefore the quantity of matter in the earth : that of the moon :: 1 : ,01245. If we assume, with some Authors, the density of the moon to that of the sun as 2,5 : 1, the quantity of matter in the earth : that in the moon :: 78 : 1, or 1 : ,0128. Also, the gravity of a body upon the earth is to that on the moon as 1 : 0,1677.

To find the relative Weights of Bodies upon the Surfaces of the Planets.

1066. The weight of a given body on a planet must be in proportion to the force with which the planet attracts it, the weight being the effect arising from that cause. Now when the force varies inversely as the square of the distance, if a body be at the surface of the sphere, the attraction (834) varies as the diameter and density conjointly; consequently the weights of equal bodies on the surfaces of different planets, vary as the radii and densities conjointly, or as the diameters and densities. Hence, the weights of equal bodies on the surfaces of the Sun, Mercury, Venus, the Earth, Mars, Jupiter, Saturn and the Georgian are as 27,7, 1,0333, 0,9771, 1, 0,3355, 2,3287, 1,0154 and 0,9285; these numbers therefore represent the forces of gravity upon the surfaces of these respective bodies.

1067. The following Table exhibits the relation of the densities, diameters, quantities of matter, and gravity on the surface of the respective bodies.

Planets.	Densities.	Diameters.	Quantities of Matter	Grav. on Surf.
Sun	0,25226	109,8	333928	27,7
Mercury	2,5833	0,4	0,16536	1,0333
Venus	1,024	0,9543	0,88993	0,9771
Earth	1	1	1	1
Mars	0,6563	0,5109	0,08752	0,3355
Jupiter	0,20093	11,59	312,101	2,3287
Saturn	0,10349	9,812	97,762	1,0154
Georgian	0,21805	4,258	16,837	0,9285
Moon	0,6149	0,2727	0,01245	0,1677

1068. The intensities of light and heat which the planets receive from the sun, vary inversely as the squares of their distances from the sun. For let L be a point from which light or heat diverges, abc a triangle upon which they fall; produce La, Lb, Lc to A, B, C, and let AB, BC, AC be respectively parallel to ab, bc, ac; then the triangle abc is similar to ABC; and the same quantity of light or heat (supposed to proceed in straight lines) which fall on abc, would, if that plane were removed, fall on ABC, and there occupying a greater space, the intensity must be so much less in proportion as the space is greater; hence, the intensity on abc : the intensity on $ABC :: ABC : abc :: AB^2 : ab^2 :: LB^2 : Lb^2$. To apply this to the sun and planets, we have (217) the distances of Mercury, Venus, the Earth, Mars, Jupiter, Saturn, and the Georgian from the sun as 4, 7, 10, 15, 52, 95 and 190, the inverse squares of which are as $\frac{10^2}{4^2}$, $\frac{10^2}{7^2}$, 1, $\frac{10^2}{15^2}$, $\frac{10^2}{52^2}$, $\frac{10^2}{95^2}$ and $\frac{10^2}{190^2}$, or as 6,25, 2,04, 1, 0,44375, 0,036875, 0,01106 and 0,00276 the relative intensities of light and heat which the respective planets receive from the sun.

FIG. 233.

1069. The apparent diameter of a body is inversely as its distance. Assuming therefore the mean diameter of the sun $=32'$, we have the apparent diameter of the Sun at Mercury $=\frac{5}{2}\times 32'=80'$; at Venus $=\frac{10}{7}\times 32'=45',7$; at

Mars $= \frac{10}{15} \times 32' \doteq 21',33$; at Jupiter $= \frac{10}{52} \times 32' = 6',15$; at Saturn $= \frac{10}{95} \times 32' = 3'.37$, and at the Georgian $= \frac{10}{195} \times 32' = 1',64$. Hence, the apparent diameter of the sun at the Georgian is only about $2\frac{1}{2}$ times greater than the apparent diameter of Jupiter seen from the earth at its mean distance.

The following Table exhibits the relative intensities of light and heat at the different planets, and the apparent diameter of the sun seen from them.

Planets.	Intensities of Light and Heat.	Apparent Diameter of the Sun.
Mercury	6,25	80′
Venus	2,04	45,7
Earth	1	32
Mars	0,44375	21,33
Jupiter	0,036875	6,15
Saturn	0,01106	3,37
Georgian	0,00276	1,64

CHAP. XXXVI.

ON THE MOTION OF THE PLANES OF THE ORBITS OF THE PLANETS, FROM THEIR MUTUAL ATTRACTIONS.

Art. 1070. BY comparing the modern with the ancient observations, it appears that the latitudes of the fixed stars vary, and that the mean inclination of the ecliptic to the equator gradually diminishes; the former can arise only from an alteration in the position of the ecliptic. M. GODIN, in a Treatise on the obliquity of the ecliptic, judged that its diminution was owing to a change in the ecliptic. He compared the position of the nodes of *Jupiter's* orbit, observed 241 years before CHRIST, with that observed by M. de la HIRE; and supposing the plane of Jupiter's orbit not to be changed, he concluded the ecliptic must. His conjecture, partly true, led him to assign the true cause of the diminution. KEPLER and TYCHO observed that the latitude of the stars was subject to a change; the former concluded that it was owing to a change in the position of the ecliptic, and that it arose from some physical cause; he suspected that it might arise from the rotation of the sun. But after Sir I. NEWTON had established the doctrine of universal gravitation, it was evident that the planets must disturb each other's motions; the consequence of which must be, that as their orbits are inclined to the ecliptic, they must tend to disturb the motion of the earth in the plane of its orbit, and therefore subject the ecliptic to a change in its position. EULER first computed these effects on the earth, and found that they would solve the above phænomena, and also give the variation of the inclination of their orbits, and retrograde motion of their nodes. The method here given of investigating these matters, is similar to that by which we determined the motion of the moon's nodes.

FIG. 234. 1071. Let NQn be the orbit of the body Q inclined to NAn the plane of the orbit of the body P, NSn the line of the nodes; draw PAS, and Aa perpendicular to the plane NQn, and AK, aK perpendicular to Nn; then AKa is the inclination of the orbit of Q to that of P; produce Sa to v; and draw Pv parallel to Aa, and consequently perpendicular to the plane NQn; join PQ, vQ, which latter will lie in the plane NQn, because v is in that plane; and draw Qu perpendicular to PS, and QL to Nn. Put $SP=a$, $SQ=b$, $Su=x$, and $s=\sin.\ AKa$; also, let $1 : m ::$ quantity of matter in P : that in the sun S. Now if SA represent the force of S towards P, then Aa is that part of the force which acts perpendicularly to the plane NQn; and $Aa=s\times AK$; but the force

of S to P varies as $\frac{1}{a^2}$, and not as SA; hence, $SA : \frac{1}{a^2} :: s \times AK$: that part of the force of P upon S which acts perpendicularly to the plane of the orbit of $Q = \frac{s}{a^2} \times \frac{AK}{SA} = \frac{s}{a^2} \times \sin. NSA$ to radius unity. Now any lines mp, zx parallel to Pv will be perpendicular to the plane NQn. Hence, if Sm, Qz represent the forces of S to P and Q to P, then mp, zx will represent that part of each force which acts perpendicularly to the plane NQn. Let therefore Sm represent $\frac{1}{a^2}$, then mp will represent $\frac{s}{a^2} \times \sin. NSA$; and let also Qz represent $\frac{1}{PQ^2}$; hence, and by similar triangles,

$$Pv : SP :: mp : Sm :: \frac{s}{a^2} \times \sin. NSA : \frac{1}{a^2} :: s \times \sin. NSA : 1$$

$$PQ : Pv :: Qz = \frac{1}{PQ^2} : zx$$

$$\therefore PQ : SP :: \frac{s}{PQ^2} \times \sin. NSA : zx = \frac{s \times SP}{PQ^3} \times \sin. NSA = \frac{sa}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} \times \sin. NSA,$$

which is that part of the force of P on Q which acts perpendicularly to the plane NQn. Hence, $s \times \sin. NSA \times \overline{\frac{a}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} - \frac{1}{a^2}}$ = the difference of the forces by which S and Q are drawn from the orbit NQn, or the whole force with which Q is drawn from the plane of its orbit about S. Now $\frac{m}{b^2}$ is the force of Q towards S; hence, if Qw be an indefinitely small arc, we have (as in Art. 927.) $\frac{m}{b^2} : s \times \sin. NSA \times \overline{\frac{a}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} - \frac{1}{a^2}} :: \frac{Qw^2}{b} : \frac{s \times b \times Qw^2 \times \sin. NSA}{m} \times \overline{\frac{a}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} - \frac{1}{a^2}}$ the velocity generated by the force drawing the body Q from the plane of its orbit, the velocity in the orbit being Qw; hence, this latter velocity : the former :: 1 : $\frac{s \times b \times Qw \times \sin. NSA}{m} \times \overline{\frac{a}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} - \frac{1}{a^2}}$.

1072. Draw wt perpendicular to the plane NQn, and take it to Qw as $\frac{s \times b \times Qw \times \sin. NSA}{m} \times \overline{\frac{a}{\overline{a^2 + b^2 - 2ax}\big|^{\frac{3}{2}}} - \frac{1}{a^2}}$ to 1, and draw the orbit $NQtn'$, and nn' will be the cotemporary motion of the nodes. Draw $n's$ perpendicular to nQ; then $Qw : wt ::$ sin. Qs, or Qn, which is QL, : $n's = \frac{wt \times QL}{Qw}$, and sin. snn'

$nn' : nn = \frac{nt \times QL}{Qw} ::$ rad. $= 1 : nn' = \frac{nt \times QL}{s \times Qw} = \frac{b \times Qw \times QL \times \text{sin. } NSA}{m} \times$

$\frac{a}{\overline{a^2+b^2-2ax}\rvert^{\frac{3}{2}}} - \frac{1}{a^2}$.

1073. Draw Dd perpendicular to PAS; then QL is the sin. $QSN =$ sin. $\overline{QSD \pm NSD} =$ sin. $QSD \times$ cos. $NSD \pm$ sin. $NSD \times$ cos. $QSD =$ sin. $QSD \times$ sin. $NSA \pm$ cos. $NSA \times$ cos. QSD. Now in a whole revolution of Q, the last term will be destroyed by the opposition of signs in the opposite quadrants, and therefore to get the *mean* motion of the nodes in a revolution, we may neglect that term; hence, if we substitute sin. $QSD \times$ sin. NSA for QL, put $b = 1$, and x for sin. QSD, we have $nn' = \frac{b \times Qw \times x \times \overline{\text{sin. } NSA}^2}{m} \times \frac{a}{\overline{a^2+1-2ax}\rvert^{\frac{3}{2}}} - \frac{1}{a^2}$. But to determine the value of this quantity for a whole revolution of Q, we must expand $\frac{1}{\overline{a^2+1-2ax}\rvert^{\frac{3}{2}}}$ and multiply it by x, and then take those terms only which contain the even powers of x, for in opposite quadrants x having a different sign, the odd powers will destroy each other in a revolution. Now by Sir I. NEWTON's Binomial Theorem, $\overline{P+PQ}\rvert^{\frac{r}{n}} = P^{\frac{r}{n}} + \frac{r}{n}AQ + \frac{r-n}{2n}BQ + \frac{r-2n}{3n}CQ$ + &c. where A, B, C, &c. represent the preceding term; and to make the expansion of the above quantity converge, the powers of a must stand in the denominators; hence, $P = a^2$, $Q = \frac{1}{a^2} - \frac{2x}{a}$, $r = -3$, $n = 2$; and by expanding the above quantity and multiplying each term by x, taking only the even powers of x, we have $nn' = \frac{Qw \times \overline{\text{sin. } NSA}^2}{m \times a^3} \times \left(3x^2 - \frac{15x^2}{2a^2} + \frac{35x^4}{2a^2} + \frac{105x^2}{8a^4} - \frac{315x^4}{4a^4} + \frac{693x^6}{8a^4} - \frac{315x^2}{16a^6} + \frac{3465x^4}{16a^6} - \frac{9009x^6}{16a^6} + \frac{6435x^8}{16a^6}\right)$. But by the principles of plane Trigonometry (see my Trig. Prop. 24.), $x^2 = \frac{1}{2} - \frac{1}{2}$ cos. $2QSD$, $x^4 = \frac{3}{8} - \frac{1}{2}$ cos. $2QSD + \frac{1}{8}$ cos. $4QSD$, $x^6 = \frac{5}{16} - \frac{15}{32}$ cos. $2QSD + \frac{3}{16}$ cos. $4QSD - \frac{1}{32}$ cos. $6QSD$, $x^8 = \frac{35}{128} - \frac{7}{16}$ cos. $2QSD + \frac{7}{32}$ cos. $4QSD - \frac{1}{16}$ cos. $6QSD + \frac{1}{128}$ cos. $8QSD$, and in a whole revolution of Q, the cosines of $2QSD$, $4QSD$, $6QSD$, $8QSD$, &c. &c. will destroy each other by the opposition of signs in the opposite quadrants, and therefore to get the *mean* motion of Q in one revolution, we may neglect those quantities, and substitute only $\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$, $\frac{35}{128}$, for x^2, x^4, x^6, x^8; hence, by collecting the terms, and substituting $360°$ for Qw, we have $nn' = \frac{3 \times 360° \times \overline{\text{sin. } NSA}^2}{2m \times a^3}$ $\times \overline{1 + \frac{15}{8a^2} + \frac{175}{64a^4} + \frac{3675}{1024a^6}}$ for the mean motion of the nodes in one revolution of Q, the place of P being given. Now according to the increase of the terms

1, $\frac{15}{8}$, $\frac{175}{64}$, $\frac{3675}{1024}$, we may assume the next term $\frac{9}{2}$; hence, we get $nn' = \frac{3\times360^\circ\times\overline{\sin. NSA}^2}{2ma^3}\times\overline{1+\frac{15}{8a^2}+\frac{175}{64a^4}+\frac{3675}{1024a^6}+\frac{9}{2a^8}}$. If any further terms should be required, the law to which they are observed to approach, will enable the computer to supply them to a very considerable degree of accuracy. This is necessary, when the difference of the distances SP, SQ is small in respect to those distances. As the $\overline{\sin. NSA}^2 = \frac{1}{2} - \frac{1}{2}\cos. 2NSA$, if we substitute this quantity for $\overline{\sin. NSA}^2$, we may, for the same reason as above, neglect $\frac{1}{2}\cos. 2NSA$ for one revolution of P; consequently the mean motion nn' of the nodes will be $\frac{3\times360^\circ}{4ma^3}\times\overline{1+\frac{15}{8a^2}+\frac{175}{64a^4}+\frac{3675}{1024a^6}+\frac{9}{2a^8}}$. This is the *mean* motion of the node of an inferior planet upon the orbit of a superior, from the action of that superior.

1074. To get the motion of the node of a superior planet from the action of an inferior, a becomes less than b, or less than unity, therefore the powers of a must go in the numerators; hence, $P = 1$, $Q = a^2 - 2ax$, and, by a similar process, we get $nn' = \frac{3\times360^\circ\times a^2}{4m}\times\overline{1+\frac{15a^2}{8}+\frac{175a^4}{64}+\frac{3675a^6}{1024}+\frac{9a^8}{2}}$. This is the *mean* motion of the node of a superior planet upon the orbit of an inferior, from the action of that inferior.

Ex. 1. To find how much the nodes of the orbit of *Jupiter* go back upon the orbit of *Saturn*, from the disturbing force of Saturn. Here we have $\frac{1}{m} = \frac{97,762}{333928}$, $a = \frac{95}{52}$; hence, in one revolution of Jupiter, $nn' = 113''$, or at the rate of about $9'',9$ in a year. EULER makes it $10''$.

Ex. 2. To find how much the nodes of the orbit of Saturn go back upon the orbit of Jupiter, from the disturbing force of Jupiter. Here we have $\frac{1}{m} = \frac{312,101}{333928}$, $a = \frac{52}{95}$; hence, in one revolution of Saturn, $nn' = 689''$, or at the rate of about $22''$ in a year. EULER makes it about $18''$.

The Motion of the Nodes of the Ecliptic upon the Orbits of all the Planets, from the Attractions of the respective Planets.

Ex. 1. To find how much the nodes of the ecliptic go back upon the orbit of the Georgian, from the disturbing force of the Georgian. Here we have $\frac{1}{m}$

... the secular motion

... back upon the orbit of ... Here we have $\frac{1}{m} = \frac{97,762}{333928}$, ... the secular motion is 34″.

... back upon the orbit of ... Here we have $\frac{1}{m} = \frac{312,101}{333928}$, ... secular motion is 693″.

... back upon the orbit of ... $\frac{1}{m} = \frac{[illegible]}{[illegible]}$, $a = 1,5$; ...

... back upon the orbit of ... Here we have $\frac{1}{m} = \frac{0,38993}{333928}$, $a = 7$ ... the secular motion is 558″.

... go back, by the action of ... $\frac{1}{m} = \frac{0,16586}{333928}$, $a = ,38$; ... is 10″.

... will be subject to a like ... the ecliptic: and by the ... of the two orbits will ... investigation, as it would ... explained.

... CE the ecliptic, and ... 1074) the attraction ... upon the orbit of ... of the orbit of A go

back upon the orbit of B; but (1075) the mean inclination of the two orbits will remain the same. Hence, if NA, NB be the orbits of two planets A and B, moving towards N, the attraction of B upon A will bring the orbit AN into the position Av, and the attraction of A upon B will bring the orbit BN into the position Bw; so that from their mutual attractions, the node will be brought to n, and the angle at n will be equal to the angle at N. But in respect to the ecliptic, the nodes of the orbit of A in FIG. 235. go backwards, and the nodes of the orbit of B go forwards; but the contrary in FIG. 236. Now it is manifest from the figures, that, as the points A and B, about which each orbit revolves, are 90° from the node N, the node of the orbit of A will be direct or retrograde upon the ecliptic, according as zN is less or greater than 90°; and the node of the orbit of B will be direct or retrograde as xN is greater or less than 90°. Now, whether zN, xN be greater or less than 90°, may be known from the triangle Nxz *, where we know the angles x and z, the inclination of the two orbits, and xz the distance of their nodes. Now to determine the motion of the node in any given time, we have, by the variation of the triangle zNz (see *Spherical Trig.*) sin. z : cot. zN :: var. z : rz = $\frac{\text{var. } z \times \text{cot. } zN}{\text{sin. } z}$ = (as var. $z = Nv \times$ sin. $x \times$ sin. xz) $\frac{Nv \times \text{sin. } x \times \text{sin. } xz \times \text{cot. } zN}{\text{sin. } z}$ the motion of the node of the orbit AN upon the ecliptic, in the time the node moves through Nv upon the orbit of B.

Ex. Let A represent *Mars*, and B *Jupiter*; then $xz = 50°.\ 22'$, the angle $Nxz = 178°.\ 41'$, and the angle $Nzx = 1°.\ 51'$, also $Nv = 14'',2$ according to M. de la LANDE (found by Art. 1073); hence, $Nz = 135°.\ 5'$, which being greater than 90°, shows that the node of Mars is retrograde upon the ecliptic from the action of Jupiter; hence, $rz = \frac{14'',2 \times \text{sin. } 178°.\ 41' \times \text{sin. } 50°.\ 22' \times \text{cot. } 135°.\ 5'}{\text{sin. } 1°.\ 51'}$ $= 7'',83$ motion of the node in a year; therefore 13°. 3'. is the secular motion. As $Nx = 44°.\ 55'$, the node of Jupiter is regressive.

1077. The angles at N and v being equal (1075), the intersection A about which the orbit moves must be 90° from N or v. Hence, as the two orbits AN, Av, and BN, Bw, diverge from each other in each direction for 90° from A and B, it is manifest, that in each case the angle at r is less than that at z, and therefore the inclination upon the ecliptic is diminished; but the angle at s is

* By spherical Trigonometry prop. 35, tan. $\frac{1}{2}$. $Nz \sim Nx$ = tan. $\frac{1}{2}$. $zx \times \frac{\text{sin. } \frac{1}{2}.\ Nxz \sim Nzx}{\text{sin. } \frac{1}{2}.\ Nxz + Nzx}$, and tan. $\frac{1}{2}$. $Nz + Nx$ = tan. $\frac{1}{2}$. $zx \times \frac{\text{cos. } \frac{1}{2}.\ Nxz \sim Nzx}{\text{cos. } \frac{1}{2}.\ Nxz + Nzx}$; hence, finding half the sum and difference of Nz, Nx, we get Nz and Nx. Thus we at once determine, whether the node of each is progressive [illegible]ive.

greater than that at x, and therefore the inclination upon the ecliptic is increased. Hence, by placing the nodes in all their different positions, we deduce this rule, given by M. de la LANDE. *Whenever the node of the orbit of the planet which attracts, is forwarder than the node of that which is attracted, the inclination of the orbit of the attracted body to the ecliptic is diminished, if the distance of the nodes be less than* 180°; *otherwise, it is increased.* We here mean the same node, that is, the ascending or descending one. Now if we arrange the planets according to the situation of their nodes, beginning with that whose node is forwardest, they will stand thus: Saturn, Georgian, Jupiter, Venus, Mars, and Mercury. Hence, *Saturn* diminishes the inclination of all the other orbits; *Jupiter* increases that of Saturn, and diminishes all the rest; and so on. This is upon supposition that the ecliptic is immoveable.

1078. Now to find how much the inclination of each varies in a given time, we may consider the triangle Nzx to vary and become vrx, where the two angles x and N remain constant; hence, by spherical Trigonometry, Prop. 54. variat. $\angle z = Nv \times \sin. x \times \sin. xz$.

Ex. The node of *Mars* goes back upon the orbit of *Jupiter* 14″,2 in a year, also the angle NxE for Jupiter is 1°. 19′, and the distance zx of their nodes = 50°. 22′; hence, Jupiter diminishes the inclination of the orbit of Mars by a quantity = 14″,2 × sin. 1°. 19′ × sin. 50°. 22′ = 0″,248 in a year, or 24″,8 in 100 years.

1079. The following Tables contain the annual movement of the nodes, and the secular change of the inclinations of the orbit of each planet from the attraction of the rest, according to the theory of M. de la GRANGE.

ANNUAL MOTION OF THE NODES.					
By the Action of	Mercury	Venus	Mars	Jupiter	Saturn
Mercury	− 0″,10	+ 0″,16	− 0″,32	− 0″,31	− 0″,11
Venus	− 5, 57	− 7, 46	−11, 80	−17, 56	− 8, 06
Earth	− 0, 87	− 6, 69	− 1, 77	− 0, 01	− 0, 00
Mars	− 0, 14	− 0, 29	− 0, 43	− 0, 39	− 0, 14
Jupiter	− 2, 18	− 5, 13	−11, 0	− 6, 95	−12, 28
Saturn	− 0, 12	− 0, 09	− 0, 47	+ 5, 88	− 0, 34
Total	− 8, 98	−19, 50	−25, 79	−19, 34	−20, 93
Precession	50, 25	50, 25	50, 25	50, 25	50, 25
Mot. in long.	41, 27	30, 75	24, 46	30, 91	29, 32

The first perpendicular line shows the body which attracts, and the first horizontal line, the body which is attracted. When we see, for instance, that Mars attracts Mars − 0″,43, it is not that Mars attracts itself, but it displaces the ecliptic, and makes the nodes of Mars move − 0″,43 upon it. And by subtracting the regression of the nodes from the precession of the equinoxes, we get the motion of the nodes in longitude. In order to adapt the effect of Venus in the second horizontal line to what it would be upon *our* supposition of its density, we must diminish all the numbers in the ratio of 1,3825 : 1,024.

SECULAR CHANGE OF THE INCLINATIONS.					
By the Action of	Mercury	Venus	Mars	Jupiter	Saturn
Mercury	0″,00	+ 1″,94	− 0″,05	− 0″,95	− 1″,10
Venus	+ 9, 46	0, 00	+17, 95	−17, 67	−26, 65
Mars	+ 0, 06	− 0, 42	0, 00	− 1, 06	− 1, 25
Jupiter	+ 9, 87	+ 2, 60	−13, 20	0, 00	+ 5, 89
Saturn	+ 1, 04	+ 0, 35	− 1, 25	− 7, 51	0, 00
Total.	+20, 43	+ 4, 47	+ 3, 45	−27, 19	−23, 11

We must diminish the second horizontal line as before, in order to adapt it to the density which we have assumed. This secular change of inclination takes into consideration the displacement of the ecliptic.

On the Change of the Plane of the Ecliptic.

FIG. 237. 1080. Let ΥL be the position of the ecliptic at any given time, BVA the orbit of *Venus*, ΥB the equator. Then as the attraction of Venus upon the earth causes the orbit of the earth to go back upon the orbit of Venus, let rcV be the next position of the ecliptic. Now as the longitude ΥA of the node of Venus's orbit is about $2^s. 14\frac{1}{2}^\circ$, and the angle $rAV = rVB$, because (1075), the inclination of the two orbits is not altered, we have $Ar = 90°$, therefore r lies backwards beyond Υ, consequently the ecliptic has gone forwards upon the equator from Υ to c; and from the motion of the ecliptic, the latitude and longitude of the stars will be affected. Hence, supposing the triangle ΥBA to become cBV, and the angles B and V to remain constant, we have (as in Art. 1076.) $\Upsilon c = \frac{AV \times \sin. B \times \sin. \Upsilon B \times \cot. \Upsilon A}{\sin. \Upsilon} =$ (because $\sin. B \times \sin. \Upsilon B$ $= \sin. A \times \sin. \Upsilon A$, and $\sin. \Upsilon A \times \cot. \Upsilon A = \cos. \Upsilon A$) $\frac{AV \times \sin. A \times \cos. \Upsilon A}{\sin. \Upsilon}$ the progression of the equinoctial points upon the equator, from the action of Venus. Now to find the value of this upon the ecliptic, draw co perpendicular to ΥL, and then we shall have $\Upsilon o = \frac{AV \times \sin. A \times \cos. \Upsilon \times \cos. \Upsilon A}{\sin. \Upsilon} = AV \times$ $\sin. A \times \cot. \Upsilon \times \cos. \Upsilon A$ the progression of the equinoctial points upon the ecliptic. For *Jupiter*, *Saturn*, and the *Georgian*, A lies above three signs from Υ and therefore r lies on the other side of Υ as in FIG. 238. therefore c and o lie on the other side of Υ; consequently they cause the equinoctial points to go *backwards*; but ΥA being less than three signs for all the other planets, they cause the equinoctial points to go *forwards*.

FIG. 238.

1081. Now to find the variation of the angle Υ, we have, (as in Art. 1076.) variat. $\angle \Upsilon = AV \times \sin. B \times \sin. \Upsilon B =$ (because $\sin. B \times \sin. \Upsilon B = \sin. A \times \sin. \Upsilon A$) $AV \times \sin. A \times \sin. \Upsilon A$. Now as Vr is $90°$, Vr is converging to Ar, and therefore the angle BcV is less than $B \Upsilon A$, consequently Venus diminishes the obliquity of the ecliptic. And as Υr is less than $90°$ for all the planets, they *all* tend to diminish the obliquity of the ecliptic. When, by the motion of the equinoxes and node A, ΥA becomes greater than 180°, its sign becomes negative, and the obliquity will be increased. Hence, when the longitude of the ascending node of a planet's orbit is *less* than 180°, it *diminishes* the obliquity of the ecliptic, but when *greater* than 180°, it *increases* it.

1082. To find the variation in latitude of any star s. Draw $Sv w$ perpendicular to Vr, and sx to rA. Then as ♈$c = \frac{AV \times \sin. A \times \cos. ♈A}{\sin.}$, therefore $co = AV \times \sin. A \times \cos.$ ♈A, and $co : vw :: \sin. r$♈ $: \sin. rw :: \cos. A$♈ $: \cos. Av$; hence, $vw = AV \times \sin. A \times \cos. Av$, *the variation of latitude*. FIG. 237.

1083. To find the variation of longitude. From the variation of the right angled triangle srx, whose hypothenuse sr is constant, we find (1047) $vx = \frac{vw \times \tan. vs}{\tan. wr}$; hence, $vx = AV \times \sin. A \times \sin. Ax \times \tan. \text{lat.} = AV \times \sin. A \times$ (sin. of longitude of the star − longitude of the node of the planet's orbit) × tan. of the star's latitude. This is one part of the variation; but as the ecliptic has moved upon the equator from ♈ to c, the longitude of all the stars will be also altered by ♈$o = AV \times \sin. A \times \cot.$ ♈ $\times \cos.$ ♈A. Hence, the whole *variation in longitude* $= AV \times \sin. A \times \sin. \overline{♈x - ♈A} \times \tan. \text{lat.} - AV \times \sin. A \times \cot.$ ♈ $\times \cos.$ ♈A, where regard must be had to the signs of $\sin. \overline{♈x - ♈A}$, tan. lat. and cos. ♈A, the last of which changes for *Jupiter*, *Saturn* and the *Georgian*, because for these planets, ♈A is greater than 90°. If the star be very near the pole of the ecliptic, the expressions fail. Hence, if

m = the inclination of the planet's orbit to the ecliptic,
w = the obliquity of the ecliptic,
L = the longitude of the ascending node of the planet's orbit,
l = the longitude of the star,
t = its latitude,
$a = AV$; then,

I. $a \times \sin. m \times \sin. L$ = the *diminution* of the obliquity, whilst the sin. L is *positive*, which is the case at present for all the planets. When L becomes greater than 180°, the planet will *increase* the obliquity (1081).

II. $a \times \sin. m \times \cos. \overline{l - L}$ = the *increase* of the latitude of a star on the *north* side of the ecliptic, within 90° longitude of A, but the *decrease* for one on the *south* side. And the contrary, if more than 90° longitude from A (1082).

III. $a \times \sin. m \times \cot. w \times \cos. L$ = the *progression* of the equinoctial points upon the ecliptic, when cos. L is *positive*, and the *regression*, when *negative* (1080).

IV. $a \times \sin. m \times \sin. \overline{l - L} \times \tan. t$ = that part of the variation of the star's longitude which belongs to each particular star (1083), where t must be written negative when the latitude is south. This expression is sufficient, when we

only want to compare the variation of the longitudes of two stars in respect to each other. The part (1083) common to all the stars is $a \times \sin. m \times \cot. w \times \cos. L$, which *diminishes* the longitude when $\cos. L$ is *positive*, and *increases* it when *negative*, and therefore it must be written negative to give it its proper effect. Hence, the whole variation in longitude $= a \times \sin. m \times \sin. \overline{l-L} \times \tan. t - a \times \sin. m \times \cot. w \times \cos. L$, regard being had to the signs of the quantities, $\sin. \overline{l-L}$, $\tan. t$; $\cos. L$; where the longitude will be *increased* or *diminished*, according as this quantity is *positive* or *negative*.

1084. These expressions agree with those in *L'Histoire de l'Acad. des Sciences*, 1754, given by M. EULER, who first discovered and explained the cause of the diminution of the obliquity of the ecliptic.

1085. The following Table contains the values of a for the respective planets, according to our determination; together with the place of the nodes of each, their secular motions in longitude, and the inclination of their orbits, according to M. de la LANDE.

Planets.	Node in 1750.	Secular Mot.	Inclination.	$a =$
Mercury	1s. 15°. 20′. 43″	1°. 12′. 10″	7°. 0′. 0″	10″
Venus	2. 14. 26. 18	0. 51. 40	3. 23. 35	558
Mars	1. 17. 38. 38	0. 46. 40	1. 51. 0	22
Jupiter	3. 7. 55. 32	0. 59. 30	1. 18. 56	693
Saturn	3. 21. 32. 22	0. 55. 30	2. 29. 50	34
Georgian	3. 12. 33. 31		0. 46. 20	0,7

The secular motion of the nodes of the *Georgian* cannot yet be determined from observation. By theory, M. de la GRANGE makes it 12′. 30″; but M. de la LANDE makes it 20′. 40″ from taking a different quantity of matter for *Venus*.

On the Motion of the Equinoxes from the disturbing forces of the Planets upon the Earth.

1086 By Article 1083, III. the motion of the equinoctial points upon the ecliptic in an hundred years at this time is, $\cot. 23°. 28' \times (10'' \times \sin. 7° \times \cos. 45°. 21' + 558'' \times \sin. 3°. 23',5 \times \cos. 74°. 26' + 22'' \times \sin. 1°. 51' \times \cos. 47°. 38',5$

$+693'' \times$ sin. $1^{\circ}. 19' \times$ cos. $97^{\circ}. 55' + 34'' \times$ sin. $2^{\circ}. 30' \times$ cos. $113^{\circ}. 22' + 0'',7 \times$ sin. $46' \times$ cos. $102^{\circ}. 33') = 17'',4$, which, being positive, shows that the motion of the equinoxes upon the ecliptic, from this cause, is according to the order of the signs; and consequently by this quantity the longitude of all the stars will be diminished in 100 years at *this* time, independent of other causes. Hence, if we take the whole secular precession to be $1^{\circ}. 23'. 45''$, as the motion of the equinoxes would be $17'',4$ progressive from the planets, the whole regression from the action of the sun and moon upon the earth, must be $1^{\circ}. 24'. 2'',4$.

1087. To find the motion of the equinoxes from the same cause, for the first century of this æra, we have, from the secular motion of the nodes, the longitude of the node of *Mercury* at that time $0^{s}. 24^{\circ}. 53'. 53''$, of *Venus* $1^{s}. 29^{\circ}. 47'. 58''$, of *Mars* $1^{s}. 4^{\circ}. 27'. 18''$, of *Jupiter* $2^{s}. 21^{\circ}. 4'. 2''$, of *Saturn* $3^{s}. 5^{\circ}. 48'. 52''$, and taking the node of the *Georgian* the same, we have the precession in that century $=$ cot. $23^{\circ}. 36' \times (10'' \times$ sin. $7^{\circ} \times$ cos. $24^{\circ}. 54' + 558'' \times$ sin. $3^{\circ}. 23',5 \times$ cos. $59^{\circ}. 48' + 22'' \times$ sin. $1^{\circ}. 51' \times$ cos. $34^{\circ}. 27' + 693'' \times$ sin. $1^{\circ}. 19' \times$ cos. $81^{\circ}. 4' + 34'' \times$ sin. $2^{\circ}. 30' \times$ cos. $95^{\circ}. 49' + 0'',7 \times$ sin. $46^{\circ} \times$ cos. $102^{\circ}. 33') = 47''$, which, being positive, shows that the motion of the equinoxes, from the attraction of the planets, was then progressive, by that quantity.

1088. The precession (1022, 1035) of the equinoxes from the sun and moon, by displacing the equator, varies as the cosine of the obliquity of the ecliptic, and therefore as the obliquity diminishes, the precession increases; this will increase the precession about $9''$ in 1700 years, from this cause. Therefore in the first 100 years of our æra, the precession of the equinoxes, from the action of the sun and moon, must have been $1^{\circ}. 24'. 2'',4 - 9'' = 1^{\circ}. 23'. 53'',4$; therefore $1^{\circ}. 23'. 53'',4 - 47'' = 1^{\circ}. 23'. 6'',4$ the whole regression for that time. Hence, $1^{\circ}. 23'. 45'' - 1^{\circ}. 23'. 6'',4 = 38'',6$ the quantity by which the regression is faster now in 100 years than it was in the first 100 years of our æra. In consequence of this, the tropical year keeps decreasing; and this will continue till the place of the nodes of *Jupiter* and *Venus*, from which the principal cause arises, be got into such a situation, that the displacements of the ecliptic and the equator together produce a *retarded* precession of the equinoxes. The regression of the equinoctial points is (at the above rate) faster now by $0,386''$ in a year than it was at the beginning of our æra; now the sun takes $9''$ to move over that space; hence, the tropical year is $9''$ shorter now than it was about 1700 years ago. The tropical year has therefore decreased at the mean rate of about *half a second* in 100 years. M. de la PLACE makes the year shorter now by $10'',33$ than it was at the time of HIPPARCHUS, who lived about 1950 years ago. These conclusions therefore agree very well.

1089. This increase of regression $38'',6$ in 17 ages, gives $2'',27$ for every 100 years, supposing it to increase uniformly; and as it was $1^{\circ}. 23'. 6'',4$ in the

first century, by the addition of 2″,27 we get the precession for every century after.

On the Variation of the Obliquity of the Ecliptic.

1090. By Art. 1083, I, the whole diminution of the obliquity of the ecliptic, at this time, in 100 years is 10″ × sin. 45°. 21′ × sin. 7° + 558″ × sin. [illegible] 26′ × sin. 3°. 23′,5 + 22″ × sin. 47°. 38′,5 × sin. 1°. 51′ + 693″ × sin. 97°. 55′,5 × sin. 1°. 19′ + 34″ × sin. 111°. 32′ × sin. 2°. 30′ + 0,7″ × sin. 102°. 33′ × sin. 46′ = 49″,35; this conclusion agrees very well with observations, from which it appears that at *this time* the obliquity diminishes at the rate of about 50″ in 100 years.

1091. Now allowing for the motion of the nodes of the planets, as in Art. 1086. we have the secular diminution for the *first* 100 years of our era 10″ × sin. 24°. 54′ × sin. 7° + 558″ × sin. 59°. 48′ × sin. 3°. 23′,5 + 22″ × sin. 34°. 27′ × sin. 1°. 51′ + 693″ × sin. 81°. 4′ × sin. 1°. 19′ + 34″ × sin. 95°. 49′ × sin. 2°. 30′ + 0,7″ × sin. 102°. 33′ × sin. 46′ = 45″,43, which is 3″,92 less than in our age. We have here supposed the inclination of the orbits to remain the same.

1092. The motion of the node of the ecliptic upon the orbit of any planet : the cotemporary variation of the inclination of the ecliptic, from the attraction of that planet (1081) :: $A\dot{V} : AV \times$ sin. $A \times$ sin. ΥA :: (because $\dot{\overline{\Upsilon A}}$: $\overline{\text{vers. sin. } \Upsilon A}$ :: rad. = 1 : sin. ΥA) $\dot{\overline{\Upsilon A}}$: sin. $A \times \overline{\text{vers. sin. } \Upsilon A}$; hence, by taking the fluents, the motion of the node of the ecliptic upon the orbit of the planet : the cotemporary variation of the inclination of the ecliptic from the time of coincidence of A and Υ :: ΥA : sin. $A \times$ vers. sin. ΥA. When $\Upsilon A = 180°$, the ratio becomes c : sin. A, c being one fourth of the circumference of a circle whose radius is unity. Now the secular motion of the longitude of the node of each planet being known, the time in which ΥA from nothing becomes 180°, will be known; hence, the motion $A\dot{V}$ in that time will be known; put that quantity $= d$, and we have the whole diminution of the obliquity of the ecliptic in the time the ascending node of a planet's orbit moves from *Aries* to *Libra* $= \frac{d \times \text{sin. } A}{c}$. Whilst the node moves from Libra to Aries, the obliquity will be increased by the same quantity. Now the value of $\frac{d \times \text{sin. } A}{c}$ for *Mercury* = 1′. 56″; for *Venus* = 1°. 13′. 12″; for *Mars* = 1′. 44″; for *Jupiter* = 30′. 41″, and for *Saturn* = 3′. 4″; the effect of the *Georgian* we here omit, as it would be extremely small. Hence, if all these could conspire they would diminish the obliquity of the ecliptic 1°. 50′. 37″; but as this is not

the case, the diminution must be less. The principal effects are produced by Venus and Jupiter; now the ascending node of Venus would have coincided with Aries about 8700 years before 1750, and that of Jupiter, 9900 years before; their effects therefore to diminish the obliquity do not begin at the same time; and for about the first 1200 years after the coincidence of Jupiter's node with Aries, the effect of Venus tended to increase the inclination; and as the annual motion of the nodes of Jupiter is not much greater than that of Venus, their effects will conspire for a very considerable part of the time in which their nodes are moving from Aries to Libra. Now the whole diminution from Venus and Jupiter together is 1°. 48'. 52"; but we cannot fix this as the limit of the variation of the equator to the ecliptic, because this calculation has been upon supposition that the inclinations of the orbits of the planets to the ecliptic remain the same, whereas they are subject to a variation. The limit of the variations of the obliquity will be greater than that which we have here given, because the inclinations of the orbits may be greater than they are at present. But a calculation (1078) of the variations of all the inclinations from their mutual attractions, and their *maxima* and *minima*, and thence the limit of the variation of the obliquity of the ecliptic to the equator, would be too long to be here introduced. As the rate of variation of the inclination appears now to be nearly uniform, it must have arrived at its maximum; consequently the inclination itself is now about its mean value; and according to M. de la GRANGE, the inclination of the ecliptic to the equator will never vary more than 5°. 23' from the year 1700. (*Mem. de l'Acad. Roy. des Scien.* 1782). It is not true therefore, what some Authors have asserted, that the ecliptic was formerly perpendicular to the equator; nor what others have asserted, that the obliquity will continue to be diminished till it coincides with the equator; nor will the small alteration which has taken place from the time of the Creation, account for the changes which some have supposed the earth has undergone.

On the Variation of a Star's Latitude and Longitude.

1003. Put A, B, C, D, E, F for the longitudes of the ascending nodes of *Mercury*, *Venus*, *Mars*, *Jupiter*, *Saturn* and the *Georgian*; m, n, r, s, v, w for the inclinations of their orbits to the ecliptic, and a, b, c, d, e, f for the respective values of H; then (1083, II.) the variation of latitude from the joint effect of all the planets $= a \times \sin. m \times \cos. \overline{l-A} + b \times \sin. n \times \cos. \overline{l-B} + c \times \sin. r \times \cos. \overline{l-C} + d \times \sin. r \times \cos. \overline{l-D} + e \times \sin. v \times \cos. \overline{l-E} + f \times \sin. w \times \cos. \overline{l-F}$. But this expression may be rendered more convenient, by substituting $\cos. A \times \cos. l + \sin. A \times \sin. l$ for $\cos. \overline{l-A}$, and making the like sub-

stitution for the other cosines; hence, the *variation of latitude* $=(a \times \sin. m \times \cos. A + b \times \sin. n \times \cos. B + c \times \sin. r \times \cos. C + d \times \sin. s \times \cos. D + e \times \sin. v \times \cos. E + f \times \sin. w \times \cos. F) \times \cos. l + (a \times \sin. m \times \sin. A + b \times \sin. n \times \sin. B + c \times \sin. r \times \sin. C + d \times \sin. s \times \sin. D + e \times \sin. v \times \sin. E + f \times \sin. w \times \sin. F) \times \sin. l$. Now if we take the values of these quantities as in the Table, we have the variation of latitude for 100 years from $1750 = 7'',558 \times \cos. l + 49'',349 \times \sin. l$.

Ex. The longitude of *Regulus* on January 1, 1760, was 4ˢ. 26°. 29′. 30″, whose sine $= 0,55205$, cos. $= -0,8838$; hence, $7'',558 \times -0,8838 + 49'',349 \times 0,55205 = 20'',56$, the *increase* of the star's latitude in 100 years, the latitude of the star being *north*.

1094. If we make the same substitution for the part of the variation in longitude which is common to every star (1083), we shall have its value $= \overline{7'',558 \times \sin. l - 49'',349 \times \cos. l} \times \tan. t$. Now (1086) the longitude of all the stars will be diminished 17″,4 from the motion of the equinoctial points, independent of the above; hence, the whole *variation in longitude* $= \overline{7'',558 \times \sin. l - 49'',349 \times \cos. l} \times \tan. t. - 17'',4$. This expression is for stars of *north* latitude; for *south* latitude, tan. t becomes negative. As the nodes are not fixed, the values of these formulæ will vary, and may be computed at any time, by assuming the places of the nodes at that time.

Ex. The longitude of *Regulus* on January 1, 1760, was 4ˢ. 26°. 29′. 30″, whose sine $= 0,55205$, cos. $= -0,8838$, and latitude 0°. 27′. 27″ N. whose tangent $= 0,007985$; hence, $\overline{7'',558 \times 0,55205 - 49'',349 \times -0,8838} \times 0,007985 - 17'',4 = 0'',377 - 17'',4 = -17'',023$, the secular variation of the longitude of Regulus.

1095. If we want only to find how much the difference of the longitudes of two stars have varied, we may leave out that part of the variation common to all the stars, and only compute the variation belonging to each particular star.

1096. The variations of the latitudes and longitudes of the stars thus determined, are found to agree very well with observations, by comparing the latitudes and longitudes of the stars as given by PTOLEMY in his Catalogue, with the latitudes and longitudes observed at this time.

Ex. The latitude of the star N°. 27. of the *Great Bear* is 54°. 27′, and that of N°. 10. *Draconis* is 81°. 48′; also,

The longitude in 1700,		Long. at the time of PTOLEMY,	
Of the first star - - -	5. 22°. 34′	Of the first star - - -	4. 29°. 50
Of the second - - - -	11. 29. 23	Of the second - - -	11. 8. 0
Diff. of Long. - - -	6. 6. 49	Diff. of Long. - - -	6. 8. 10

Hence, the difference of the longitudes of these two stars has decreased 1°. 21′ since the time of PTOLEMY, and this agrees nearly with the above theory, by computing the variation of the longitude of each star for the above interval of time.

1097. In the constellation *Auriga*, at the time of PTOLEMY the longitude of the star, N°. 1, was ♊ 25°. 36′, and latitude 30°. N.; and in 1700 its latitude was 30°. 49′ N. The longitude of N°. 12. was ♊ 19°. 50′, and latitudes 8°. 30′ and 8°. 51′ at the above times. The former latitude was therefore increased 19′ and the latter 21′, which agrees very well with the theory.

On the Motion of the Orbits of the Satellites of Jupiter, *from their mutual Attractions.*

1098. The motion of the nodes of the satellites are found in the same manner as those of the primary planets, taking Jupiter for the central body, instead of the sun.

Ex. 1. To find the motion of the node of the second satellite of Jupiter upon the orbit of the third, from the attraction of the third. Here $\frac{1}{m} = 0,0000687$ (1073), $a =$ [illegible]; hence, in one revolution of the second satellite, the motion of its node $= 42'',386$; and considering 105 revolutions in a year, its annual motion is [illegible]°. 14′.

Ex. 2. To find the motion of the nodes of the third satellite of Jupiter upon the orbit of the second, from the attraction of the second. Here $\frac{1}{m} =$ 0,0002417, $a =$ [illegible]; hence, in one revolution of the third satellite, the motion of its node $= 23'',61$; and considering 51 revolutions in a year, its annual motion is $20',4$.

And let a', $e' \times a'$, L', Γ' and γ' be the same in respect to p; and let $\frac{a'}{a} = n$;

$$b = \frac{1}{\overline{1+n^2}|^{\frac{3}{2}}} \times \left(1 + 1 - \frac{1}{4^2} \times \overline{\frac{2n}{1+n^2}}\Big|^2 + \frac{1}{4^2} \times \frac{1}{8^2} \times \overline{\frac{2n}{1+n^2}}\Big|^4 + \frac{1}{4^2} \times \frac{1}{8^2} \times \frac{1}{12^2} \times \overline{\frac{2n}{1+n^2}} + \&c.\right)$$

$$c = \frac{3n}{\overline{1+n^2}|^{\frac{5}{2}}} \times \left(1 + \frac{3}{4.3^2-1} \times \overline{\frac{2n}{1+n^2}}\Big|^2 + \frac{3}{4.3^2-1} \times \frac{3}{4.5^2-1} \times \overline{\frac{2n}{1+n^2}}\Big|^4 + \frac{3}{4.3^2-1} \times \frac{3}{4.5^2-1} \times \frac{3}{4.7^2-1} \times \overline{\frac{2n}{1+n^2}}\Big|^6 + \&c.\right)$$

1102. Let v be the number of revolutions of P from the given epoch, v being negative for any time *before* the epoch; then

The increase of the equation of the center is

$$m \times v \times 360^\circ \times e' \times \sin. \overline{L'-L} \times c \times \overline{1+n^2} - 3bn.$$

The mean motion of the aphelion according to the order of the signs is

$$m \times v \times 360^\circ \times (\tfrac{1}{4}\, nc - \tfrac{1}{2} \cdot \frac{e'}{e} \times \cos. \overline{L'-L} \times c \times \overline{1+n^2} - 3bn).$$

The diminution of the inclination of the orbit upon the fixed plane is

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc\gamma' \times \sin. \overline{\Gamma'-\Gamma}.$$

The retrograde motion of the node upon the fixed plane is

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc \times \overline{1 - \frac{\gamma'}{\gamma}} \times \cos. \overline{\Gamma'-\Gamma}.$$

If we thus collect the effects of all the planets upon P, the sum will give the whole variation of the elements of the orbit of P. But these formulæ only serve for a limited time from the given epoch.

1103. If we fix upon the year 1700 for the epoch, we shall have,

$$\tfrac{1}{4}\, m \times v \times 360^\circ \times nc\gamma \times \sin. \Gamma$$

for the diminution of the obliquity of the ecliptic, produced by the action of p upon the earth; v being the number of revolutions of the earth from 1700, γ' the inclination of the orbit of the planet p upon the ecliptic, and Γ' the longitude of the ascending node. The sum of all the diminutions from all the planets will give the whole diminution. When Γ' is greater than 180°, which it is not at present for any of the planets, the sin. Γ' becomes negative, and the obliquity will then be increased by that planet.

1104. Hence, by collecting the effects of all the planets, if v be the number of years *after* 1700, the whole diminution will be the difference between $932'',56$ and $932'',56 \times \cos. 17'',7686v - 3140'',34 \times \sin. 32'',8412v$, supposing the variation at this time to be $50''$ in 100 years. For any number of years *before* 1700, the sign of the second term must be changed.

1105. In like manner, all the planets together will produce a precession of the equinoxes equal to $50'',53353v - 3292'',28 \sin. 17'',7686v - 9315'',65 \cos. 32'',8412v + 9315'',65$ in v years *after* 1700; for any time *before*, the sign of v becomes negative. The inequality of the precession of the equinoxes changes the secular motion of the sun in respect to the equinoxes; this motion is $46'$ in this age, but it was $45'. 23''$ in the beginning of our æra. Hence, the place of the sun, calculated with the uniform secular motion of $46'$, as in our Tables, will require a secular equation. And this secular motion of the sun gives, for the increase of the year by going back from 1700, $36'',114 \times \sin. 32'',8412v + 6'',9039 \times \cos. 17'',7686v - 6'',9039$. Hence, at the time of HIPPARCHUS the year was about $10'',33$ longer than at this time (1088).

CHAP. XXXVII.

ON THE EFFECTS PRODUCED ON THE MOTIONS OF THE PLANETS IN THE PLANES OF THEIR ORBITS, FROM THEIR MUTUAL ATTRACTIONS.

Art. 1106. LET S be the center of force, PM the orbit of a body described FIG. 239.
by virtue of two forces, one (f) tending from the body at P in the direction PS, and the other (F) acting in a direction Pw perpendicular to PS. Let SQ be a line given in position; draw PA perpendicular to SQ, and complete the parallelogram $PASV$, and draw Vr, At perpendicular to PS. Of the whole force acting on P, let M be that part which acts in the direction PA, and m the part acting in the direction PV; also, let $r = PS$, $p = PA$, $q = PV$, $\dot{t} =$ the fluxion of the time, $v =$ the angle PSQ, $x =$ its sine, $y =$ its cosine; then $Mx =$ that part of the force M which acts in the direction PS, and $my =$ that part of the force y which acts in the same direction; hence, $Mx + my = f$; also, $My =$ that part of the force M which acts in a direction parallel to tA, and $mx =$ that part of the force m which acts in a direction parallel to rV; hence, $mx - My = F$. Now by the principles of motion*, $\ddot{p} = -M\dot{t}^2$, and $\ddot{q} = -m\dot{t}^2$, the fluxion of the time being constant. But $p = rx$, and $q = ry$; and as $\dot{x} = y\dot{v}$, and $\dot{y} = -x\dot{v}$, we have,

$$\dot{p} = x\dot{r} + ry\dot{v},$$

$$(A)\ \ddot{p} = x\ddot{r} + 2y\dot{r}\dot{v} + ry\ddot{v} + rx\dot{v}^2 = -Mt\ ;$$

$$\dot{q} = y\dot{r} - rx\dot{v},$$

$$(B)\ \ddot{q} = y\ddot{r} - 2x\dot{r}\dot{v} - rx\ddot{v} - ry\dot{v}^2 = -m\dot{t}^2\ ;$$

Multiply (A) by y and (B) by x, and subtract the latter from the former, and we get

$$(C)\ 2\dot{r}\ddot{v} + r\ddot{v} = -\dot{t}^2 \times \overline{My - mx} = F\dot{t}^2.$$

* If F represent any accelerative force, v the velocity which the body would acquire in 1'' by that force, $\dot{s}$ the fluxion of the space described in a given time $\dot{t}$ with that velocity, then if we measure the velocity v by the space described uniformly in 1'', we have 1'' : $\dot{t}$ :: v : $\dot{s} = v\dot{t}$, therefore $\ddot{s} = \pm v\ddot{t}$; but $\dot{v}$ varies as $F \times \dot{t}$; let therefore $\dot{v} = F \times \dot{t}$, and $\ddot{s} = \pm F\dot{t}^2$.

Multiply (A) by x and (B) by y and add them together, and we get

$$\ddot r - r\dot v^2 = -\dot t^2 \times \overline{Mx+my} = -f\dot t^2, \text{ or}$$
$$(D)\ r\dot v^2 - \ddot r = f\dot t^2.$$

From the equations (C) and (D), the curve PM described by the body P may be found. These fluxional equations are the same as those determined by CLAIRAUT, EULER and MAYER, in their Treatises upon the theory of the moon, the integration of which is a problem of great difficulty; and as the first of these Authors has proceeded in a manner the most easy to be understood by the generality of readers, I shall here enter into a very full explanation of all his principles of investigation.

1107. Let the force f consist of two parts, one of which $=\frac{C}{r^2}$, C being a constant quantity, and the other $=D$; or let $f=\frac{C}{r^2}+D$; then $2\dot r\dot v + r\ddot v = F\dot t^2$; and $r\dot v^2 - \ddot r = \overline{\frac{C}{r^2}+D} \times \dot t^2$. Multiply the first of these equations by $\frac{r}{\dot t}$, and we get $\frac{2r\dot r\dot v + r^2\ddot v}{\dot t} = Fr\dot t$, whose fluent ($\dot t$ being constant) is $\frac{r^2\dot v}{\dot t} = a + \int Fr\dot t$, a being a constant quantity. Multiply this equation by $Fr\dot t$, and we have $Fr^3\dot v = aFr\dot t + Fr\dot t \times \int Fr\dot t$, whose fluent is $\int Fr^3\dot v = a\int Fr\dot t + \frac{1}{2}\overline{\int Fr\dot t}^2$ *; multiply this by 2, and add a^2 to both sides, and it becomes $a^2 + 2\int Fr^3\dot v = a^2 + 2a\int Fr\dot t + \overline{\int Fr\dot t}^2$; whose square root is $\sqrt{a^2 + 2\int Fr^3\dot v} = a + \int Fr\dot t$. Hence, $\frac{r^2\dot v}{\dot t} = \sqrt{a^2 + 2\int Fr^3\dot v}$, consequently $\dot t = \frac{r^2\dot v}{\sqrt{a^2 + 2\int Fr^3\dot v}} =$ (if we put $e = \frac{\int Fr^3\dot v}{a^2}$) $\frac{r^2\dot v}{a\sqrt{1+2e}}$.

1108. Let us next take the other equation $r\dot v^2 - \ddot r = \overline{\frac{C}{r^2}+D} \times \dot t^2$. Now in this equation, $\dot t$ is constant and $\dot r$ variable; but it is well known, that in a fluxional equation of the second order, where $\dot r$ is variable and $\dot t$ constant, that if we substitute $-\frac{\dot r\ddot t}{\dot t}$ for $\ddot r$, the equation will be changed into one in which $\dot r$ is constant and $\dot t$ variable; if therefore for $\ddot r$ we substitute $\ddot r - \frac{\dot r\ddot t}{\dot t}$, the equation will be changed into one in which $\dot r$ and $\dot t$ are both variable. Substituting there-

* For if $y = \int Fr\dot t$, then $\dot y = Fr\dot t$, therefore $y\dot y = Fr\dot t \times \int Fr\dot t$, and the fluent $= \frac{1}{2}y^2 = \frac{1}{2}\overline{\int Fr\dot t}^2$.

fore $\ddot r - \frac{\dot r \ddot t}{\dot t}$ for $\ddot r$ in the above equation, it becomes $r\dot v^2 - \ddot r + \frac{\dot r \ddot t}{\dot t} = \frac{C}{r^2} + D \times \dot t^2$, in which equation we may assume $\dot r$ or $\dot v$ constant, as it may be found convenient. But (1107) $\dot t = \frac{r^2 \dot v}{a\sqrt{1+2e}}$, therefore $\ddot t = \frac{2r\dot r\dot v \times a \times \overline{1+2e} - ar^2\dot v\dot e}{a^2 \times \overline{1+2e} \times \sqrt{1+2e}}$; hence, $\frac{\ddot t}{\dot t} = \frac{2\dot r}{r} - \frac{\dot e}{1+2e}$; by substitution, therefore we get $r\dot v^2 - \ddot r + \frac{2\dot r^2}{r} - \frac{\dot r\dot e}{1+2e} = \frac{\frac{C}{r^2} + D \times r^4 \dot v^2}{a^2 \times \overline{1+2e}}$; divide both sides by $r^2\dot v^2$, and transpose $-\frac{\dot r\dot e}{1+2e}$, and we get $\frac{1}{r} - \frac{\frac{\ddot r}{r^2} - \frac{2\dot r^2}{r^3}}{\dot v^2} = \frac{C + Dr^3 + \frac{a^2\dot r\dot e}{r^2\dot v^2}}{a^2 \times \overline{1+2e}}$. Now $\frac{\ddot r}{r^2} - \frac{2\dot r^2}{r^3} =$ flux. $\frac{\dot r}{r^2}$; put also $1 + P = \frac{1 + \frac{Dr^3}{C} + \frac{a^2\dot r\dot e}{Cr^2\dot v^2}}{1+2e} = \left(\text{as } e = \frac{\int Fr^3\dot v}{a^2}, \text{ and therefore } \dot e = \frac{Fr^3\dot v}{a^2}\right)$ $\frac{1 + \frac{Dr^3}{C} + \frac{Fr\dot r}{C\dot v}}{1+2e}$, and then $P = \frac{\frac{Dr^3}{C} + \frac{Fr\dot r}{C\dot v} - 2e}{1+2e}$; hence, by substitution we get $\frac{1}{r} - \frac{\text{flux.}\ \frac{\dot r}{r^2}}{\dot v^2} = \frac{C}{a^2} \times \overline{1+P}$, or $1 - \frac{a^2}{rC} + \frac{a^2}{C\dot v^2} \times$ flux. $\frac{\dot r}{r^2} + P = 0$; put $1 - \frac{a^2}{Cr} = s$, then $\dot s = \frac{a^2\dot r}{Cr^2}$, and $\ddot s = \frac{a^2}{C} \times$ flux. $\frac{\dot r}{r^2}$, or $\frac{C}{a^2} \times \ddot s =$ flux. $\frac{\dot r}{r^2}$; hence, the equation becomes $s + \frac{\ddot s}{\dot v^2} + P = 0$.

1109. To find the fluent of this fluxional equation $s + \frac{\ddot s}{\dot v^2} + P = 0$, supposing $P = \cos.\ mv$, or of $s + \frac{\ddot s}{\dot v^2} + \cos.\ mv = 0$. Multiply it by $\cos.\ v \times \dot v$, and it becomes $\frac{\cos.\ v \times \ddot s}{\dot v} + s \times \cos.\ v \times \dot v + \cos.\ mv \times \cos.\ v \times \dot v = 0$; now the fluent of the two first terms is $\cos.\ v \times \frac{\dot s}{\dot v} + s \times \sin.\ v$*; also, $\cos.\ mv \times \cos.\ v \times \dot v = \frac{1}{2}$. cos.

* For the fluxion of $\cos.\ v \times \frac{\dot s}{\dot v} + s \times \sin.\ v$ is $\frac{\cos.\ v \times \ddot s}{\dot v} + \dot{\overline{\cos.\ v}} \times \frac{\dot s}{\dot v} + \sin.\ v \times \dot s + s \times \dot{\overline{\sin.\ v}}$ $= \frac{\cos.\ v \times \ddot s}{\dot v} - \sin.\ v \times \dot s + \sin.\ v \times \dot s + s \times \cos.\ v \times \dot v = \frac{\cos.\ v \times \ddot s}{\dot v} + s \times \cos.\ v \times \dot v$, because $\dot{\overline{\cos.\ v}} = -\sin.\ v \times \dot v$, and $\dot{\overline{\sin.\ v}} = \cos.\ v \times \dot v$. Here $\dot v$ is supposed constant, as in the last Art.

$\overline{m+1}\,.\,v\times\dot v+\frac{1}{2}\,.\,\cos.\,\overline{m-1}\,.\,v\times\dot v$, whose fluent is $\frac{1}{2\,.\,\overline{m+1}}\times\sin.\,\overline{m+1}\,.\,v+\frac{1}{2\,.\,\overline{m-1}}\times\sin.\,\overline{m-1}\,.\,v$; but $\sin.\,\overline{m+1}\,.\,v=\sin.\,mv\times\cos.\,v+\sin.\,v\times\cos.\,mv$, and $\sin.\,\overline{m-1}\,.\,v=\sin.\,mv\times\cos.\,v-\sin.\,v\times\cos.\,mv$; hence, the last quantity becomes $\frac{1}{2\,.\,\overline{m+1}}\times\sin.\,mv\times\cos.\,v+\frac{1}{2\,.\,\overline{m+1}}\times\sin.\,v\times\cos.\,mv+\frac{1}{2\,.\,\overline{m-1}}\times\sin.\,mv\times\cos.\,v-\frac{1}{2\,.\,\overline{m-1}}\times\sin.\,v\times\cos.\,mv$; the whole fluent therefore becomes, $\cos.\,v\times\frac{\dot s}{\dot v}+s\times\sin.\,v+\frac{m}{m^2-1}\times\sin.\,mv\times\cos.\,v-\frac{1}{m^2-1}\times\sin.\,v\times\cos.\,mv=g$, a correction. Multiply the last equation by $\frac{\dot v}{\cos.\,v^2}$, and it becomes $\frac{\dot s}{\cos.\,v}+\frac{s\times\sin.\,v\times\dot v}{\cos.\,v^2}+\frac{m}{m^2-1}\times\frac{\sin.\,mv\times\dot v}{\cos.\,v}-\frac{1}{m^2-1}\times\frac{\sin.\,v\times\cos.\,mv\times\dot v}{\cos.\,v^2}=g\times\frac{\dot v}{\cos.\,v^2}$; now the fluent of the two first quantities is $\frac{s}{\cos.\,v}$, for if this quantity be put into fluxions it will be found to give those terms; also, the fluent of the two next terms is $-\frac{1}{m^2-1}\times\frac{\cos.\,mv}{\cos.\,v}$, for if this quantity be put into fluxions it will produce those two terms; but this last will want a correction, because, when $v=0$, it becomes $-\frac{1}{m^2-1}$; therefore that part of the fluent corrected is $\frac{1}{m^2-1}-\frac{1}{m^2-1}\times\frac{\cos.\,mv}{\cos.\,v}$; lastly, the fluent of $g\times\frac{\dot v}{\cos.\,v^2}=g\times\tan.\,v$*; hence, the fluent becomes $\frac{s}{\cos.\,v}+\frac{1}{m^2-1}-\frac{1}{m^2-1}\times\frac{\cos.\,mv}{\cos.\,v}-g\times\tan.\,v=c$, a correction; consequently $s-g\times\sin.\,v-c\times\cos.\,v+\frac{1}{m^2-1}\times\cos.\,v-\frac{1}{m^2-1}\times\cos.\,mv=0$, the two last terms of which arose from the value of P, that is, from introducing the disturbing forces D and F, without which, the whole force would have varied inversely as the square of the distance of the body from the center of force, and the orbit described would have been an ellipse about the center of force in its focus. Hence, if instead of assuming $P=\cos.\,mv$, we assume $P=a'\times\cos.\,mv+b'\times\cos.\,nv+$ &c. and for

* For $\dot v=\frac{\dot{\overline{\tan.\,v}}}{\sec.\,v^2}=\cos.\,v^2\times\dot{\overline{\tan.\,v}}$, therefore $\frac{\dot v}{\cos.\,v^2}=\dot{\overline{\tan.\,v}}$.

s we substitute $1-\frac{a^2}{Cr}$, we obtain $\frac{a^2}{Cr}=1-g\times\sin.v-(c-\frac{a'}{m^2-1}-\frac{b'}{n^2-1}-\&c.)$ $\times\cos.v-\frac{a'}{m^2-1}\times\cos.mv-\frac{b'}{n^2-1}\times\cos.nv-\&c.=0$, which is the equation of the curve described by the two forces $\frac{C}{r^2}+D$ and F, the former tending to the center of force, and the latter acting in a direction perpendicular to the radius vector, upon the above supposition for the value of P, which supposition is always applicable in the case of the planets.

1110. It has been proved (868) that if r be equal to the radius vector from the focus of an ellipse whose semi-parameter is equal to p, c equal the distance of the focus from the center divided by the semi-axis major, and v equal to the true anomaly, then $r=\frac{p}{1-c\times\cos.v}$, therefore $\frac{p}{r}=1-c\times\cos.v$. But if we estimate the motion of the body from some other point B instead of A, and put the angle $BEM=v$, and $BEA=m$, then $AEM=v-m$, and we must put $v-m$ instead of v; hence, $\frac{p}{r}=1-c\times\cos.\overline{v-m}=1-c\times\overline{\cos.v\times\cos.m+\sin.v\times\sin.m}=$ (if $h=c\times\cos.m$, $k=c\times\sin.m$) $1-h\times\cos.v-k\times\sin.v$. It appears therefore, that the part $\frac{a^2}{Cr}=1-g\times\sin.v-c\times\cos.v$ of the equation of the curve in the last Article, expresses an ellipse, whose semi-parameter is $\frac{a^2}{C}$, described by the force $\frac{C}{r^2}$. The second part therefore expresses the alteration arising from the disturbing forces D and F. And if we estimate the motion from A, $g=0$, or that correction (1109) will be unnecessary, and the equation (putting $p=\frac{a^2}{C}$ the semi-parameter) becomes $\frac{p}{r}=1-(c-\frac{a'}{m^2-1}-\frac{b'}{n^2-1}-\&c.)\times\cos.v-\frac{a'}{m^2-1}\times\cos.mv-\frac{b'}{n^2-1}\times\cos.nv-\&c.$ FIG. 198.

1111. The equation of the ellipse which would have been described without the disturbing forces, is $\frac{1}{r}=\frac{1}{p}-\frac{c}{p}\times\cos.v$, where $c=$ the excentricity divided by the semi-axis major, and $p=$ the semi-parameter; and if we put $u=c-\frac{a'}{m^2-1}-\frac{b'}{n^2-1}-\&c.$ and $\frac{1}{p}\times(-\frac{a'}{m^2-1}\times\cos.mv-\frac{b'}{n^2-1}\times\cos.nv-\&c.)=S$, the equation of the orbit described by P becomes $\frac{1}{r}=\frac{1}{p}-\frac{u}{p}\times\cos.v+S$, where $\frac{1}{r}=\frac{1}{p}-\frac{u}{p}\times\cos.v$ is the equation of a new ellipse. Hence, the

ellipse which would have been described without the disturbing forces is changed into another ellipse very nearly, the deviation from an ellipse being only that which arises from the small quantity S. The effect therefore of the disturbing forces is to alter the excentricity of the ellipse, to change the mean distance, and to cause a small alteration in this new ellipse, the dimensions of which must be found from observations.

1112. If at the same time that the planet describes the angle v, the apside describe the angle $v - mv$, then the motion of the planet in respect to the apside $= mv$, and the true anomaly of the planet in this moveable ellipse $= mv$; therefore the equation of the moveable ellipse is $\frac{1}{r} = \frac{1}{d} - \frac{w}{d} \times \cos. mv$. Hence, as the apsides of the orbits of the planets are moveable, we must assume $\frac{1}{r} = \frac{1}{d} - \frac{w}{d} \times \cos. mv$, when we determine the value of P, d being half the parameter, and $w =$ the excentricity divided by the semi-axis major.

1113. Having determined the value of r from the general equation of the curve, we can get the time; for, $\dot{t} = \frac{r^2 \dot{v}}{a\sqrt{1+2e}}$; now $\frac{a^2}{C} = p$ the semi-parameter, and let us suppose $C = 1$; then the orbit being supposed to have but a small excentricity, and the mean distance being assumed $= 1$, we may suppose $p = 1$; hence, $a = 1$. Therefore $\dot{t} = \frac{r^2 \dot{v}}{\sqrt{1+2e}} = r^2 \dot{v} \times \overline{1+2e}|^{-\frac{1}{2}} = r^2 \dot{v} \times \overline{1-e}$, neglecting the other terms of the series.

1114. By the property of the ellipse (1112), $r = \frac{1}{d} - \frac{w}{d} \times \cos. mv$, or assuming $d = 1$, $\frac{1}{r} = 1 - w \times \cos. mv + S$, S being the correction of $\frac{1}{r}$ from the disturbing forces (1111). Put $1 - w \times \cos. mv = l$, then $r^2 = \overline{l+S}|^{-2} = l^{-2} - 2l^{-3}S$, neglecting the other terms; but $l^{-2} = \overline{1 - w \times \cos. mv}|^{-2} = 1 + 2w \times \cos. mv$, and $l^{-3} = \overline{1 - w \times \cos. mv}|^{-3} = 1 + 3w \times \cos. mv$; hence, $r^2 = 1 + 2w \times \cos. mv - 2S - 6wS \times \cos. mv$; substitute this value of r^2 into $\dot{t} = r^2 \dot{v} \times \overline{1-e}$, and we obtain $\dot{t} = (1 + 2w \times \cos. mv - 2S - 6wS \times \cos. mv)\, \dot{v} \times \overline{1-e} = \dot{v} + 2w \times \cos. mv \times \dot{v} - 2S \times \dot{v} - 6wS \times \cos. mv \times \dot{v} - e\dot{v} - 2ew \times \cos. mv \times \dot{v}$, neglecting those terms where the product of the two small quantities S and e enter. Now the two first terms are independent of e and S, and therefore we have nothing to do with them in our present enquiry, which is only to find the equations arising from the disturbing forces; but the other terms, depending upon e and S, arise from the disturbing forces; therefore the required correction of the fluxion of the time is $-\overline{2S+e} \times \dot{v} - 2w \times \overline{3S+e} \times \cos. mv \times \dot{v}$. Now if the orbit had been a perfect circle, and there had been no disturbing force, we should have had $\dot{t} = \dot{v}$, and

$t = v$, and the motion being uniform, t would have been the mean longitude; and t being here constant, it must also express the mean longitude of the body P. Let therefore $\alpha =$ the fluent of $2w \times \cos. mv \times \dot{v}$ arising from the elliptic form of the orbit, and $\beta =$ the fluent of $-\overline{2S+e} \times \dot{v} - 2w \times \overline{3S+e} \times \cos. mv \times \dot{v}$ arising from the disturbing forces; then $t = v + \alpha + \beta$, therefore $t - \alpha - \beta = v$; that is, to find the true place of the body, first correct the mean longitude by the elliptic equation (222), and then apply the fluent of $-\overline{2S+e} \times \dot{v} - 2w \times \overline{3S+e} \times \cos. mv \times \dot{v}$ with a contrary sign. If the orbit be a circle, or if the excentricity be very small, for the correction we may assume only the fluent of $-\overline{2S+e} \times \dot{v}$ with its sign changed.

1115. The quantities here found are in terms of the radius supposed to be unity; and as an arc of $57°,29578$ is equal to radius, the correction in the last Article must be multiplied by $57°,29578$ in order to reduce it into degrees. Also, the forces D and F are functions of the force of the sun at the distance unity, or at the mean distance of the planet which is disturbed.

1116. Let the sum of the masses of the earth E (or the body attracted) and sun S be unity, $M =$ the mass of the planet P disturbing the motion of the earth; then $\frac{1}{ES^2} =$ the attraction of E to S, and $\frac{M}{EP^2} =$ the attraction of E to P; complete the parallelogram $PSEW$, and put $z =$ the angle PSE. The attraction of E to $S = \frac{1}{ES^2}$, the attraction of E to $P = \frac{M}{EP^2}$, and the attraction of S to $P = \frac{M}{SP^2}$; and by the resolution of forces, $EP : ES :: \frac{M}{EP^2} : \frac{M \times ES}{EP^3}$ = the force of E to P in the direction ES; and $EP : EW$, or SP, $:: \frac{M}{EP^2} : \frac{M \times SP}{EP^3}$ the part of the force of E to P which acts in a direction parallel to SP; therefore $\frac{M \times SP}{EP^3} - \frac{M}{SP^2} =$ the disturbing force of P upon E in a direction parallel to SP, or EW; and this force, tending to draw E *from* S, must be written negative; and resolving this force into two others, one in the direction SE, and the other perpendicular to it, we have $F = -\left[\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right] \times \sin. z =$ the force which acts perpendicularly to SE; this must be written with a contrary sign, or $+$, when the body attracting is nearest to the sun, because it then acts in an opposite direction; also, $-\left[\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right] \times \cos. z =$ the force which acts in the direction ES; hence, $D = \frac{M \times ES}{EP^3} - \left[\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right] \times \cos. z =$ the disturbing force in the direction of the radius. The value of z is FIG. 240.

$\int Fr^3\dot{v}$, a being unity (1113); also, $P = \dfrac{\dfrac{Dr^2}{C} + \dfrac{Fr\dot{r}}{C\dot{v}} - 2e}{1+e}$, and as e is a very small quantity, we may assume $P = \dfrac{Dr^2}{C} + \dfrac{Fr\dot{r}}{C\dot{v}} - 2e$; and as we assume the sum of the masses of the sun and body attracted $= 1$, we have $C = 1$, and therefore $P = Dr^2 + \dfrac{Fr\dot{r}}{\dot{v}} - 2e$.

Hence, we have these four equations,

$$D = \frac{M \times ES}{EP^3} - \left\{\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right\} \times \cos. z$$

$$F = \mp \left\{\frac{M \times SP}{EP^3} - \frac{M}{SP^2}\right\} \times \sin. z.$$

$$e = \int Fr^3\dot{v}.$$

$$P = Dr^2 + \frac{Fr\dot{r}}{\dot{v}} - 2e.$$

1117. Let E be at A the higher apside in conjunction with P at B, and $z =$ the angle ESP of elongation, $v = ASE$, and let the planet P describe a circle, and E an ellipse whose center of force is S and the other focus C, and let $1-n$ be to 1 as the mean motion of the planet P to the mean motion of the earth E, or the body attracted; bisect SC in O, and let $AO = 1$. As the motion of the earth's apogee is very small, we may, for our present purpose, consider it as fixed, and putting $u = SO$, we have, $\frac{1}{r} = \frac{1}{d} - \frac{u}{d} \times \cos. v$. Now the mean motion is very nearly (227) about the point C, and $2u \times \sin. v = Cd$, which measures the angle CES; hence, $ECA = v + 2u \times \sin. v$; therefore $v - z : v + 2u \times \sin. v$ $:: 1-n : 1$, and $z = nv - 2u \times \overline{1-n} \times \sin. v$. If the disturbing planet be inferior, and its mean motion be to that of the earth as $1+n : 1$, then $z = nv + 2u \times \overline{1+n} \times \sin. v$.

We proceed now to the application of these principles.

On the Equations of the Earth's Motion produced by the Action of Jupiter.

1118. Put $EP=s$, $SP=b$, $SE=r$, and draw Eu perpendicular to SP; then $Su=r\times\cos. z$; hence, $s=\sqrt{b^2+r^2-2br\times\cos. z}$; put $2br\times\cos. z-r^2=l$, and we have $s=\sqrt{b^2-l}$; also,

$$D=\frac{Mr}{s^3}-\left(\frac{Mb}{s^3}-\frac{M}{b^2}\right)\times\cos. z.$$

$$F=-\left(\frac{Mb}{s^3}-\frac{M}{b^2}\right)\times\sin. z.$$

$$e=\int Fr^3\dot{v}.$$

$$P=Dr^2+\frac{Fr\dot{r}}{\dot{v}}-2e.$$

Now $\frac{1}{s^3}=\frac{1}{\sqrt{b^2-l}\,^3}=\overline{b^2-l}\,^{-\frac{3}{2}}=\frac{1}{b^3}+\frac{3l}{2b^5}+\frac{15l^2}{8b^7}+\frac{35l^3}{16b^9}+\frac{315l^4}{128b^{11}}+$ &c. substitute for l its value $2br\times\cos. z-r^2$, and for $\cos. z^2$, $\cos. z^3$, $\cos. z^4$, &c. put $\frac{1}{2}+\frac{1}{2}\cos. 2z$, $\frac{3}{4}\cos. z+\frac{1}{4}\cos. 3z$, $\frac{3}{8}+\frac{1}{2}\cos. 2z+\frac{1}{8}\cos. 4z$, &c. and we get $\frac{1}{s^3}=\frac{1}{b^3}+\frac{9r^2}{4b^5}+\frac{225r^4}{64b^7}+\left(\frac{3r}{b^4}+\frac{45r^3}{8b^6}\right)\times\cos. z+\left(\frac{15r^2}{4b^5}+\frac{105r^4}{16b^7}\right)\cos. 2z+\frac{35r^3}{8b^6}\times\cos. 3z+\frac{315r^4}{64b^7}\times\cos. 4z+$ &c. which we might put under the general form $A+B\cos. z+C\cos. 2z+D\cos. 3z+$&c. The value of $\frac{1}{s^3}$ being obtained, substitute it for its value in the expression for D, and we get the second part of $D=\left(\left(\frac{Mb}{s^3}-\frac{M}{b^2}\right)\times\cos. z\right)=M\times\left(\left(\frac{9r^2}{4b^4}+\frac{225r^4}{64b^6}\right)\times\cos. z+\left(\frac{3r}{b^3}+\frac{45r^3}{8b^5}\right)\times\cos. z\times\cos. z+\left(\frac{15r^2}{4b^4}+\frac{105r^4}{16b^6}\right)\times\cos. 2z\times\cos. z+\frac{35r^3}{8b^5}\times\cos. 3z\times\cos. z+\frac{315r^4}{64b^6}\times\cos. 4z\times\cos. z\right)$; substitute for $\cos. z\times\cos. z$, $\cos. 2z\times\cos. z$, $\cos. 3z\times\cos. z$,

and cos. $4z \times$ cos. z their values*, and we get $\left(\frac{Mb}{s^3}-\frac{M}{b^2}\right) \times \cos. z = M \times \Big[\frac{3r}{2b^3} + \frac{45r^3}{16b^5} + \left(\frac{3r}{2b^3}+\frac{5r^3}{b^5}\right) \times \cos. z + \left(\frac{33r^2}{8b^4}+\frac{435r^4}{64b^6}\right) \times \cos. 2z + \left(\frac{15r^2}{8b^4}+\frac{735r^4}{128b^6}\right) \times \cos. 3z + \frac{35r^3}{16b^5} \times \cos. 4z + \frac{315r^4}{128b^6} \times \cos. 5z\Big]$; this we must subtract from the above value of $\frac{1}{s^3}$ multiplied by Mr, and we have $D = M \times \Big[-\frac{r}{2b^3} - \frac{9r^3}{16b^5} + \frac{225r^5}{64b^7} - \left(\frac{9r^2}{8b^4}+\frac{75r^4}{64b^6}\right) \cos. z - \left(\frac{3r}{2b^3}+\frac{5r^3}{4b^5}-\frac{105r^5}{16b^7}\right) \times \cos. 2z - \left[\frac{15r^2}{8b^4}+\frac{175r^4}{128b^6}\right] \times \cos. 3z - \left[\frac{35r^3}{16b^5}-\frac{315r^5}{64b^7}\right] \times \cos. 4z - \frac{315r^4}{128b^6} \times \cos. 5z\Big]$. The first part of this is constant, and expresses the mean gravity, it having no respect to the relative situations of E and P.

1119. The value of F is $-\left[\frac{Mb}{s^3}-\frac{M}{b^2}\right] \times \sin. z$, which is equal to the above first value of $\left[\frac{Mb}{s^3}-\frac{M}{b^2}\right] \times \cos. z$, multiplied by $-\sin. z$ and divided by cos. z; hence, $F = -M \times \Big[\left[\frac{9r^2}{4b^4}+\frac{225r^4}{64b^6}\right] \times \sin. z + \left[\frac{3r}{b^3}+\frac{45r^3}{8b^5}\right] \times \cos. z \times \sin. z + \left[\frac{15r^2}{4b^4}+\frac{105r^4}{16b^6}\right] \times \cos. 2z \times \sin. z + \frac{35r^3}{8b^5} \times \cos. 3z \times \sin. z + \frac{315r^4}{64b^6} \times \cos. 4z \times \sin. z\Big]$; substitute for $\cos. z \times \sin. z$, $\cos. 2z \times \sin. z$, $\cos. 3z \times \sin. z$ and $\cos. 4z \times \sin. z$, their values†, and we get $F = -M \times \Big[\left[\frac{3r^2}{8b^4}+\frac{15r^4}{64b^6}\right] \times \sin z. + \left[\frac{3r}{2b^3}+\frac{5r^3}{8b^5}\right] \times \sin. 2z + \left[\frac{15r^2}{8b^4}+\frac{105r^4}{128b^6}\right] \times \sin. 3z + \frac{35r^3}{16b^5} \times \sin. 4z + \frac{315r^4}{128b^6} \times \sin. 5z\Big]$. The values of D and F being determined, we shall know the value of P; hence, we obtain S (1112), and thence (1114) the correction required.

In like manner, for the action of the superior on the inferior planet, we have

* By Trigonometry, cos. $A \times$ cos. $B = \frac{1}{2}$ cos. $\overline{A+B} + \frac{1}{2}$ cos. $\overline{A-B}$; hence, cos. $z \times$ cos. $z = \frac{1}{2} + \frac{1}{2}$ cos. $2z$; cos. $2z \times$ cos. $z = \frac{1}{2}$ cos. $3z + \frac{1}{2}$ cos. z; cos. $3z \times$ cos. $z = \frac{1}{2}$ cos. $4z + \frac{1}{2}$ cos. $2z$; and cos. $4z \times$ cos. $z = \frac{1}{2}$ cos. $5z + \frac{1}{2}$ cos. $3z$.

† By Trigonometry, cos. $A \times$ sin. $B = \frac{1}{2}$ sin. $\overline{A+B} - \frac{1}{2}$ sin. $\overline{A-B}$; hence, cos. $z \times$ sin. $z = \frac{1}{2}$ sin. $2z$; cos. $2z \times$ sin. $z = \frac{1}{2}$ sin. $3z - \frac{1}{2}$ sin. z; cos. $3z \times$ sin. $z = \frac{1}{2}$ sin. $4z - \frac{1}{2}$ sin. $2z$, and cos. $4z \times$ sin. $z = \frac{1}{2}$ sin. $5z - \frac{1}{2}$ sin. $3z$.

$$D = M\left\{\frac{3r^2}{4b^4}+\frac{45r^4}{64b^6}+\frac{\cos. z}{r^2}+\left[\frac{2r}{b^3}+\frac{3r^3}{2b^5}\right]\cos. z+\left[\frac{9r^2}{4b^4}+\frac{25r^4}{16b^6}\right]\cos. 2z+\frac{5r^3}{2b^5}\cos. 3z\right\}.$$

1120. To find the values of the powers of r in terms of v. On account of the small excentricity of the earth's orbit, we may neglect all the powers of u above the first; and from the slow motion of the apogee, we may suppose it fixed; hence, (1111), $\frac{1}{r}=\frac{1}{p}-\frac{u}{p}\cos. v$. is the equation from which we must deduce r; therefore $\frac{r}{p}=1+u\times\cos. v$, in which the semi-parameter p and excentricity u are known from observation; therefore $\frac{r^2}{p^2}=1+2u\times\cos. v$, $\frac{r^3}{p^3}=1+3u\times\cos. v$, &c. and by taking the fluxion of the first of these, we have $\frac{\dot{r}}{p^2\dot{v}}=-u\times\sin. v$.

1121. To find the value of sin. z, sin. $2z$, cos. z, cos. $2z$, in terms of v; for, in this case, it will not be necessary to go any farther in the values of D (1118) and F (1119). As $z=nv-2u\times\sin. \overline{1-n}\times\sin. v$ (1117), we have by Trigonometry, considering the cosine of the last term $=1$, and the term itself $=$ to its sine on account of its smallness,

$$\sin. z=\sin. nv-u\times\overline{1-n}\times\sin. \overline{n+1}\times v+u\times\overline{1-n}\times\sin. \overline{n-1}\times v,$$
$$\cos. z=\cos. nv-u\times\overline{1-n}\times\cos. \overline{n+1}\times v+u\times\overline{1-n}\times\cos. \overline{n-1}\times v,$$
$$\sin. 2z=\sin. 2nv-2u\times\overline{1-n}\times\sin. \overline{2n+1}\times v+2u\times\overline{1-n}\times\sin. \overline{2n-1}\times v,$$
$$\cos. 2z=\cos. 2nv-2u\times\overline{1-n}\times\cos. \overline{2n+1}\times v+2u\times\overline{1-n}\times\cos. \overline{2n-1}\times v.$$

1122. Let us assume $M=\frac{1}{1067}$, according to Sir I. NEWTON, the mass of the sun being unity, which we may consider as the mass of the sun and earth, on account of the extreme smallness of the earth in respect to the sun; and, by observation, the value of $\frac{1}{b}=0{,}192451$; therefore $\frac{1}{b^3}=0{,}0071264$, $\frac{1}{b^4}=0{,}0013714$, $\frac{1}{b^5}=0{,}0002639$, $\frac{1}{b^6}=0{,}0000508$, $\frac{1}{b^7}=0{,}00000977$, &c. also, $1-n=\frac{1}{11{,}857}$; therefore $n=0{,}915659$; and $u=0{,}01683$; consequently $u\times\overline{1-n}$

$=\frac{1}{352}$, $n-1=-0,0843$, $n+1=1,91560$, $2n-1=0,831[illegible]$ and $2n+1=2,831[illegible]$.
Also, we assume $r=1$, its mean value.

1123. To find the value of $e=\int Fr^2 \dot{v}$. We have (1119) $Fr^2=-M\times\left[\left(\frac{3r^5}{8b^4}+\frac{15r^7}{64b^6}\right)\times\sin. z+\left(\frac{3r^4}{2b^3}+\frac{5r^6}{8b^5}\right)\times\sin. 2z\right]=-M\times 0,0005268\times\sin. z-M\times 0,0108545\times\sin. 2z$. Now for $\sin. z$ and $\sin. 2z$, substitute their values found in Art. 1121. neglecting the terms $-u\times\overline{1-n}\times\sin.\overline{n+1}\times v$ and $-2u\times\overline{1-n}\times\sin.\overline{2n+1}\times v$, because, when the fluent is taken, the denominators will be so large as to render the terms too small for consideration. This substitution being made, and substituting also for M its value, we have, after taking the fluent,

$$e=0,000000[illegible]\times\cos. nv+0,0000004555\times\cos. 2nv-0,000000[illegible]74\times\cos.\overline{n-1}\times v+0,0000004457\times\cos.\overline{2n-1}\times v.$$

1124. To find the value of Dr^2. The first term $M\times\left[\frac{r}{2b}+\frac{9r^3}{16b^3}+\frac{225r^5}{64b^7}\right]$ in the value of D (1118) not being multiplied into the sine or cosine of z, may be here neglected, it not being a variable quantity depending on the place of the body, but only a constant part of the whole force of the earth to the sun, and we are here considering only the variable quantities which produce the irregularities of the earth's motion in different parts of its orbit. We have therefore only to multiply the next two terms of the value of D into r^2, and we obtain $-M\times\left[\frac{9r^4}{8b^4}+\frac{75r^6}{64b^6}\right]\times\cos. z-M\times\left[\frac{3r^3}{2b^3}+\frac{5r^5}{4b^5}-\frac{105r^7}{16b^7}\right]\times\cos. 2z=-M\times 0,0016023\times\cos. z-M\times 0,019554\times\cos. 2z$; substitute for M its value, and for $\cos. z$ and $\cos. 2z$ their values found in Art. 1121. neglecting $-u\times\overline{1-n}\times\cos.\overline{n+1}\times v$ and $-2u\times\overline{1-n}\times\cos.\overline{2n+1}\times v$, on account of their small effects, and we get (for our present purpose) the value of

$$Dr^2=-0,0000015017\cos. nv-0,000010267\cos. 2nv-0,0000005265\cos.\overline{n-1}\times v-0,000000269\cos.\overline{2n-1}\times v.$$

1125. To find the value of $\frac{Frr}{\dot{v}}$. Assuming $p=1$, we have (1120) $\frac{\dot{r}r}{\dot{v}}=-$

1129. By Art. 1114, the correction of the mean longitude, so far as regards the disturbing forces, is the above correction of the time with its sign changed, which correction is in terms of the true longitude; but here for the *true* longitude we may substitute the *mean*, without producing any sensible error: if therefore x represent the mean longitude of the earth, we have the *correction of the mean longitude*

$$= -0,00003429 \sin. nx + 0,00001295 \sin. 2nx + 0,00000194 \sin. \overline{n+1} \times x - 0,00000785 \sin. \overline{2n-1} \times x.$$

1130. The mean motion of Jupiter : that of the earth :: $1-n$: 1, or as $\overline{1-n} \times x : x$; hence, nx = the mean longitude of the earth — that of Jupiter, which put $= y$. If therefore (1115) we multiply each of the co-efficients by 57°,29578, we get *the equations of the mean motion of the earth arising from the action of Jupiter*

$$= -7',1 \times \sin. y + 2',7 \times \sin. 2y + 0',4 \times \sin. \overline{y-x} - 1',5 \times \sin. \overline{2y-x}.$$

1131. In the foregoing solution, we have supposed the orbit of Jupiter to be a circle, and consequently b was constant; but if b represent the semiparameter, b' the true distance, u' the excentricity of the orbit divided by the semi-axis-major, and v' the true anomaly, then $b' = \frac{b}{1-u' \cos. v'}$; the mean anomaly also of Jupiter $= v' + 2u' \cos. v'$, proved as in Art. 1117. and the mean motion of the earth being to that of Jupiter as $1+n$ to 1, the mean motion of the earth $= \overline{1+n} \cdot v' + \overline{1+n} \times 2u' \cos. v'$; from this subtract v' the true motion of Jupiter, and we get $z = nv' + \overline{1+n} \times 2u' \cos. v'$. Also, $\frac{b'}{b} = 1 + u' \cos. v'$. Proceed therefore for this value of z and $\frac{b'}{b}$, as we have done for that of z and $\frac{r}{p}$ in the foregoing operation, assuming those terms only where u' enters, by which we shall obtain the equations arising from the excentricity of Jupiter's orbit. We have here omitted those terms where the powers of the excentricity above the first have entered; but if we had taken any of the higher powers into consideration, we might, in the same manner, have got the equations thence arising. If the reader wish to see an example where the excentricity of each orbit is considered, and also the square of the excentricity of the orbit of the body which is disturbed, he may consult the *Mem. de l'Acad. Roy. des Scien.* 1761, where he will find M. de la LANDE has computed the equations of

therefore $\frac{\dot{s}}{\dot{z}} \times \overline{1-g\cos. z} + mgs \sin. z = 0$. In this equation, substitute $A+B\cos. z + C\cos. 2z + \&c.$ for s, and $-B\sin. z.\dot{z} - 2C\sin. 2z.\dot{z} - \&c.$ for $\dot{s}$; and substituting also for $\sin. nz \times \cos. z$ its value $\frac{1}{2}\sin. \overline{n+1}.z + \frac{1}{2}\sin. \overline{n-1}.z$, and for $\cos. nz \times \sin. z$ its value $\frac{1}{2}\sin. \overline{n+1}.z - \frac{1}{2}\sin. \overline{n-1}.z$, according as $n = 1, 2, 3, 4, 5, \&c.$ we get

$\overline{\text{Cos. } z} = \text{cos. } z$

$\overline{\text{Cos. } z^2} = \frac{1}{2} (\text{cos. } 2w + \frac{1}{2}.\frac{2}{1})$.

$\overline{\text{Cos. } z^3} = \frac{1}{4} (\text{cos. } 3w + \frac{3}{1} \text{cos. } w)$.

$\overline{\text{Cos. } z^4} = \frac{1}{8} (\text{cos. } 4w + \frac{4}{1} \text{ cos. } 2w + \frac{1}{2}.\frac{4.3}{1.2})$.

$\overline{\text{Cos. } z^5} = \frac{1}{16} (\text{cos. } 5w + \frac{5}{1} \text{ cos. } 3w + \frac{5.4}{1.2} \text{ cos. } w)$.

$\overline{\text{Cos. } z^6} = \frac{1}{32} (\text{cos. } 6w + \frac{6}{1} \text{ cos. } 4w + \frac{6.5}{1.2} \text{ cos. } 2w + \frac{1}{2}.\frac{6.5.4}{1.2.3})$.

$\overline{\text{Cos. } z^7} = \frac{1}{64} (\text{cos. } 7w + \frac{7}{1} \text{ cos. } 5w + \frac{7.6}{1.2} \text{ cos. } 3w + \frac{7.6.5}{1.2.3} \text{ cos. } w)$.

&c. &c. &c.

1135. As therefore the values of $\overline{\text{cos. } z^2}$, $\overline{\text{cos. } z^3}$, $\overline{\text{cos. } z^4}$, &c. &c. are expressed in terms of cos. z, cos. $2z$, cos. $3z$, cos. $4z$, &c. we have, by substitution, $\overline{1-g \text{ cos. } z}|^{-m} = A + B \text{ cos. } z + C \text{ cos. } 2z + D \text{ cos. } 3z + E \text{ cos. } 4z + \&c.$ A, B, C, D, E, &c. being the coefficients arising from the substitution. Now from the substitution it appears that

$$A = \left\{ \begin{array}{l} 1 + \frac{2}{4} \times \frac{m.\overline{m+1}}{1.2} g^2 + \frac{4.3}{4.8} \times \frac{m.\overline{m+1}.\overline{m+2}.\overline{m+3}}{1.2.3.4} g^4 + \frac{6.5.4}{4.8.12} \times \\ \frac{m.\overline{m+1}.\overline{m+2}.\overline{m+3}.\overline{m+4}.\overline{m+5}}{1.2.3.4.5.6} g^6 + \&c. \end{array} \right.$$

$$B = mg \times \left\{ \begin{array}{l} 1 + \frac{3}{4} \times \frac{\overline{m+1}.\overline{m+2}}{2.3} g^2 + \frac{5.4}{4.8} \times \frac{\overline{m+1}.\overline{m+2}.\overline{m+3}.\overline{m+4}}{2.3.4.5} g^4 + \frac{7.6.5}{4.8.12} \\ \times \frac{\overline{m+1}.\overline{m+2}.\overline{m+3}.\overline{m+4}.\overline{m+5}.\overline{m+6}}{2.3.4.5.6.7} g^6 + \&c. \end{array} \right.$$

The values of the other coefficients C, D, E, &c. we shall find in terms of A and B, and therefore having computed A and B, we shall very easily obtain C, D, E, &c.

1136. Let $s = A + B \text{ cos. } z + C \text{ cos. } 2z + D \text{ cos. } 3z + E \text{ cos. } 4z + F \text{ cos. } 5z + \&c. = \overline{1-g \text{ cos. } z}|^{-m}$, then log. $s = -m \times \text{log. } \overline{1-g \text{ cos. } z}$, and $\frac{\dot{s}}{s} = \frac{-mg \text{ sin. } z.\dot{z}}{1-g \text{ cos. } z}$,

$$\left.\begin{array}{l}
\frac{ds}{dz} = -B \sin. z - 2C \sin. 2z - 3D \sin. 3z - 4E \sin. 4z - 5F \sin. 5z - \&c. \\
-\frac{gs}{z} \times \cos. z = +\frac{1}{2} gB \sin. 2z + \frac{2}{2} gC \sin. 3z + \frac{3}{2} gD \sin. 4z + \frac{4}{2} gE \sin. 5z + \&c. \\
\qquad + \frac{2}{2} gC \sin. z + \frac{3}{2} gD \sin. 2z + \frac{4}{2} gE \sin. 3z + \frac{5}{2} gF \sin. 4z + \frac{6}{2} gG \sin. 5z + \&c. \\
mgs \sin. z = + mgA \sin. z + \frac{1}{2} mgB \sin. 2z + \frac{1}{2} mgC \sin. 3z + \frac{1}{2} mgD \sin. 4z + \frac{1}{2} mgE \sin. 5z + \&c. \\
\qquad - \frac{1}{2} mgC \sin. z - \frac{1}{2} mgD \sin. 2z - \frac{1}{2} mgE \sin. 3z - \frac{1}{2} mgF \sin. 4z - \frac{1}{2} mgG \sin. 5z - \&c.
\end{array}\right\} = 0.$$

Make the coefficients of sin. z, sin. $2z$, &c. respectively $= 0$, and we get

$$C = \frac{2B - 2mgA}{\overline{2-m} \times g},\quad D = \frac{4C - \overline{m+1} \,.\, gB}{\overline{3-m} \times g},\quad E = \frac{6D - \overline{m+2} \,.\, gC}{\overline{4-m} \times g},\quad F = \frac{8E - \overline{m+3} \,.\, gD}{\overline{5-m} \times g},\quad G = \frac{10F - \overline{m+4} \,.\, gE}{\overline{6-m} \times g},$$

where the law of continuation is manifest; and putting $m = \frac{1}{2}$, which is its value in the present case, we find

$$C = \frac{4B - 6gA}{g},\quad D = \frac{8C - 5gB}{3g},\quad E = \frac{12D - 17gC}{5g},\quad F = \frac{16E - 9gD}{7g},\quad G = \frac{20F - 11gE}{9g},\ \&c.$$

1137. The principal part of this operation therefore consists in finding the values of A and B. One of the methods which EULER proposes for this purpose is this. In each series for A and B, compute the value of a few of the first terms, for instance, of ten terms; and let the following terms be $O + OPg^2 + OPQg^4 + OPQRg^6 +$ &c. and then we have only to find the sum of this series, or of the series $1 + Pg^2 + PQg^4 + PQRg^6 +$ &c. and as the factors $P, Q, R,$ &c. approach to unity, we may consider this as a recurring series, and which may therefore be represented by a fraction. Let therefore $\frac{1 + \alpha g^2 + \beta g^4}{1 - \gamma g^2 - \delta g^4} = 1 + Pg^2 + PQg^4 + PQRg^6 +$ &c. and we have

$$\left.\begin{array}{lllll} 1 + Pg^2 & + PQg^4 & + PQRg^6 & + PQRSg^8 & + \&c. \\ -\gamma & -\gamma P & -\gamma PQ & -\gamma PQR & - \&c. \\ & -\delta & -\delta P & -\delta PQ & - \&c. \\ -1 - \alpha & -\beta & & & \end{array}\right\} = 0;$$

hence, $P - \gamma - \alpha = 0$, $PQ - \gamma P - \delta - \beta = 0$, $PQR - \gamma PQ - \delta P = 0$, $PQRS - \gamma PQR - \delta PQ = 0$; from the two last equations we find $\gamma = \frac{R \times \overline{Q - S}}{Q - R}$, $\delta = Q \times \overline{R + \gamma}$; therefore $\alpha = P - \gamma$ and $\beta = PQ - \gamma P - \delta$ are known. But as the factors $P, Q, R, S,$ &c. approach to unity, at an infinite distance we have $PQRS$ &c. $- \gamma PQR$ &c. $- \delta PQ$ &c. $= 0$, and the quantities $PQRS$ &c. PQR &c. PQ &c. having each an infinite number of factors and the first only one more than the second, and the second one more than the third, and the last factors also equal to unity, we have these quantities $PQRS$ &c. PQR &c. PQ &c. ultimately equal; therefore $1 - \gamma - \delta = 0$, or $\gamma + \delta = 1$; hence, $\gamma = R + \frac{R - 1}{Q - 1}$; $\delta = 1 - \gamma$; $\alpha = P - \gamma$; $\beta = PQ - \gamma P - \delta$. Thus we obtain the value of $O \times \frac{1 + \alpha g^2 + \beta g^4}{1 - \gamma g^2 - \delta g^4}$, which represents the value of all the terms after those which were computed. To adapt this to our present purpose, we must assume $m = g$.

1138. The values of A and B may also be thus obtained. Let $\overline{1 - g \cos. z}^{-\frac{3}{2}} = A + B \cos. z + C \cos. 2z + D \cos. 3z +$ &c. and then $\overline{1 - g \cos. z}^{-\frac{3}{2}} \times \dot{z} = A\dot{z} + B \cos. z . \dot{z} + C \cos. 2z . \dot{z} +$ &c. therefore $\int \overline{1 - g \cos. z}^{-\frac{3}{2}} \times \dot{z} = Az + B \times \sin. z + \frac{1}{2} C \sin. 2z +$ &c. make $z = c$ an arc of 180°, and all the terms after Az will become $= 0$; hence, $A = \int \frac{\overline{1 - g \cos. z}^{-\frac{3}{2}} \times \dot{z}}{c}$. Now the fluent

of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \dot{z}$ is the area of a curve whose ordinate is $\overline{1-g\cos. z}|^{-\frac{3}{2}}$
FIG. 241. and abscissa z. Let therefore $PZ =$ an arc of 180°, $PW = z$, $WH =$ $\overline{1-g\cos. z}|^{-\frac{3}{2}}$, and AK the locus of all the points H, and the area $PAHW$ will be the fluent of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \dot{z}$, but this we can find only by an approximation. Let the abscissa PZ be divided into a great many (a) equal parts $PQ = QR = RS = ST =$ &c. and let each of these parts be called unity; then as the ordinates are supposed to be very near, $PQ \times QB$, or (as $PQ = 1$) $QB =$ area $PAQB$ nearly; and thus $RC =$ area $QBCR$ nearly; $SD =$ area $RCDS$ nearly, &c. therefore any area $PAET = QB + RC + SD + TE$ nearly; divide therefore c into $a =$ parts, and let the successive values of z be $\frac{c}{a}$, $\frac{2c}{a}$, $\frac{3c}{a}$, ... $\frac{ac}{a}$; then will $\overline{1-g\cos. z}|^{-\frac{3}{2}}$ become $\overline{1-g\cos. \frac{c}{a}}|^{-\frac{3}{2}} = QB =$ area $PABQ$; $\overline{1-g\cos. \frac{2c}{a}}|^{-\frac{3}{2}} = RC =$ area $QBCR$; $\overline{1-g\cos. \frac{3c}{a}}|^{-\frac{3}{2}} = SD$ $=$ area $RCDS$, &c. nearly; put these quantities $= H, I, K, L,$ &c. respectively, and the fluent of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \dot{z} = H + I + K + L +$ &c. nearly. Now let us consider what is the fluent of all the $A\dot{z}$ on the same supposition. As we have found the fluent of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \dot{z}$ by taking the successive increments and adding them together, we must find the value of the fluent of $A\dot{z}$ in the same manner. Now as the increment of z is called unity, the whole value of $A\dot{z}$ thus generated is $A \times$ number of these increments $= Aa$ when $z = c$. Now the fluent of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \dot{z} = \int A\dot{z}$ (neglecting the other terms which vanish when $z = c$); hence, $H + I + K + L +$ &c. $= Aa$, and $A = \frac{H + I + K + L + \&c.}{a}$

1139. To find the value of B, multiply $\overline{1-g\cos. z}|^{-\frac{3}{2}} = A + B\cos. z + C\cos. 2z +$ &c. by $\cos. z \times \dot{z}$, and we have $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z \times \dot{z} = A \cos. z . \dot{z} + B\overline{\cos. z}^2 . \dot{z} + C\cos. z . \cos. 2z . \dot{z} +$ &c. $= A\cos. z . \dot{z} + B \times \overline{\frac{1}{2} + \frac{1}{2}\cos. 2z} \times \dot{z} +$ &c. $= A \times \cos. z . \dot{z} + \frac{1}{2}B\dot{z} + \frac{1}{2}B\cos. 2z . \dot{z} +$ &c. Now the fluent of $\frac{1}{2}B\dot{z}$ is $\frac{1}{2}Bz$, and the fluent of all the other terms will be in terms of sin. z, sin. $2z$, sin. $3z$, &c. and therefore when $z = c = 180°$ they all vanish; hence, when z becomes c, $\int \overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z . \dot{z} = \frac{1}{2}Bc$; therefore $B = \frac{\int \overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z . \dot{z}}{\frac{1}{2}c}$. If therefore any ordinate WH

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$\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z$, and proceeding exactly as before, we get $B = \dfrac{H' + I' + K' + L' + \&c.}{\frac{1}{2}a}$, where H', I', K', L', &c. represent QB, RC, SD, TE, &c. the values of $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z$, when we substitute $\frac{c}{a}$, $\frac{2c}{a}$, &c. for z. The more parts a into which we divide c, the nearer is the approximation.

1140. But we may approximate to the area $PAKZ$ more nearly in this manner. $BQ \times PR =$ area $PACR$ very nearly; $SD \times RT =$ area $RCET$ very nearly; &c. in which case the number of such quantities $= \frac{1}{2}a$. Now let us, for instance, divide c into 20 parts, or let $a = 20$; then as we now assume for z the first, third, fifth, &c. ordinates, the respective values of z, if we take them in parts of a right angle ρ instead of two right angles c, will be $\frac{1}{10}\rho$, $\frac{3}{10}\rho$, $\frac{5}{10}\rho$, $\frac{7}{10}\rho$, $\frac{9}{10}\rho$, $\frac{11}{10}\rho$, $\frac{13}{10}\rho$, $\frac{15}{10}\rho$, $\frac{17}{10}\rho$, $\frac{19}{10}\rho$, the cosines of which we must substitute for z in $\overline{1-g\cos. z}|^{-\frac{3}{2}}$ in order to get the first, third, fifth, &c. ordinates, and the cos. z being negative for the last five values, the quantity $-g\cos. z$ becomes positive; but $\cos. \frac{19}{10}\rho = \sin. \frac{9}{10}\rho$, $\cos. \frac{17}{10}\rho = \sin. \frac{7}{10}\rho$, $\cos. \frac{15}{10}\rho = \sin. \frac{5}{10}\rho$, $\cos. \frac{13}{10}\rho = \sin. \frac{3}{10}\rho$, and $\cos. \frac{11}{10}\rho = \sin. \frac{1}{10}\rho$; substituting therefore these sines instead of the cosines in the value of A (1138), and $\frac{1}{2}a$ instead of a, and we get

$$A = \frac{1}{10}\left\{\begin{array}{l} \overline{1-g\cos. \frac{1}{10}\rho}|^{-\frac{3}{2}} + \overline{1-g\cos. \frac{3}{10}\rho}|^{-\frac{3}{2}} + \overline{1-g\cos. \frac{5}{10}\rho}|^{-\frac{3}{2}} + \&c. \\ \overline{1+g\sin. \frac{1}{10}\rho}|^{-\frac{3}{2}} + \overline{1+g\sin. \frac{3}{10}\rho}|^{-\frac{3}{2}} + \overline{1+g\sin. \frac{5}{10}\rho}|^{-\frac{3}{2}} + \&c. \end{array}\right.$$

This is the other expression for A* which EULER has given in his *Recherches des Inegalitès de Saturn et de Jupiter*, which he says is sufficiently accurate. The number of the quantities is only ten, and are very easily computed in numbers. If c be divided into a greater number of parts, the conclusion will be more accurate.

1141. To obtain the value of B, instead of the quantity $\overline{1-g\cos. z}|^{-\frac{3}{2}}$, we have $\overline{1-g\cos. z}|^{-\frac{3}{2}} \times \cos. z$; substitute sin. z for cos. z, as before, and without the vinculum cos. z becomes negative for the last half of the terms; hence,

* In the case of the first and third satellites of Jupiter, M. BAILLY has shown that this does not differ more than the 2000th part of the whole from the truth.

$$B=\frac{1}{5}\left\{\begin{array}{l}\overline{1-g\cos.\tfrac{1}{10}\varrho}\,\Big|^{-\frac{3}{2}}\times\cos.\tfrac{1}{10}\varrho+\overline{1-g\cos.\tfrac{3}{10}\varrho}\,\Big|^{-\frac{3}{2}}\times\cos.\tfrac{3}{10}\varrho+\&c.\\ -\overline{1+g\sin.\tfrac{1}{10}\varrho}\,\Big|^{-\frac{3}{2}}\times\sin.\tfrac{1}{10}\varrho-\overline{1+g\sin.\tfrac{3}{10}\varrho}\,\Big|^{-\frac{3}{2}}\times\sin.\tfrac{3}{10}\varrho-\&c.\end{array}\right.$$

1142. But there is another method by which we may approximate to the area of this curve, by supposing a parabolic curve $y=a'+b'x+c'x^2+d'x^3+\&c.$ to pass through the extremities of any number $a, b, c, d, e, \ldots\, l, m, n,$ of ordinates, x representing the abscissa and y the ordinate. For instance, take three ordinates $a, b, c,$ and the parabolic curve passing through them is $y=a'+b'x+c'x^2$, the area of which, in terms of $a, b, c,$ is $\frac{1}{3}a+\frac{4}{3}b+\frac{1}{3}c$* $=$ the area $PACR$; for the same reason, $\frac{1}{3}c+\frac{4}{3}d+\frac{1}{3}e=$ area $RCET$; &c. &c. hence, the whole area $=\frac{1}{3}a+\frac{4}{3}b+\frac{2}{3}c+\frac{4}{3}d+\frac{2}{3}e\ldots\ldots\frac{2}{3}l+\frac{4}{3}m+\frac{1}{3}n$, that is, the whole area $=\frac{1}{3}$ of the sum of the first and last ordinates $+\frac{4}{3}$ of the sum of the alternate ordinates, beginning at the second, $+\frac{2}{3}$ of the sum of the alternate ordinates, beginning at the third. Thus we may find the values of A and B from the known ordinates of the curve. Multiply these values of A and B by $\dfrac{1}{\overline{b^2+r^2}\,\big|^{\frac{3}{2}}}$, and we obtain the values of A' and B'.

1143. M. de la GRANGE has thus investigated the values of the coefficients.

Let $V=b^2+r^2-2br\cos.z$, and suppose

$$\frac{1}{V^s}=P+Q\cos.z+R\cos.2z+\&c.$$

Take the fluxion, and we have

$$\frac{2sbr\sin.z}{V^{s+1}}=Q\sin.z+2R\sin.2z+\&c.$$

Multiply the first of these by V, and the second by its equal $b^2+r^2-2br\cos.z$, and

$$\frac{2sbr\sin.z}{V^s}=\overline{b^2+r^2-2br\cos.z}\times\overline{Q\sin.z+2R\sin.2z+\&c.},$$

$$\text{Or, } 2sbr\sin.z\times\overline{P+Q\cos.z+R\cos.2z+\&c.}$$

$$=\overline{b^2+r^2-2br\cos.z}\times\overline{Q\sin.z+2R\sin.2z+\&c.}$$

* This is proved in the Chapter upon Interpolation.

1144. Now sin. $z \times$ cos. $2z = \frac{1}{2}$ sin. $3z - \frac{1}{2}$ sin. z, and cos. $z \times$ sin. $2z = \frac{1}{2}$ sin. $3z + \frac{1}{2}$ sin. z; in the last equation therefore, substitute for sin. $z \times$ cos. $2z$, and cos. $z \times$ sin. $2z$, these values, and we get

$$sbr \times \overline{2P - R} \times \text{sin. } z + \&c. = \overline{b^2 + r^2 \times Q - 2brR} \times \text{sin. } z + \&c.$$

all the other terms containing sin. $2z$, sin. $3z$, &c. Equating therefore the coefficients of sin. z, we get

$$sbr \times \overline{2P - R} = \overline{b^2 + r^2} \times Q - 2brR;$$

hence, $R = \dfrac{\overline{b^2 + r^2} \times Q - 2sbrP}{\overline{2 - s} \times br}$.

1145. Let $\frac{1}{V^{s+1}} = P' + Q'$ cos. $z + R'$ cos. $2z$ + &c. Multiply the first by V, and the second by $b^2 + r^2 - 2br$ cos. z, and we have

$$\frac{1}{V^s} = \overline{b^2 + r^2 - 2br \text{ cos. } z} \times \overline{P' + Q' \text{ cos. } z + \&c.}$$

But cos. $z \times$ cos. $z = \frac{1}{2} + \frac{1}{2}$ cos. $2z$; hence,

$$\frac{1}{V^s} = \overline{b^2 + r^2} \times P' - brQ' + \&c.$$

the other terms being in terms of sin. z, sin. $2z$, &c. And comparing this with our first assumed value of $\frac{1}{V^s}$, we have

$$\overline{b^2 + r^2} \times P' - brQ' = P.$$

1146. Next, multiply $\frac{1}{V^{s+1}} = P' + Q'$ cos. $z + R'$ cos. $2z$ + &c. by $2sbr$ sin. z, and

$\frac{2sbr \text{ sin. } z}{V^{s+1}} = 2sbrP'$ sin. z + $2sbrQ'$ sin. z $\times$ cos. z + $2sbrR'$ $\times$ sin. z $\times$ cos. $2z$ + &c. = (as sin. z $\times$ cos. $2z$ = $\frac{1}{2}$ sin. $3z - \frac{1}{2}$ sin. z) $\overline{2sbrP' - sbr \times R'}$ $\times$ sin. z + &c.

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Compare this with the value of $\frac{2sbr \sin. z}{V^{s+1}}$ in Art. 1143. and we have

$$2sbrP' - sbrR' = Q.$$

But there must be the same relation between P', Q', R', as between P, Q, R, only writing $s+1$ instead of s. Hence, (1144), $R' = \frac{\overline{b^2+r^2} \times Q' - 2 \times \overline{s+1} \times brP'}{\overline{1-s} \times br}$; substitute this quantity for R' in $2sbrP' - sbrR' = Q$, and we get

$$\frac{s}{1-s} \times \overline{4brP' - \overline{b^2+r^2} \times Q'} = Q.$$

From this equation, and the above equation $\overline{b^2+r^2} \times P' - brQ' = P$, we obtain

$$P' = \frac{\overline{b^2+r^2} \times P - \frac{1-s}{s} \times brQ}{\overline{b^2-r^2}^2}$$

$$Q' = \frac{4brP - \frac{1-s}{s} \times \overline{b^2+r^2} \times Q}{\overline{b^2-r^2}^2}.$$

Knowing therefore the two first quantities P, Q in the value of $\frac{1}{V^s}$, we can find the two first quantities P', Q' in the value of $\frac{1}{V^{s+1}}$. In like manner, if $\frac{1}{V^{s+2}} = P'' + Q'' \cos. z + \&c.$ we have

$$P'' = \frac{\overline{b^2+r^2} \times P' + \frac{s}{1+s} \times brQ'}{\overline{b^2-r^2}^2},$$

$$Q'' = \frac{4brP' + \frac{s}{1+s} \times \overline{b^2+r^2} \times Q'}{\overline{b^2-r^2}^2}.$$

If we substitute in these expressions for P' and Q' their values in terms of P and Q, we shall get P'' and Q'' in terms of P and Q. Thus we get the coefficients of the terms of the superior powers in terms of the coefficients of those of the inferior.

1147. Let $\frac{1}{V_s} = P + Q \cos. z + R \cos. 2z + S \cos. 3z + \&c.$ to find $P, Q, R,$ &c. If e be a number whose hyperbolic log. $= 1$, then $e^{mz\sqrt{-1}} + e^{-mz\sqrt{-1}} = 2 \cos. mz$. Hence, the value of V, or $b^2 + r^2 - 2br \cos. z = \overline{b - re^{z\sqrt{-1}}} \times \overline{b - re^{-z\sqrt{-1}}}$; therefore $\frac{1}{V_s} = \frac{1}{\overline{b - re^{z\sqrt{-1}}}|^s} \times \frac{1}{\overline{b - re^{-z\sqrt{-1}}}|^s}$; these two factors expanded give these two series,

$$\frac{1}{b^s} + \frac{sre^{z\sqrt{-1}}}{b^{s+1}} + \frac{s \,.\, \overline{s+1} \,.\, r^2 e^{2z\sqrt{-1}}}{2b^{s+2}} + \frac{s \,.\, \overline{s+1} \,.\, \overline{s+2} \,.\, r^3 e^{3z\sqrt{-1}}}{2 \,.\, 3b^{s+}} + \&c.$$

$$\frac{1}{b^s} + \frac{sre^{-z\sqrt{-1}}}{b^{s+1}} + \frac{s \,.\, \overline{s+1} \,.\, r^2 e^{-2z\sqrt{-1}}}{2b^{s+2}} + \frac{s \,.\, \overline{s+1} \,.\, \overline{s+2} \,.\, r^3 e^{-3z\sqrt{-1}}}{2 \,.\, 3b^{s+3}} + \&c.$$

Multiply these two series together, and for $e^{z\sqrt{-1}} + e^{-z\sqrt{-1}}$ put $2 \cos. z$, for $e^{2z\sqrt{-1}} + e^{-2z\sqrt{-1}}$ put $2 \cos. 2z$, for $e^{3z\sqrt{-1}} + e^{-3z\sqrt{-1}}$ put $2 \cos. 3z$, &c. put also $\alpha = s$, $\beta = s. \frac{s+1}{2}$, $\gamma = \frac{s \,.\, \overline{s+1} \,.\, \overline{s+2}}{2 \,.\, 3}$, &c. and the product, or $\frac{1}{V^s}$, becomes,

$$\frac{1}{b^{2s}} \times \left(1 + \alpha^2 \frac{r^2}{b^2} + \beta^2 \frac{r^4}{b^4} + \gamma^2 \frac{r^6}{b^6} + \&c.\right)$$

$$+ \frac{2}{b^{2s}} \times \left(\alpha \frac{r}{b} + \alpha\beta \frac{r^3}{b^3} + \beta\gamma \frac{r^5}{b^5} + \&c.\right) \cos.$$

$$+ \frac{2}{b^{2s}} \times \left(\beta \frac{r^2}{b^2} + \beta\gamma \frac{r^4}{b^4} + \gamma\delta \frac{r^6}{b^6} + \&c.\right) \cos. 2z$$

&c. &c. &c.

And comparing this with

$$\frac{1}{V^s}=P+Q\cos. z+R\cos. 2z, \text{ \&c.}$$

we have

$$P=\frac{1}{b^{2s}}\left(1+\alpha^2\frac{r^2}{b^2}+\beta^2\frac{r^4}{b^4}+\gamma^2\frac{r^6}{b^6}+\text{\&c.}\right)$$

$$Q=\frac{2}{b^{2s}}\left(\alpha\frac{r}{b}+\alpha\beta\frac{r^3}{b^3}+\beta\gamma\frac{r^5}{b^5}+\text{\&c.}\right)$$

As the quantity $b^2+r^2-2br\cos. z$ is also $=\overline{r-be^{z\sqrt{-1}}}\times\overline{r-be^{-z\sqrt{-1}}}$, we may change r for b and b for r, so as to make the greater of the two enter into the denominators.

1148. In the present instance, we want to find P and Q when $s=\frac{3}{2}$; but in this case the coefficients α, β, γ, &c. would not be converging; we will begin therefore by taking $s=-\frac{1}{2}$, which will give the series a sufficient degree of convergency. Let us therefore compute P and Q in the equation

$$\frac{1}{V^{-\frac{1}{2}}}=P+Q\cos. z+R\cos. 2z+\text{\&c.}$$

from the series expressing their values in the last Article, writing $-\frac{1}{2}$ for s.

1149. Let $\frac{1}{V^{\frac{1}{2}}}=P'+Q'\cos. z+R'\cos. 2z+\text{\&c.}$ then (1146)

$$P'=\frac{\overline{b^2+r^2}\times P+3brQ}{\overline{b^2-r^2}^2},\quad Q'=\frac{4brP+3\times\overline{b^2+r^2}\times Q}{\overline{b^2-r^2}^2}.$$

1150. Let $\frac{1}{V^{\frac{3}{2}}}=P''+Q''\cos. z+R''\cos. 2z+\text{\&c.}$ then (1146)

$$P''=\frac{\overline{b^2+r^2}\times P'-brQ'}{\overline{b^2-r^2}^2},\quad Q''=\frac{4brP'-\overline{b^2+r^2}\times Q'}{\overline{b^2-r^2}^2}.$$

Substitute for P' and Q' their values found above in terms of P and Q, and we get

1154. The value of $e = \int F\dot{v}$, the radius r being unity; multiply therefore the above value of F by $\dot{v}$, and take the fluent, and we have

$$e = M\left(\left(-Ab + \frac{1}{b^2} + \frac{1}{2}\,Cb\right)\frac{1}{n}\cos.\,nv - \left(Bb - Db\right)\frac{1}{4n}\cos.\,2nv - \left(Cb - Eb\right)\right.$$
$$\left.\frac{1}{6n}\cos.\,3nv - \left(Db - Fb\right)\frac{1}{8n}\cos.\,4nv\right).$$

1155. As $r = 1$, we have $\dot{r} = 0$, and the value of $P = D - 2e$; but the first term $A - \frac{1}{2}\,Bb$ in the value of D may be omitted, it not being a variable quantity, and therefore cannot be concerned in producing any of the irregularities of the earth's motion in different parts of its orbit; hence, we here assume

$$P = M\left(\left(B + \overline{\frac{2}{n} - 1} \times Ab - \overline{\frac{2}{n} - 1} \times \frac{1}{b^2} - \overline{\frac{1}{n} + \frac{1}{2}} \times Cb\right)\cos.\,nv\right.$$
$$+ \left(C + \overline{\frac{1}{2n} - \frac{1}{2}} \times Bb - \overline{\frac{1}{2n} + \frac{1}{2}} \times Db\right)\cos.\,2nv$$
$$+ \left(D + \overline{\frac{1}{3n} - \frac{1}{2}} \times Cb - \overline{\frac{1}{3n} + \frac{1}{2}} \times Eb\right)\cos.\,3nv$$
$$\left.+ \left(E + \overline{\frac{1}{4n} - \frac{1}{2}} \times Db - \overline{\frac{1}{4n} + \frac{1}{2}} \times Fb\right)\cos.\,4nv\right).$$

1156. By observation, we have $b = 0{,}72348$, $n = 0{,}625$; hence, $b^2 = 0{,}52342$, $1 + b^2 = 1{,}52342$, $2b = 1{,}44696$; therefore $\overline{1 + b^2 - 2b\cos.\,z}\,\Big|^{-\frac{3}{2}} = \overline{1{,}52342 - 1{,}44696\cos.\,z}\,\Big|^{-\frac{3}{2}} = A + B\cos.\,z + C\cos.\,2z + D\cos.\,3z + E\cos.\,4z + F\cos.\,5z +$ &c. and by the rules (1133, &c.) we have already given, we find $A = 4{,}9996$, $B = 8{,}8861$; hence (1136), $C = 7{,}425$, $D = 6{,}0368$, $E = 4{,}8572$, $F = 3{,}9281$; from whence $Ab = 3{,}6171$, $Bb = 6{,}4289$, $Cb = 5{,}3718$, $Db = 4{,}367$, $Eb = 3{,}5143$, $Fb = 2{,}8419$.

1157. Substitute these numbers into the above values of e and P, and we have

$$e = M\,(1{,}5699\cos.\,nv - 0{,}8274\cos.\,2nv - 0{,}4954\cos.\,3nv - 0{,}305\cos.\,4nv)$$

$$P = M\,(1{,}3598 \text{ cos. } nv + 3{,}6765 \text{ cos. } 2nv + 2{,}584 \text{ cos. } 3nv + 1{,}8828 \text{ cos. } 4nv)$$

1158. Assuming $d=1$, if $P = a' \text{ cos. } mz + b' \text{ cos. } nz + \&c.$ then $S = -\frac{a'}{m^2-1} \times \text{cos. } mz - \frac{b'}{n^2-1} \times \text{cos. } nz - \&c.$ (1111); in this case, the divisors m^2-1, n^2-1, &c. become $n^2-1 = -0{,}609375$, $4n^2-1 = 0{,}5625$, $9n^2-1 = 2{,}5156$, $16n^2-1 = 5{,}25$; also $a' = 1{,}3598$, $b' = 3{,}6765$, $c' = 2{,}584$, $d' = 1{,}8828$. Hence,

$$S = M\,(2{,}2314 \text{ cos. } nv - 6{,}536 \text{ cos. } 2nv - 1{,}027 \text{ cos. } 3nv - 0{,}3586 \text{ cos. } 4nv).$$

1159. The expression for the correction of the time is the fluent of $-\overline{2S+e} \times \dot{v}$; substituting therefore for S and e their values here found, we have

$$-\overline{2S+e} \times \dot{v} = -M\,(6{,}0327 \text{ cos. } nv.\dot{v} - 13{,}8994 \text{ cos. } 2nv.\dot{v} - 2{,}5494 \text{ cos. } 3nv.\dot{v} - 1{,}0222 \text{ cos. } 4nv.\dot{v}).$$

Hence, the *correction of the time*

$$= -\int \overline{2S+e} \times \dot{v} = -M\left(\frac{6{,}0327}{n} \text{ sin. } nv - \frac{13{,}8994}{2n} \text{ sin. } 2nv - \frac{2{,}5494}{3n} \text{ sin. } 3nv - \frac{1{,}0222}{4n} \text{ sin. } 4nv.\right)$$

$$= -M\,(9{,}6475 \text{ sin. } nv - 11{,}1174 \text{ sin. } 2nv - 1{,}3597 \text{ sin. } 3nv - 0{,}4089 \text{ sin. } 4nv).$$

1160. The correction of the longitude, so far as regards the disturbing forces (1114), is the above correction of the time with its sign changed; substituting therefore the mean longitude x for v, and putting $t = nx =$ the mean longitude of Venus − that of the earth, we have the correction of the mean longitude

$$= M\,(9{,}6475 \text{ sin. } t - 11{,}1174 \text{ sin. } 2t - 1{,}3597 \text{ sin. } 3t - 0{,}4089 \text{ sin. } 4t).$$

Now (1067) the mass of the sun being to that of Venus as 333928 : 0,88993, we have $M = \frac{0{,}88993}{333928} = 0{,}000002665$; substitute therefore this value for M,

this, the *mean* velocity of the planet. Now since $r=1$ considered as the mean radius, and that the mean force in the direction of the radius is (Art. 1118.) $M\left[-\frac{r}{2b^3}-\frac{9r^3}{16b^5}+\frac{225r^5}{64b^7}\right]$, this quantity will express the square of the mean velocity. Hence, the square of the velocity of the inferior planet $= M\left\{-\frac{r}{2b^3}-\frac{9r^3}{16b^5}+\frac{225r^5}{64b^7}-n\left[\frac{3r^2}{2b^3}+\frac{15r^4}{32b^5}\right]+n\left[\frac{3r^3}{4b^4}+\frac{15r^5}{32b^6}\right]\cos z.\right.$ $+\,n\left[\frac{3r^2}{2b^3}+\frac{5r^4}{8b^5}\right]\cos.\ 2z+n\left[\frac{5r^3}{4b^4}+\frac{35r^5}{64b^6}\right]\cos.\ 3z+\frac{35r^4n}{32b^5}\cos.\ 4z+\frac{63r^5n}{64b^6}$ $\left.\cos.\ 5z\right\}$, considering $r=1$.

For the retardation of a superior planet by the action of an inferior, we have (putting $b=1$) the force acting in the direction of the radius $=M\left\{1+\frac{3r^2}{4}+\right.$ $\frac{45r^4}{64}+\frac{\cos.\ z}{r^2}+\left[2r+\frac{3r^3}{2}\right]\cos.\ z+\left[\frac{9r^2}{4}+\frac{25r^4}{16}\right]\cos.\ 2z+\frac{5r^3}{2}\cos.$ $\left.3z.\right\}$ and the square of the velocity perpendicular to the radius $=M\left[1+\frac{3r^2}{4}\right.$ $+\frac{45r^4}{64}-n\left[\frac{3r^2}{2}-\frac{15r^4}{32}\right]-n\,(2r-2r^2)\cos.\ z-n\left[\frac{3r^2}{2}+\frac{5r^4}{8}\right]\cos.\ 2z\,-$ $\left.\frac{5nr^3}{4}\cos.\ 3z.\right]$, where the difference of the motions : motion of the superior :: 1 : n.

The terms where cos. z, cos. $2z$, &c. enter, give the inequalities of the planet's motion from the cause here considered. And dividing them by the radius vector, we get the angular irregularities.

If we divide this quantity by the force before found, we shall get the radius of curvature of the orbit at E, which will appear to be nearly as cos. $2z$. But as this property belongs to the ellipse (882), if the superior planet should remain stationary, it would describe an ellipse (FIG. 203) about E the inferior planet, the minor axis of which passes through the superior; but if the superior planet be considered as revolving about the sun, the same ellipse Ca would be described in a moveable plane about E, by taking CEr : CEs :: difference of the angular motions : the angular motion of the inferior planet, and $Cs=Cr$. For the action of a superior on the inferior, the same takes place with this difference, that the major axis is directed to the inferior planet. See Art. 885.

term in the first part of D, is $\frac{3r^2}{4b^4}$; hence, $n=4$, and $\frac{\frac{1}{2}m}{b^4}=\frac{3r^2}{nb^4}$, and $m=\frac{3r^2}{2}$; the second term is $\frac{45r^4}{64\,b^6}$; hence, $n=6$, and $\frac{\frac{1}{2}m}{b^6}=\frac{45r^4}{64b^6}$, and $m=\frac{45r^4}{32}$. Hence, for the first term, $\overline{\frac{1}{4}n-\frac{1}{2}}\,.\,m=\frac{1}{2}m=\frac{3r^2}{4}$; for the second term, $\overline{\frac{1}{4}n-\frac{1}{2}}\,.\,m=m=\frac{45\,r^4}{32}$; therefore the motion of the apsides $=M\left\{\frac{3r^2}{4}+\frac{45r^4}{2}\right\}\times 360^\circ=7'.\ 6''$ in every revolution of Saturn. In like manner, the motion of the aphelion of the earth from the action of Jupiter is found $=13''$ yearly. And as (Art. 854.) the motion of the aphelia of the inferior planets in the same time are as the periodic times, the annual motion of the aphelion of Mars is $25''.\ 5'''$; of Venus $8''$; of Mercury $3''$.

Hence also we may find the motion of the aphelia of orbits arising from the attraction of an oblate spheroid, the attraction to which is $\frac{2pc^3}{3a^2}+\frac{4pc^2b}{3a^2}+\frac{2pc^4b}{5a^4}-\frac{6pc^4bt^2}{5a^4}$; and neglecting the 4th term as very small in respect to the 3d, and the 2d as being small in respect to the first, the attraction $=\frac{2pa^3}{3a^2}+\frac{2pc^4b}{5a^4}$, and the last term is the force arising from the spheroidical form of the body, and therefore $r=4$; also, $\frac{2pc^3}{3a^2}=$ attraction to the sphere, or gravity; hence, $1:\frac{1}{2}m::\frac{2pc^3}{3a^2}:\frac{2pc^4b}{5a^4}$, and $m=\frac{6cb}{5a^2}$. Now for Jupiter, $c=1$, $b=\frac{1}{13}$, and $c:a::1:25$ for the fourth satellite; hence, the motion of the apside (from this cause) for every revolution of the satellite, is $=1',595$, or $35'$ for every revolution of Jupiter. And as this motion of the aphelia vary in the inverse square of the distance from Jupiter, in the first satellite where $c:a::1:5{,}25$, the motion of the apside in one revolution is $36'$.

On the Motion of the Moon's Apogee.

1163. The action of one planet upon another to disturb its motion about the sun, is similar to the action of the sun upon the moon to disturb its motion about the earth. The general equation of the curve therefore which the disturbed planet describes, may represent that which the moon describes about the earth, and thence the irregularities of the moon's motion may be investigated. But on account of the length of the subject, we cannot here give an

investigation of all the lunar inequalities; we propose however to show how the motion of the moon's apogee may be found; and that being understood, together with the method of deducing the equations of the motions of the planets, the reader will be prepared to enter upon any further researches of this kind.

1164. Let E represent the earth, M the moon, S the sun, and at A the higher apside, let the moon be in conjunction with the sun at B; put the mean distance of the moon from the earth $=1$, $d=$ the semi-parameter, $r=EM$, $ES=b$, $MS=s$, $u=$ the excentricity of the moon's orbit, $v=AEM$, $x=BES$, $z=SEM$. Now (1118) FIG. 242.

$$D=\frac{M\times r}{b^3}-\left[\frac{M\times b}{s^3}-\frac{M}{b^2}\right]\ \cos.\ z.$$

$$F=-\left[\frac{M\times b}{s^3}-\frac{M}{b^2}\right]\ \sin.\ z.$$

But $MS=\overline{b^2+r^2-2br\cos.z}|^{\frac{1}{2}}=$ (as r is very small in respect to b) $b\times\overline{1-\frac{r}{b}\cos.z}$, and $\frac{1}{s^3}=\frac{1}{MS^3}=\frac{1}{b^3\times\overline{1-\frac{r}{b}\cos.z}|^3}=\frac{1}{b^3}\times\overline{1+\frac{3r}{b}\cos.z}$. Hence,

$D=\frac{Mr}{b^3}-\frac{3Mr}{b^3}\times\overline{\cos.z^2}$, neglecting that term where b^4 enters into the denominator, as being extremely small when compared with the rest. Also, $F=-\frac{3Mr}{b^3}\times\cos.z\times\sin.z$; but $\cos.z^2=\frac{1}{2}-\frac{1}{2}\cos.2z$; and $\cos.z\times\sin.z=\frac{1}{2}\sin.2z$; hence,

$$D=-\frac{\frac{1}{2}Mr}{b^3}-\frac{\frac{3}{2}Mr}{b^3}\cos.\ 2z.$$

$$F=-\frac{\frac{3}{2}Mr}{b^3}\sin.\ 2z.$$

But $e=\int\frac{Fr^3\dot{v}}{a^2}=\int\frac{Fr^3\dot{v}}{p}$, C being assumed $=1$; and $P=Dr^2+\frac{Fr\dot{r}}{\dot{v}}-2e$; substituting therefore for F and D their above values, and putting $a=\frac{Md^3}{b^3}$, we obtain

[illegible] $\cos. 2z$, we may consider $2z$ as [illegible] $- 2b' \sin. 2mv$; hence, by Trigo- [illegible] $(2a' \sin. mv + 2b' \sin. 2mv) - \cos.$ [illegible] but as a' and b' are small, we may consi- [illegible] equal to unity, and sin. $(2a' \times \sin. mv +$ [illegible], and substituting for the two products [illegible] terms of half the sine of their sum — half the [illegible]

$$[illegible] \sin. \overline{\frac{2}{n} - m} . v - a' \sin. \overline{\frac{2}{n} + m} . v +$$

$$[illegible] \overline{\frac{2}{n} - m} . v - b' \sin. \overline{\frac{2}{n} + 2m} . v.$$

[illegible] $\times \cos. (2a' \sin. mv + 2b' \sin. 2mv) + \sin. \frac{2v}{n} \times$ [illegible]; assuming therefore the same cosine $= 1$ as [illegible] for the products of the two sines, their values in [illegible] their difference - half the cosine of their sum, we [illegible]

$$[illegible] + a' \cos. \overline{\frac{2}{n} - m} . v - a' \cos. \overline{\frac{2}{n} + m} . v$$

$$[illegible] \cos. \overline{\frac{2}{n} - 2m} . v - b' \cos. \overline{\frac{2}{n} + 2m} . v.$$

[illegible] $-\frac{\alpha r^3}{2d^3}$ is $-\frac{\alpha\alpha'}{2} - \frac{3\alpha\beta'}{2} \cos. mv - \frac{3\alpha w^2}{2} \cos. 2mv$, which is [illegible] substituting the value of P.

[illegible] found the values of $\frac{r^4}{d^4}$ and of sin. $2z$, we have $e = -\frac{3\alpha d}{2p} \int \frac{r^4}{d^4}$ [illegible] $-\frac{3\alpha d}{2p} \int (\gamma' + 4\delta' \cos. mv + 5w^2 \cos. 2mv) \times (\sin. \frac{2v}{n} + a' \sin.$ [illegible] $\overline{\frac{2}{n} + m} . v + b' \sin. \overline{\frac{2}{n} - 2m} . v - b' \sin. \overline{\frac{2}{n} + 2m} . v) \times v$. Now [illegible] general multiplier $-\frac{3\alpha dv}{2p}$, and actually multiplying the other

cos. $\overline{\frac{2}{n} + 2m} \cdot v$; hence, $\frac{3ar^3}{d^3}$ cos. $2z = 3a \times \Big(\Big(\alpha' + 3\beta'$ cos.

$\Big) \times \Big($cos. $\frac{2v}{n} + \acute{a}$ cos. $\overline{\frac{2}{n} - m} \cdot v - \acute{a}$ cos. $\overline{\frac{2}{n} + m} \cdot v + b'$ cos.

cos. $\overline{\frac{2}{n} + 2m} \cdot v\Big)\Big)$; now if we multiply these together, and

…se terms which are of the same species as those which we be-

. neglecting some terms of the same species whose coefficients are

we have $\frac{3ar^3}{2d^3}$ cos. $2z = \alpha' \times$ cos. $\frac{2v}{n} + \alpha'\acute{a}$ cos. $\overline{\frac{2}{n} - m} \cdot v - \alpha'\acute{a}$ cos.

$3\beta'$ cos. $mv \times$ cos. $\frac{2v}{n} + 3w^2$ cos. $2mv \times$ cos. $\frac{2v}{n} + 3\beta\acute{a}$ cos. $mv \times$ cos.

$\cdot v + \alpha'b'$ cos. $\overline{\frac{2}{n} - 2m} \cdot v$; but

$$3\beta' \text{ cos. } mv \times \text{cos. } \frac{2v}{n} = \tfrac{3}{2}\, \beta' \left(\text{cos. } \overline{\tfrac{2}{n} - m} \cdot v + \text{cos. } \overline{\tfrac{2}{n} + m} \cdot v\right)$$

$$3w^2 \text{ cos. } 2mv \times \text{cos. } \frac{2v}{n} = \tfrac{3}{2}\, w^2 \left(\text{cos. } \overline{\tfrac{2}{n} - 2m} \cdot v + \text{cos. } \overline{\tfrac{2}{n} + 2m} \cdot v\right)$$

$$3\beta\acute{a} \text{ cos. } mv \times \text{cos. } \overline{\tfrac{2}{n} - m} \cdot v = \tfrac{3}{2}\, \beta\acute{a} \left(\text{cos. } \overline{\tfrac{2}{n} - 2m} \cdot v + \text{cos. } \frac{2v}{n}\right);$$

putting therefore $\pi = \frac{3}{4}\alpha'$, $\rho = \frac{3}{4}\beta' + \frac{3}{4}\alpha'\acute{a}$, $\sigma = \frac{3}{4}\beta' - \frac{3}{4}\alpha'\acute{a}$, $\tau = \frac{3}{4}\alpha'b' + \frac{3}{4}\beta\acute{a} + \frac{3}{4}w^2$, we have (neglecting the last term on account of its smallness)

$$-\frac{3ar^3}{2d^3} \text{ cos. } 2z = -\pi a \text{ cos. } \frac{2v}{n} - \rho a \text{ cos. } \overline{\tfrac{2}{n} - m} \cdot v$$

$$- \sigma a \text{ cos. } \overline{\tfrac{2}{n} + m} \cdot v - \tau n \text{ cos. } \overline{\tfrac{2}{n} - 2m} \cdot v$$

1174. Having found the values of all the terms which compose the quantity P, let us add them together, and collect the coefficients of the like terms; and

putting $A=2a+a'+\pi$, $B=2b-b'+\rho$, $C=2c+b'+\sigma$, $D=2d+c'-\tau$, $E=\frac{1}{2}\epsilon'$, $L=2p-\frac{e^2}{2}$, we have

$$P=L\alpha-E\alpha\cos. mv-A\alpha\cos.\frac{2v}{n}-B\alpha\cos.\overline{\frac{2}{n}-m}\,.\,v$$

$$-C\alpha\cos.\overline{\frac{2}{n}+m}\,.\,v+D\alpha\cos.\overline{\frac{2}{n}-2m}\,.\,v.$$

1175. Substitute this value of P in the general equation of the curve in Art. 1110. and (dividing by p) we get

$$\frac{1}{r}=\frac{1+L\alpha}{p}-\frac{1}{p}\left(c-\frac{E\alpha}{1-m^2}+\frac{A\alpha}{\frac{4}{n^2}-1}-\frac{B\alpha}{1-\overline{\frac{2}{n}-m}^2}+\&c.\right)\cos. v$$

$$-\frac{E\alpha}{p\times\overline{1-m^2}}\times\cos. mv+\frac{A\alpha}{p\times\overline{\frac{4}{n^2}-1}}\times\cos.\frac{2v}{n}$$

$$-\frac{B\alpha}{p\times\overline{1-\overline{\frac{2}{n}-m}^2}}\times\cos.\overline{\frac{2}{n}-m}\,.\,v+\frac{C\alpha}{p\times\overline{\overline{\frac{2}{n}+m}^2-1}}\times\cos.\overline{\frac{2}{n}+m}\,.\,v$$

$$+\frac{D\alpha}{p\times\overline{1-\overline{\frac{2}{n}-2m}^2}}\times\cos.\overline{\frac{2}{n}-2m}\,.\,v,$$

the Equation of the Lunar Orbit. Make $1+L\alpha=\frac{p}{d}$, $w=\frac{E\alpha d}{p\times\overline{1-m^2}}$, $c-\frac{E\alpha}{1-m^2}+\frac{A\alpha}{\frac{4}{n^2}-1}-\&c.=0$, where d, p, c, w, m, have such values as will satisfy these equations; put also $p'=\frac{A\alpha d}{p\times\overline{\frac{4}{n^2}-1}}$, $q'=\frac{B\alpha d}{p\times\overline{1-\overline{\frac{2}{n}-m}^2}}$, $r'=\frac{C\alpha d}{p\times\overline{\overline{\frac{2}{n}+m}^2-1}}$, s'

$= \frac{Dad}{p \times \overline{1-\frac{2}{n}-2m}^{\,2}}$, and the equation becomes $\frac{d}{r} = 1 - w \cos.\, mv + p' \cos.\, \frac{2v}{n} -$ $q' \cos.\, \overline{\frac{2}{n}-m}\,.\,v + r' \cos.\, \overline{\frac{2}{n}+m}\,.\,v + s' \cos.\, \overline{\frac{2}{n}-2m}\,.\,v$. Now $\frac{d}{r} = 1 - w \cos.\, mv$ expresses (1112) the curve described by a moveable ellipse, and the other terms being small compared with these, first find the value of r from this equation, and substituting it into the value of P, a second equation is found, approximating more nearly to the truth; thus (were we to proceed in the theory of the moon) we might correct the equation of the curve, or the equation which gives the relation between the distance and the angle described. But we here propose only to show the method of finding the mean motion of the apogee.

1176. Now $\frac{d}{r} = 1 - w \cos.\, mv$ are the principal terms of the equation, and (1112) denote a moveable ellipse, containing the great equation of the moon's motion, that is, the equation of the center; also the motion of the apogee. And as this equation does not depend upon the situation of the sun, the motion of the apogee which is denoted by it, may be considered as the mean effect of the disturbing force. This motion of the apogee is constantly progressive, and (1112) is in proportion to the motion of the body as $1 - m$ to 1; if therefore 1 represent the mean motion of the moon, the mean motion of the apogee will be represented by $1 - m$. The other terms are small, and depending on the position of the sun in respect to the moon, they will produce some of the smaller equations of the moon's motion, and the equations of the motion of the apogee. Hence, we may consider $\frac{d}{r} = 1 - w \cos.\, mv$ as an equation representing the basis of the lunar orbit.

1177. By observation, $w = 0{,}05505$ the mean excentricity of the moon's orbit, according to Sir I. NEWTON. Also, $1 - \frac{1}{n} : 1 :: 0{,}0748 : 1$, or $1 - \frac{1}{n} = 0{,}0748$. And if $p =$ the periodic time of the moon, $P =$ that of the earth, we have (818) $p^2 : P^2 :: \frac{d^3}{1} : \frac{b^3}{M}$, setting aside the effect of the disturbing forces; hence, $\frac{p^2}{P^2} = \frac{Md^3}{b^3} = \alpha$, and $\alpha \left[= \frac{p^2}{P^2}\right] = 0{,}005595$ very nearly.

1178. The equation $c - \frac{E\alpha}{1-m^2} + \frac{A\alpha}{\frac{4}{n^2}-1} - \frac{B\alpha}{\overline{1-\frac{2}{n}-m}^{\,2}} + \&c. = 0$, would (as E, B, &c. are in terms of w) give the relation between c and w, from which

we get the ratio of the excentricity of the orbit which would have been described without any disturbing forces to the excentricity of the true orbit; but as c enters not into any future part of the process, it is unnecessary to determine that point.

1179. The equation $1+L\alpha=\frac{p}{d}$ is used to determine the value of d, that quantity entering into the values of p', q', r', s'.

1180. The equation $w=\frac{F.\alpha}{[illegible]}$ contains an element of great importance in the theory of the moon; as from it we obtain the value of m, and thence (1148) the motion of the apogee.

1181. As the motion of the apogee is slow, we may first assume $m = 1$, and afterwards correct that assumption. From these values therefore of $\frac{1}{n}$, a and m, we obtain from the equations in the preceding articles, [illegible]

$p' = 0,0555$, $r' = 1,0151$, $d' = 0,0557$, $w = 0,00303$, $a = 0,00824$, $b' = 0,00017$, $[illegible] = 1,3136$, $[illegible] = 0,1974$, $m = 0,1124$, [illegible] $= 0,00718$, $a = 0,0007$, $b = 0,0416$, $c = 0,00458$.

1182. The coefficients a, b, c, d, as they are expressed in terms of $\frac{d}{p}$, cannot be known till after the resolution of the equation $1+L\alpha=\frac{p}{d}$, in which L itself depends upon $\frac{d}{p}$. Now p represents the semi-parameter of the orbit that would have been described without any disturbing forces, and d the true semi-parameter; and as the magnitude of the orbit can suffer but a very small alteration from the disturbing forces, we may assume $\frac{d}{p} = 1$, and afterwards correct it. Hence, we find a $= 0,8229$, b $= 0,2107$, c $= 0,0543$, d $= 0,0869$, and p $= 1,001$; consequently $A = 3,1595$, $B = 0,5172$, $C = 0,2627$, $D = 0,1721$, $L = 1,4975$. Substitute this value of L into the equation $1+L\alpha=\frac{p}{d}$, and we get $\frac{p}{d} = 1,00838$; hence $\frac{d}{p} = 0,9917$. Now, as the quantities expressing the values of a, b, c, d, have the quantity $\frac{d}{p}$ as a multiplier, the above values of these quantities must be diminished in the ratio of 1 : 0,9917; therefore a $= 0,81607$, b $= 0,20895$, c $= 0,053849$, d $= 0,086179$; consequently we get a more correct value of $A = 3,1557$, $B = 0,5162$, $C = 0,2624$, $D = 0,1708$. Hence, $p' = 0,00722$, $q' = 0,01035$, $r' = 0,00203$, $s' = 0,00097$, of which values, q' and s' are those where the substitution of $\frac{1}{n}$ for

m causes the greatest error, on account of the divisors $1-\overline{\frac{2}{n}-m}^{2}$ and $1-\overline{\frac{2}{n}-2m}^{2}$.

1183. From the equation $w=\frac{Ead}{p\times\overline{1-m^2}}$ we get $1-m^2=\frac{Ead}{wp}$; now $E=0{,}0838$, and $\frac{d}{p}\times a=0{,}9917\times0{,}005595=0{,}00555$; therefore $1-m^2=0{,}008388$, consequently $1-m=0{,}004186$. According to this conclusion therefore, the mean motion of the apogee : that of the moon :: 0,004186 : 1, whereas the ratio ought to be that of 0,008455 : 1; this conclusion therefore gives the motion of the apogee only about one half of what it ought to be according to observation. We must therefore see whether we have not omitted any terms, which, in the present case, may have been too considerable to be neglected.

1184. In substituting the value of $\frac{d}{r}$ into e, into $\frac{3r^2\dot{r}}{2d^3\dot{v}}\times\sin.\ 2z$, and into $\frac{3r^3}{2d^3}\times\cos.\ 2z$, we assumed it equal to $1-w\cos.\ mv$ (1166); whereas, if we assume $\frac{d}{r}=1-w\cos.\ mv+p'\cos.\frac{2v}{n}-q'\cos.\overline{\frac{2}{n}-m}\,.\,v+$ &c. (1175), the introduction of these terms $p'\cos.\frac{2v}{n}-q'\cos.\overline{\frac{2}{n}-m}\,.\,v+$ &c. will be found to have a very considerable effect in the value of $\frac{1}{2}g'$, or of E, and consequently of $1-m$. Now that term which will produce any considerable effect on E is $-q'\cos.\overline{\frac{2}{n}-m}\,.\,v$ in the terms $\frac{r^3}{d^3}$, $\frac{r^4}{d^4}$ and $\frac{r^2\dot{r}}{\dot{v}}$, when joined with $\sin.\frac{2v}{n}$, $\cos.\frac{2v}{n}$ in the values of $\sin.\ 2z$, $\cos.\ 2z$. We will take each case separately.

1185. Assume $\frac{d}{r}=1-w\cos.\ mv-q'\cos.\overline{\frac{2}{n}-m}\,.\,v$; hence, $\frac{d^4}{r^4}=1-4w\cos.\ mv-4q'\cos.\overline{\frac{2}{n}-m}\,.\,v$, therefore $\frac{r^4}{d^4}=1+4w\cos.\ mv+4q'\cos.\overline{\frac{2}{n}-m}\,.\,v$; the two first terms we have already considered; we have therefore to assume $\frac{r^4}{d^4}=4q'\cos.\overline{\frac{2}{n}-m}\,.\,v$, and taking only the first term, $\sin.\frac{2v}{n}$, in the value of $\sin.\ 2z$, we have $\frac{r^4}{d^4}\times\sin.\ 2z\times\dot{v}=4q'\cos.\overline{\frac{2}{n}-m}\,.\,v\times\sin.\frac{2v}{n}\times\dot{v}=2q'\sin.\ mv\,.\,\dot{v}+$

... term we may omit, it not being of the species ... $= -\frac{3ad}{p}\int 2q' \sin. mv \times \dot{v} = \frac{3adq'}{pm} \cos. mv$; the ... in the value of P is $-\frac{6adq'}{pm} \cos. mv$.

... in the quantity $\frac{r^3}{d^3}$, we get the term $3q' \cos. \overline{\frac{2}{n} - m} . v$ ... considered; and taking only the first term, $\cos. \frac{2v}{n}$, in the ... we have $\frac{r^3}{d^3} \times \cos. 2z = 3q' \cos. \overline{\frac{2}{n} - m} . v \times \cos. \frac{2v}{n} = \frac{3}{2} q' \cos.$ ... $\overline{ - n}$ ..., which last term we omit, as not being of the species ...; hence, we assume $\frac{3ar^3}{2d^3} \cos. 2z = \frac{9}{4} aq' \cos. mv$, which is ... $\frac{3ar^3}{2d^3} \cos. 2z$ in the value of P.

... assuming $d = 1$, in the quantity r^3 we introduce the term $3q'$ ... therefore $\frac{r^2\dot{r}}{\dot{v}} = \overline{\frac{2}{n} - m} \times q' \sin. \overline{\frac{2}{n} - m} . v$; hence, we assume ... $\overline{\frac{2}{n} - m} \times \sin. \overline{\frac{2}{n} - m} . v$; and taking only the first term, $\sin. \frac{2v}{n}$, in ... $\sin. 2z$, we have $\frac{3ar^2\dot{r}}{2d^3\dot{v}} \times \sin. 2z = \frac{3aq'}{2} \times \overline{\frac{2}{n} - m} \times \sin. \overline{\frac{2}{n} - m} . v \times$ ... $\overline{\frac{2}{n} - m} \times \cos. mv - \frac{3aq'}{4} \times \overline{\frac{2}{n} - m} \times \cos. \overline{\frac{4}{n} - m} . v$, which last ... omit, as not being of the species we here want; hence, we assume $\frac{3ar^2\dot{r}}{2d^3\dot{v}}$ ... $2z = \frac{3aq'}{4} \times \overline{\frac{2}{n} - m} \times \cos. mv$, which is the correction of $\frac{3ar^2\dot{r}}{2d^3\dot{v}} \times \sin. 2z$ ... value of P.

...88. The three corrections therefore which P receives make $-\left(\frac{6d}{pm} + \frac{9}{4} - \frac{3}{4}\right.$ $\left.\overline{\frac{2}{n} - m}\right) aq' \cos. mv$; but in the value of P before, this term was $-\frac{3}{2} a\beta' \cos.$ mv; and we put $E = \frac{1}{2}\beta'$; we must now therefore put $E = \frac{1}{2}\beta' + \left(\frac{6d}{pm} + \frac{9}{4} - \frac{3}{4} \times\right.$

1191. Let $PEpD$ be the earth at rest without any rotation about its axis, S a body at rest attracting it; draw $SEOD$, and Pp perpendicular to it; then, by the attraction of S, the earth will put on the form of a spheroid, whose minor axis is Pp, and major ED. Let P be the attraction of the spheroid at P, E its attraction at E, independent of the disturbing force of S; then (975) the attraction of M in the direction MR is $P \times \frac{QO}{PO}$, and in the direction MQ it is $E \times \frac{OR}{OE}$. Let m represent the addititious force of S upon the point P, and n that, upon the point E; then (848) as the addititious force varies as the distance from O, $PO : MO :: m : m \times \frac{MO}{PO}$ the addititious force at M; hence, $MO : MR$, or QO, $:: m \times \frac{MO}{PO} : m \times \frac{QO}{PO}$ the addititious force at M in the direction MR; also, $EO : OM :: n : n \times \frac{OM}{EO}$ the addititious force at M; hence, $OM : MQ$, or OR, $:: n \times \frac{OM}{EO} : n \times \frac{OR}{EO}$ the addititious force at M which acts in the direction MQ; therefore (845) $2n \times \frac{OR}{EO}$ = the whole disturbing force on M by the action of S in the direction QM. Hence, the whole force of M in the direction MQ is $\overline{E-2n} \times \frac{OR}{OE}$, and in the direction MR it is $\overline{P+m} \times \frac{QO}{PO}$; let these two forces be represented by Mg, Mr, and complete the parallelogram $Mrqg$, and produce Mq to G; then gq, or Mr, $: Mg :: QG : QM$, that is, $\overline{P+m} \times \frac{QO}{PO} : \overline{E-2n} \times \frac{OR}{OE} :: QG = \frac{OE^2}{OP^2} \times OQ$ (978) $: QM$, or $P+m : E-2n :: OE : OP$; therefore a spheroid having two axes in such a ratio, the direction of the whole force at every point will be perpendicular to the surface, consequently the fluid will be at rest.

1192. By Art. 855, it appears, that the addititious force of the sun on a particle at the earth's surface : the force of gravity at the earth's surface :: the square of the periodic time of a body revolving at the earth's surface : the square of the periodic time of the earth :: 1 : 38604600 = w, and this (without any sensible error in the present case) we may take for the ratio on any part of the earth's surface, considered first as a sphere, and then only disturbed by the sun and moon. Let therefore $P = w$, $m = 1$, and put $OP : OE :: 1 : 1 + d$; then as $m : n :: OP : OE :: 1 : 1 + d$, we have $n = 1 + d$; also (977, 978) $1 + \frac{2}{5}d : 1 + \frac{4}{5}d :: w : E = w \times \overline{1+\frac{2}{5}d}$; hence, by substituting in the last Article, we shall have $w \times \overline{1+\frac{2}{5}d} - 2 + 2d : 1 + d :: 1$, and, by neglecting the terms

smallness, we get $d = \frac{15}{4w - 10} =$

$\frac{15}{4w} = \frac{1}{10294560}$ the part of the

of the two semidiameters; hence,

$d = 2,033$ feet for the effect of the

the effect of the sun is to that of the

ameters and densities conjointly; and if

mean semidiameters to be 32'. 12' and 31'.

91; and (1038) the densities are as 1 :

: 5,412 feet, the height to which the

therefore the sun and moon are in conjunc-

they both tend to raise the tides in the same

7,445 feet. Thus we have determined the

of the tides, upon the supposition of an equi-

that the earth was a sphere and not a spheroid, spheroid and the sun in the equator, then the mean have assumed will be to the radius at the equator : 465. Now as the addititious force varies as the at the equator in the ratio of 464 : 465; also, as varies inversely as the distance (984), the gra- the same ratio; hence, the ratio of gravity w to the diminished in the ratio of $465^2 : 464^2$, consequently d ratio of $464^2 : 465^2$. But w, in this case, instead of in the ratio of the radius at the mean distance to PO, d will be increased in the ratio of $464^3 : 465^3$.

supposed, that the high tide was under the luminary, general equilibrium of the waters; but the high tide is at luminary, and the waters rise and fall by a reciproca- motion of the waters in the open seas is hindered by shallow lands; in consequence of which, the tides in some of the of the conjunction of the luminaries, are found to rise of about three feet. Thus the theory alone will afford no ons. We shall now therefore proceed to explain, as briefly as D. Bernoulli has written upon this subject, as, by correct- observation, he has been enabled to deduce practical rules for and height of the high tides.

be the surface of the earth undisturbed by the sun or moon, be a sphere, $EPQp$ the spheroidical figure arising from the

tides. Let $OM=1$, $Pg=s$, $Eb=r$, $p=3,14159$, &c. then the content of the sphere $=\frac{4p}{3}\times 1^3$, and the content of the spheroid $=\frac{4p}{3}\times\overline{1+r}\times\overline{1-s}^2=\frac{4p}{3}\times\overline{1+r-2s}$, neglecting the terms where s^2 enters on account of their smallness; hence, $\frac{4p}{3}\times 1^3=\frac{4p}{3}\times\overline{1+r-2s}$, therefore $2s=r$, or $2Pg=Eb$. Hence, the altitude of the high tide above the level of the water if there had been no tide, is double of the depression of the low tide below; therefore the middle between the highest and lowest tides is not the height of the sea if it were undisturbed.

1196. Draw Ozv, and wzy perpendicular to QE; then as we may consider the angle zvw a right one, the triangles zvw, zyO will be similar, therefore $zO:zy::zw:zv=zw\times\frac{zy}{zO}$. Let $Oy=s$, $Oz=b$, then $zy=\sqrt{b^2-s^2}$; and if $OE-OP=m$, then (1195) $bE=\frac{2}{3}m$, and $Pg=\frac{1}{3}m$; and by the property of the ellipse, $yw=\frac{OP}{OE}\times\sqrt{Ey\times yQ}=\frac{b-\frac{1}{3}m}{b+\frac{2}{3}m}\times\sqrt{\overline{b+\frac{2}{3}m-s}\times\overline{b+\frac{2}{3}m+s}}=$ (by dividing $b-\frac{1}{3}m$ by $b+\frac{2}{3}m$ and extracting the root of the quantity under the radical sign, omitting the terms where the powers of m above the first enter) $\overline{1-\frac{m}{b}}\times\overline{\sqrt{b^2-s^2}+\frac{\frac{2}{3}bm}{\sqrt{b^2-s^2}}}=\sqrt{b^2-s^2}+\frac{3s^2-b^2}{3b\sqrt{b^2-s^2}}\times m$ very nearly, from which subtract $\sqrt{b^2-s^2}=zy$, and there remains $\frac{3s^2-b^2}{3b\sqrt{b^2-s^2}}\times m=zw$; hence, $zv=\frac{3s^2-b^2}{3b^2}\times m$. Now $Eb=\frac{2}{3}m$; hence, $Eb-zv=\frac{2}{3}m-\frac{3s^2-b^2}{3b^2}\times m=\frac{b^2-s^2}{b^2}\times m$; consequently the falling of the water from the highest point is as the square of the sine of the hour angle from the time of the high tide, E being a point in the equator to which the luminary is vertical. When $zv=0$, we have $3s^2-b^2=0$, hence, $s=b\sqrt{\frac{1}{3}}=$ (if $b=1$) $0,57736$ the cosine of $54°. 44'$ the angle EOM, the distance of the high tide from the point where the water is at the same height at which it would have been if there had been no tide.

1197. If we suppose both the sun and moon to be in conjunction at E, and if $m=OE-OP$ arising from the sun, and $n=$ the difference caused by the moon, then if we take these quantities at the time when the sun and moon are at their mean distances, at which time we may consider their apparent semidiameters as equal, the effects produced will be as their densities (853); therefore their densities will be as $m:n$. Hence, (1196), $zv=\frac{3s^2-b^2}{3b^2}\times m+\frac{3s^2-b^2}{3b^2}\times n$ the altitude from the joint effects of the sun and moon when in conjunction or opposition.

1198. If the sun and moon be not in conjunction, but the sun be vertical to

E and the moon to F, then if r be the cosine of the angle FOz, the altitude zw of the tide at z will be $\frac{3s^2-b^2}{3b^2}\times m+\frac{3r^2-b^2}{3b^2}\times n$. Now to find at what point the tide will be highest, we must make the fluxion$=0$; hence, $ms\dot{s}+nr\dot{r}=0$. Put $A=$ the arc bz, $a=$ the arc Fz, C and c their respective sines; then $\dot{s}=\frac{C}{b}\times\dot{A}$, and $\dot{r}=\frac{c}{b}\times\dot{a}$; hence, by substitution, $\frac{msC}{b}\times\dot{A}+\frac{nrc}{b}\times\dot{a}=0$; but, as $A+a$ is constant, $\dot{A}+\dot{a}=0$, therefore $\dot{a}=-\dot{A}$; consequently $msC=nrc$. But $sC=\frac{b}{2}\times\sin.\,2A$, and $rc=\frac{b}{2}\times\sin.\,2a$; therefore $m\times\sin.\,2A=n\times\sin.\,2a$, or $m:n::\sin.\,2a:\sin.\,2A$. Hence, we have only to divide an arc $2bF$ into two parts, that the ratio of the sines may be given, and the half of each part will give bz and Fz. To do that, let $bac=2bF$, draw be parallel to ac, and take
FIG. 245. $ab:be::m:n$, and join ae, and it will divide it in the ratio required. And to compute the two parts, in the triangle abe we have the angle abe the supplement of the given angle cab, and the ratio of $ab:be$ as $m:n$; hence, $n+m:n-m::\tan.\,\frac{1}{2}.\,\overline{eab+aeb}:\text{tang.}\,\frac{1}{2}.\,\overline{eab-aeb}$, therefore we know the angles themselves. Thus we get the point where the tide is highest. If the arcs A and a be very small, so that the sines may be taken for the arcs, then $m\times 2A=n\times 2a$, or $m\times A=n\times a$, and hence, $m:n::a:A$. M. D. BERNOULLI has solved this problem analytically.

1199. M. D. BERNOULLI proceeds from hence to find the ratio of the densities $m:n$ in the following manner. Conceive on one day the sun and moon to be in conjunction at E, and the high tide at E when they are on the meridian. Now a mean lunar day being $24h.\ 50'$, let us suppose the next day when the sun is returned to E that the moon is got to F, so that the earth has to describe an angle of $50'$ of time before the moon comes to the meridian. Now the greatest tide at z has been found, from the mean of a great number of observations, to be $35'$ after the sun passed the meridian; hence, as these arcs bz, Fz are so small that they may be taken as their sines, we have, by the last Article, $m:n::15':35'$, therefore $n=\frac{35}{15}m=2\frac{1}{3}m$. The arc Fz shows the time of the high tide from the transit of the moon over the meridian. We may compute this ratio at any position of the sun and moon, only by considering how much in time the moon recedes from the sun in a day, and how much the high tide one day precedes that on the next, and thence find Fz and bz. When the sun and moon are in quadratures, if $A=90°$, then $a=0°$, for in this case $m\times\sin.\,2A=0=n\times\sin.\,2a$ agreeable to the conditions; hence, in this situation, the time of the high tide would be at the passage of the moon over the meridian. Afterwards the point z of high tide will lie on the other side of F. Hence, from syzygies to quadratures the high tide precedes the time of the

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moon's passage over the meridian, and from quadratures to syzygies it follows it. Now in quadratures, the difference of the times of the two tides was found to be 85′; therefore we have, $m : n :: 85-50 : 85$; consequently $n = \frac{85}{35} \times m = 2,4m$. From the mean of all the results, M. D. Bernoulli concluded $n = 2\frac{1}{2}m$; which agrees very well with that deduced in Art. 1038. from the precession of the equinoxes, and the nutation of the earth's axis. The method used by Sir I. Newton to determine the ratio of $m : n$, was by observing the greatest (a) and least (b) ascent of the waters from low to high tide at the vernal and autumnal equinox in conjunction or opposition and in quadratures, and then as the effects of the sun and moon are in proportion to their densities, their mean diameters being nearly equal, we have, $n+m : n-m :: a : b$; hence, $n : m :: \frac{1}{2}.a+b : \frac{1}{2}.a-b$. By this method, Sir I. Newton found $n : m :: 3\frac{1}{2} : 1$ from observations made at Bristol. From observations of a like kind made at St. Malo, by M. Thoughoud, it appeared that $n : m :: 1\frac{1}{2} : 1$. The ratio of m and n here found represents the proportional disturbing forces of the sun and moon at their mean distances, where we supposed their apparent semidiameters equal, and consequently they will also represent their densities. But at any other distances, we must add to this ratio, that of the cubes of their apparent diameters. Hence, when the moon is in its perigee and the earth in its aphelion, the ratio of $m : n$ will be very nearly as $1 : 3$; and when the moon is in its apogee and the earth in its perihelion, it will be nearly as $1 : 2$; we will therefore take these two ratios for the limits of the ratio of $m : n$. The great difference therefore in the results of the ratio of $n : m$ from the observations at Bristol and St. Malo, cannot be accounted for from the difference which may possibly take place, and therefore the method of Sir I. Newton gives a conclusion which is subject to too great a degree of inaccuracy to be depended upon. From the agreement of the conclusions found by M. D. Bernoulli with that deduced in Art. 1038. we may suppose the densities of the sun and moon as $1 : 2\frac{1}{2}$ as being probably very near the truth.

1200. Hence, by computing the angle ZOz, or TOz, for every day from the new or full moon, we might get the time of the high tide when compared with the passage of the sun or moon over the meridian; and thus, from theory, we might construct a Table, showing the times of the high tides, if, as we have hitherto supposed, the whole effect of the sun and moon upon the waters took place immediately upon the operation of the cause, and that an equilibrium of the waters was the consequence. But although the sun and moon exert their greatest influence when they are in the meridian, yet they continue to act some time after, from which, and the inertia of the water, it happens that the high tide is not at that time; a Table therefore constructed upon theory alone, must necessarily want to be corrected. We shall therefore explain the principles upon which M. D. Bernoulli has applied this correction.

CHAP. XXXVIII.

ON THE TIDES.

Art. 1190. THE phænomenon of the tides is a circumstance which must have been observed in the first ages of the world. According to Mr. COSTARD, in his History of Astronomy, HOMER is the most ancient author who has mentioned it. HERODOTUS, speaking of the *Red Sea*, or the *Arabian Gulph*, says, that "there is a flux and reflux of water in it every day." And DIODORUS SICULUS describes it to be "a great and rapid tide." These writers, however, do not attempt to guess at the cause. PYTHEAS of *Marseilles*, who lived about the time of ALEXANDER *the Great*, was the first person who suspected that the phænomenon was owing to the moon. PLINY says directly, that it is caused by the sun and moon; *Æstus maris accedere et reciprocare maxime mirum est; pluribus quidem modis accidit, verum causa in sole lunaque. Bis inter duos exortus lunæ affluunt, bisque remeant, vicenis quaternisque semper horis*; and from further observations which he has made upon the tides, it appears, that they must have been very accurately observed in his time. GALILEO thought that the tides were owing to the rotatory motion of the earth about its axis, and its revolution about the sun; but the phænomena can by no means be solved from these causes, as the former motion could only make the earth put on the form of an oblate spheroid subject to no change, and the latter would produce no effect on the surface. DES CARTES imagined the tides to be caused by the pressure of the moon; but according to this hypothesis, the tides ought to be lowest when the moon is on the meridian; nor could the effect be the same when the moon is below as when above the horizon. Dr. WALLIS supposed the phænomenon might be solved by the earth and moon revolving about their center of gravity; but as the tides depend on the situation of the moon to the sun, and are greatest when they are in conjunction, and least when in quadratures, it is manifest, that they must be partly owing to the sun, and therefore as the sun and moon appear to act in like manner to produce the effect, this cannot be the true principle. KEPLER was the first who assigned the true physical cause; he says, that the waters of the sea gravitate towards the moon, and causes the tides (220). Lastly, Sir I. NEWTON has shown, that from the principles of gravity, the phænomena of the tides may be solved; but the effects from theory must be interrupted variously from local circumstances, as the theory supposes the whole surface of the earth to be covered with water, which would, in such a case, have a free motion.

1191. Let $PEpD$ be the earth at rest without any rotation about its axis, S a body at rest attracting it; draw $SEOD$, and Pp perpendicular to it; then, by the attraction of S, the earth will put on the form of a spheroid, whose minor axis is Pp, and major ED. Let P be the attraction of the spheroid at P, E its attraction at E, independent of the disturbing force of S; then (975) the attraction of M in the direction MR is $P\times\frac{QO}{PO}$, and in the direction MQ it is $E\times\frac{QR}{QE}$. Let m represent the addititious force of S upon the point P, and n that upon the point E; then (843) as the addititious force varies as the distance from O, $PO : MO :: m : m\times\frac{MO}{PO}$ the addititious force at M; hence, $MO : MR$, or QO, $:: m\times\frac{MQ}{PO} : m\times\frac{QO}{PO}$ the addititious force at M in the direction MR; also, $EO : OM :: n : n\times\frac{OM}{EO}$ the addititious force at M; hence, $OM : MQ$, or OR, $:: n\times\frac{OM}{EO} : n\times\frac{OR}{EO}$ the addititious force at M which acts in the direction MQ; therefore (843) $2n\times\frac{OR}{EO}$ = the whole disturbing force on M by the action of S in the direction QM. Hence, the whole force of M in the direction MQ is $E-2n\times\frac{OR}{OE}$, and in the direction MR it is $\overline{P+m}\times\frac{QO}{PO}$; let these two forces be represented by Mg, Mr, and complete the parallelogram $Mrag$, and produce Mq to G; then ga, or $Mr : Mg :: QG : QM$, that is, $\overline{P+m}\times\frac{QO}{PO} : E-2n\times\frac{OR}{OE} :: QG = \frac{OE^2}{OP^2}\times OQ$ (978) $: QM$, or $\overline{P+m} : \overline{E-2n} :: OE : OP$; therefore a spheroid having two axes in such a ratio, the direction of the whole force at every point will be perpendicular to the surface, consequently the fluid will be at rest. FIG. 243.

1192. By Art. 855, it appears, that the addititious force of the sun on a particle at the earth's surface : the force of gravity at the earth's surface :: the square of the periodic time of a body revolving at the earth's surface : the square of the periodic time of the earth :: 1 : 38604600 = w, and this (without any sensible error in the present case) we may take for the ratio on any part of the earth's surface, considered first as a sphere, and then only disturbed by the sun and moon. Let therefore $P = w$, $m = 1$, and put $OP : OE :: 1 : 1 + d$; then as $m : n :: OP : OE :: 1 : 1 + d$, we have $n = 1 + d$; also (977, 978) $1+\frac{1}{2}d : 1+\frac{3}{4}d :: w : E = w\times\overline{1+\frac{1}{4}d}$; hence, by substituting in the last Article, [illegible] $1 + d$ [illegible]; and, by neglecting the terms

moon's passage over the meridian, and from quadratures to syzygies it follows it. Now in quadratures, the difference of the times of the two tides was found to be 85′; therefore we have, $m : n :: 85-50 : 85$; consequently $n = \frac{85}{35} \times m = 2\frac{2}{5}m$. From the mean of all the results, M. D. Bernoulli concluded $n = 2\frac{1}{2}m$; which agrees very well with that deduced in Art. 1038. from the precession of the equinoxes, and the nutation of the earth's axis. The method used by Sir I. Newton to determine the ratio of $m : n$, was by observing the greatest (a) and least (b) ascent of the waters from low to high tide at the vernal and autumnal equinox in conjunction or opposition and in quadratures, and then as the effects of the sun and moon are in proportion to their densities, their mean diameters being nearly equal, we have, $n+m : n-m :: a : b$; hence, $n : m :: \frac{1}{2}.a+b : \frac{1}{2}.a-b$. By this method, Sir I. Newton found $n : m :: 3\frac{1}{2} : 1$ from observations made at Bristol. From observations of a like kind made at St. Malo, by M. Thoroud, it appeared that $n : m :: 1\frac{1}{2} : 1$. The ratio of m and n here found represents the proportional disturbing forces of the sun and moon at their mean distances, where we supposed their apparent semidiameters equal, and consequently they will also represent their densities. But at any other distances, we must add to this ratio, that of the cubes of their apparent diameters. Hence, when the moon is in its perigee and the earth in its aphelion, the ratio of $m : n$ will be very nearly as $1 : 3$; and when the moon is in its apogee and the earth in its perihelion, it will be nearly as $1 : 2$; we will therefore take these two ratios for the limits of the ratio of $m : n$. The great difference therefore in the results of the ratio of $n : m$ from the observations at Bristol and St. Malo, cannot be accounted for from the difference which may possibly take place, and therefore the method of Sir I. Newton gives a conclusion which is subject to too great a degree of inaccuracy to be depended upon. From the agreement of the conclusions found by M. D. Bernoulli with that deduced in Art. 1038. we may suppose the densities of the sun and moon as $1 : 2\frac{1}{2}$ as being probably very near the truth.

1200. Hence, by computing the angle EOz, or TOz, for every day from the new or full moon, we might get the time of the high tide when compared with the passage of the sun or moon over the meridian; and thus, from theory, we might construct a Table, showing the times of the high tides, if, as we have hitherto supposed, the whole effect of the sun and moon upon the waters took place immediately upon the operation of the cause, and that an equilibrium of the waters was the consequence. But although the sun and moon exert their greatest influence when they are in the meridian, yet they continue to act some time after, from which, and the inertia of the water, it happens that the high tide is not at that time; a Table therefore constructed upon theory alone, must necessarily wait to be corrected. We shall therefore explain the principles upon which M. D. Bernoulli has applied this correction.

FIG. 246. 1201. When the sun and moon are in syzygies b at Brest, it appears, from the mean of a great number of observations, that the high tide happens at 3h. 28′; and when the sun is at b and the moon in quadratures at g, it happens at 8h. 40′; the difference is 5h. 12′. This difference was observed to be the same at Dunkirk, and at other ports, although the absolute times were different. Now let us consider what is the difference from theory. When the sun and moon are in syzygies at b, the high tide is at b at that time, or at 12 o'clock. When the moon is in quadratures at g, it is low tide on the earth at b; now whilst the earth is revolving about its axis so as to bring this point at b up to the moon, the moon is got to v about 3° from g, and by our computation (1198) the point z of high tide is about 2° beyond v; hence, the point on the earth at b at low tide must describe an arc bz of 95° before it be at the high tide, which arc it describes in 6h. 20′. This interval of the two tides therefore from theory does not agree with observation. Take the point F at 20° before b, and suppose the moon to be in F and the high tide by theory to be at z'; then, by computation, this happens at 11h. 2′, lunar time. Now let us take F at 20° before g, and suppose the moon at F and the high tide at z''; then, by computation, this is found to happen at 3h. 59½′ lunar time, which gives an interval of 4h. 57½′ lunar time, or 5h. 8′ solar time. This therefore agrees very well with the interval between the times of the high tides when the moon is in conjunction and quadratures. Hence, to get the true interval of the times of the tides, we must compute from our theory, by supposing the moon 20° behind its true place, and then we shall get the true interval, agreeing with observation. Hence, the following Table was computed, the first column of which shows the moon's true distance from the sun when the moon passes the meridian, for every 10° from conjunction to opposition; the next three columns show the times of the high sea in respect to the passage of the moon over the meridian, for the perigee, mean distance and apogee of the moon, and the last three show the absolute hours.

A TABLE

For finding the Time of the High Tides.

Dist. ☉ à ☾.	Time of High Tide before and after the passage of the moon over the meridian.			True Time of the High Tide nearly.		
	Perig. of ☾.	Mean Dist. ☾	Apog. of ☾.	Per. of ☾	Me.Dis. ☾	Apo. of ☾
0°	18′ after.	22′ after.	27½′ after.	0h. 18′	0h. 22′	0h. 27½′
10	9½ after.	11½ after.	14 after.	0. 49½	0. 51½	0. 54
20	0	0	0	1. 20	1. 20	1. 20
30	9½ before.	11½ before.	14 before.	1. 50½	1. 48½	1. 46
40	18 before.	22 before.	27½ before.	2. 22	2. 18	2. 12½
50	26 before.	31½ before.	39½ before.	2. 54	2. 48½	2. 40½
60	33 before.	40 before.	50 before.	3. 27	3. 20	3. 10
70	37½ before.	45 before.	56 before.	4. 2½	3. 55	3. 44
80	38½ before.	46½ before.	58 before.	4. 41½	4. 33½	4. 22
90	33½ before.	40½ before.	50½ before.	5. 26½	5. 19½	5. 9½
100	21 before.	25 before.	31 before.	6. 19	6. 15	6. 9
110	0	0	0	7. 20	7. 20	7. 20
120	21 after.	25 after.	31 after.	8. 21	8. 25	8. 31
130	33½ after.	40½ after.	50½ after.	9. 13½	9. 20½	9. 30½
140	38½ after.	46½ after.	58 after.	9. 58½	10. 6½	10. 18
150	37½ after.	45 after.	56 after.	10. 37½	10. 45	10. 56
160	33 after.	40 after.	50 after.	11. 13	11. 20	11. 30
170	26 after.	31½ after.	39½ after.	11. 46	11. 51½	11. 59½
180	18 after.	22 after.	27½ after.	0. 18	0. 22	0. 27½

1202. This Table gives the true interval of the tides, and also the time very nearly, upon supposition that the luminaries are in the equator, and that the effect could take place as supposed in the theory. But from the inertia of the water, and the obstructions it meets with in its passage from rocks, islands, shores, &c. this Table cannot exhibit the absolute time of the high tide at every port, which must vary according to the effect of these circumstances, although it shows the difference of the times. To determine therefore the true time at any port, we must find from observation what is the difference between the true time and that shown by the Table, and then that difference added to the time shown by the Table will give the time of the high tide. For the same obstacles remaining, there must always be nearly the same retardations; the greater however the tides are, the less the same causes will retard, and the less they are, the more they will retard; and accordingly it is found from observation, that the highest tides always come sooner to their height, and the lowest later, than the calculations give it, the above difference being determined from observations on the mean high tides. The declination of the luminaries, as it alters the quantity of the tides, and also their direction, will cause some small variation of the difference; and the different direction of the winds must have also some effect.

1203. But besides the small variation of difference arising from the declination of the sun and moon for the reasons in the last article, the time will also be altered from hence, that as *bg* is not the equator, the arcs upon it will not be the measures of the hour angles. As the moon's orbit makes but a small angle with the ecliptic, we may suppose them to coincide. Hence, when the moon is in the equator, the arc of the moon's orbit included between two meridians : the corresponding arc of the equator :: rad. : cos. $23\frac{1}{2}°$, or as 100 : 92; and when the declination is the greatest, these arcs are as cos. dec. : rad. or as 92 : 100. Hence, the numbers in the second, third, and fourth columns must in the former case be multiplied by $\frac{92}{100}$, and in the latter by $\frac{100}{92}$. Very nearly in the middle point between these situations the arcs will be equal; and for any intermediate points, we may compute the multiplier by the Note, Art. 128.

FIG. 244. 1204. From the value of zv in Art. 1197. we may compute the altitude of the sea at any time; and by Art. 1198. we can find the points where the tide is highest, and at 90° from thence it is lowest. Thus we can find the altitude of the highest and lowest tides for all times, supposing (as in Art. 1201.) the moon to be 20° behind its true place. But this can only give the relative altitudes, the true altitudes varying very much in different parts, from their situation. M. D. BERNOULLI therefore puts A and B for the mean height of the high tides when the luminaries are in conjunction or opposition, and when they are in quadratures, to be determined from observation; the height of the tide being the ascent of the water from the low to the high tide. Hence, $n+m=A$, $n-m=B$; therefore

$n = \frac{A+B}{2}$; $m = \frac{A-B}{2}$; and putting these values for n and m, he has constructed the following Table, the first column of which shows the distances of the sun and moon, at the time the moon passes the meridian, and the other three show the height of the high tides for the perigee, mean distance, and apogee of the moon.

A TABLE

To show the Height of the High Tides.

Dist.	Alt. in Perigee.	Alt. at Mean Dist.	Alt. in Apogee.
0°	0,995*A* + 0,149*B*	0,883*A* + 0,117*B*	0,795*A* + 0,082*B*
10	1,104*A* + 0,038*B*	0,970*A* + 0,030*B*	0,874*A* + 0,021*B*
20	1,138*A* + 0,000*B*	1,000*A* + 0,000*B*	0,901*A* + 0,000*B*
30	1,104*A* + 0,038*B*	0,970*A* + 0,030*B*	0,874*A* + 0,021*B*
40	0,995*A* + 0,149*B*	0,883*A* + 0,117*B*	0,795*A* + 0,082*B*
50	0,853*A* + 0,319*B*	0,750*A* + 0,250*B*	0,676*A* + 0,176*B*
60	0,668*A* + 0,527*B*	0,587*A* + 0,413*B*	0,529*A* + 0,290*B*
70	0,460*A* + 0,749*B*	0,413*A* + 0,587*B*	0,327*A* + 0,412*B*
80	0,284*A* + 0,958*B*	0,250*A* + 0,750*B*	0,225*A* + 0,527*B*
90	0,133*A* + 1,127*B*	0,117*A* + 0,883*B*	0,105*A* + 0,621*B*
100	0,034*A* + 1,238*B*	0,030*A* + 0,970*B*	0,027*A* + 0,682*B*
110	0,000*A* + 1,277*B*	0,000*A* + 1,000*B*	0,000*A* + 0,703*B*
120	0,034*A* + 1,238*B*	0,030*A* + 0,970*B*	0,027*A* + 0,682*B*
130	0,133*A* + 1,127*B*	0,117*A* + 0,883*B*	0,105*A* + 0,621*B*
140	0,284*A* + 0,958*B*	0,250*A* + 0,750*B*	0,225*A* + 0,527*B*
150	0,460*A* + 0,749*B*	0,413*A* + 0,587*B*	0,372*A* + 0,412*B*
160	0,668*A* + 0,527*B*	0,587*A* + 0,413*B*	0,529*A* + 0,290*B*
170	0,853*A* + 0,319*B*	0,750*A* + 0,250*B*	0,676*A* + 0,176*B*
180	0,995*A* + 0,149*B*	0,883*A* + 0,117*B*	0,795*A* + 0,082*B*

It is manifest from this Table, that the highest tides are when the moon has passed conjunction 20°, or about $1\frac{1}{2}$ days after, and the lowest tides when the moon has so far passed her quadratures.

1205. We come in the next place to consider the effect arising from the declination of the moon. It appears by Art. 1196. that the fall of the water from E to $v = \frac{b^2 - s^2}{b^2} \times m$, therefore at P it is $= m$; hence, $m - \frac{b^2 - s^2}{b^2} \times m = \frac{s^2}{b^2} \times m =$ the rise of the water from P to v; put therefore $c =$ the cosine of the angle EOv to radius unity, and we have $c^2 m =$ the height of the water above the lowest point.

1206. Let $GBAH$ be a meridian of the earth passing through the moon vertical to B, AG the axis of the earth, DMK the equator, and CEL any parallel to it, and assume E any place; then will the diameter BH be the axis of the spheroid, which, as it differs but very little from a sphere, may be regarded as such, so far as respects the triangles on its surface. Put therefore radius $= 1$, $S =$ sine of AB, $C =$ its cosine, $s =$ sine of AE, $c =$ its cosine, $y =$ cosine of the angle BAE, and $q =$ cosine of BE; then, by Spherical Trigonometry, $q = Ssy + Cc$; therefore, by the last Article, $\overline{Ssy + Cc}^2 \times m =$ the height of the water above the lowest point. Hence, we may consider the following cases. FIG. 247.

I. Let e be the point where the water is lowest; then $Ssy + Cc = 0$, hence, $y = -\frac{Cc}{Ss}$ the cosine of BAe.

II. When $C = s$, and $c = S$, then $y = -1$, therefore the angle $CAe = 180°$, and consequently e coincides with L. Hence, when the latitude of the place $=$ the complement of the moon's declination, the low tide happens at L, distant from the high tide at C twelve hours; in this case therefore there is only one high and one low tide in twenty-four hours.

III. When s is less than C, or when the distance of the place from the pole is less than the moon's declination, then $\overline{Ssy + Cc}^2 \times m$ never can become $= 0$, within the limits of y. Hence, the altitude diminishes from the passage of the moon over the meridian to the opposite meridian; and consequently from the parallel whose distance from the pole $=$ the moon's declination, to the pole, there is only one high and one low tide in twenty-four lunar hours. And if we make $y = 1$, and $y = -1$, we have $\overline{Ss + Cc}^2 \times m - \overline{Cc - Ss}^2 \times m = 4SsCcm$ for the difference of the altitudes of the two tides.

IV. When the declination of the moon is equal to the latitude of the place,

$S=s$, $C=c$, and make $y=1$; hence, $Ss+Cc=S^2+C^2=1$; therefore the greatest altitude $=m$; also, y (Case I.) $=\frac{Cc}{Ss}=\frac{C^2}{S^2}=$ cos. B.&c.

V. When the moon is in the equator, $S=1$, $C=0$, and the altitude of the tide $=s^2m$, which therefore varies as the square of the cosine of latitude. Hence, in this case, at the pole there is no tide.

VI. The height of the tide when the moon passes the meridian $=\overline{Ss+Cc}^2\times m$ and when the moon is at the opposite meridian the height is $\overline{-Ss+Cc}^2\times m$. Hence, when the moon is in the equator $C=0$, and the height of both tides are equal. To a place on the north of the equator, when the moon has south declination, C becomes negative, and the latter tides are the greatest; but when the moon has north declination, C is positive, and the former is the greatest. Hence, to us in this case, the high tide is greater when the moon is above the horizon than when below. In all cases, e is nearer to or further from C, according as $y\left[-\frac{Cc}{Ss}\right]$ is positive or negative. The difference of the two tides is always $=4SsCcm$.

VII. The height of the two tides when the moon passes the meridian being $\overline{Ss+Cc}^2\times m$ and $\overline{-Ss+Cc}^2\times m$, the mean height is $\overline{S^2s^2+C^2c^2}\times m$.

VIII. Hence, the same north and south declination of the moon give the same mean altitude. This is confirmed by observations.

IX. In latitude 45°, $S^2=C^2=\frac{1}{2}$; hence, the mean altitude $=\frac{1}{2}\times\overline{s^2+c^2}\times m=\frac{1}{2}m$; therefore whatever be the declination of the moon, the mean altitude is, in this latitude, always the same. Hence, in our latitude, the mean altitude will vary but very little.

X. Under the equator, the mean height $=S^2m$, which therefore varies as the square of the cosine of the moon's declination.

1207. As the tides rise from the collecting of the waters on the whole surface of the main sea, if there be any quantities of water separated from it, the variation must be proportionally smaller. For if ro be a small surface of water detached from the rest, its surface will put on the figure xs similar to $d't$, consequently the variation xr from the mean altitude must be very small. Hence, there have never been any tides observed in the Çaspian sea; for from the dimensions of that sea, the greatest altitude will not be above $1\frac{1}{2}$ inches at the eastern and western extremities, according to M. de la LANDE, who has corrected an error made by M. D. BERNOULLI in his computation; and it is manifest, that the middle of the sea will never be affected. Very small tides have been observed in the Black sea, which, from its connection with the Mediterranean sea only by a very small passage, may be considered as a de-

FIG. 244.

tached sea. The Mediterranean sea is connected to the main sea only by a narrow passage at Gibraltar, so that only a small quantity of tide from the open sea can flow in, and the sea itself is not large enough to produce any very sensible tides; accordingly we find the tides there to be but very small. The best observations are those which have been made by M. le Chevalier d'ANGOS at Toulon, from which it appears, that the tides produce a variation of about one foot in the height of the water. There were frequently greater variations, but those appeared to arise from high winds.

1208. The general phænomena of the tides from observation agree very well with the conclusions deduced from the theory of gravity; indeed much more accurately than could have been expected, when we consider how many circumstances there are which take place, and which cannot be reduced to computation. The theory supposes the whole surface of the earth to be covered with deep waters—that there is no inertia of the waters—that the major axis of the spheroid is constantly directed to the moon, and that there is an equilibrium of all the parts. But the inertia of the waters will make them continue to rise after they have passed the moon, although the action of the moon begins to decrease, and they come to their greatest altitude in the open seas about three hours after, at which time there is not a general equilibrium, but the waters rise and fall by a reciprocation; hence, the longest axis is not directed to the moon, nor is the figure a perfect spheroid—The waters have not a free motion on account of the shallow places, rocks, islands and continents, the force of currents and winds; also, as the waters approach the equator where the earth has a greater velocity about its axis, they must necessarily be left behind and obstruct the regular motion of the water when it moves from west to east, but conspire with that from east to west. All these circumstances must affect the measures of the phænomena as deduced from theory; it may however in many cases give the relative measures without any great error, so that by accurate observations once made on their absolute quantity in some one particular case, the measures, in all other cases, may be ascertained to a considerable degree of accuracy.

1209. If a place communicate with two seas, or has two inlets to the same sea, two tides may arrive at that place at different times, and produce various phænomena. An instance of this kind takes place at Batsha, a port in the kingdom of Tunquin in the East Indies, in 20°. 50′ north latitude. The day in which the moon passes the equator the water stagnates; as the moon recedes from the equator towards the north, the water begins to rise and fall once a day; and it is high water at the setting of the moon, and low water at its rising. This daily tide increases for six or eight days, and then decreases for the same time by the same degrees, and the motion ceases when the moon has

returned to the equator. When it has passed the equator, and approaches the south pole, the water rises and falls as before, but it is now high water at the rising, and low water at the setting of the moon. Sir I. NEWTON thus accounts for this phænomenon. There are two inlets to this port, one from the Chinese ocean between the continent and the Manillas, the other from the Indian ocean between the continent and Borneo; and he supposes that a tide may arrive at Batsha, through one inlet, at the third hour of the moon, and through the other inlet, six hours after; and supposing these tides to be equal, one flowing in whilst the other flows out, the water must stagnate. Now they are equal when the moon is in the equator; but as the moon begins to decline on the same side of the equator with Batsha, the diurnal tides exceed the nocturnal (as appears by the foregoing principles), so that two greater and two lesser tides must arrive at Batsha by turns. The difference of these will produce a motion of the water, which will rise to its greatest height at the mean time between the two greatest tides, and fall lowest at the mean time between the two lowest tides; so that it will be high water about the sixth hour at the setting of the moon, and low water at its rising. When the moon has got to the other side of the equator, the nocturnal tide will exceed the diurnal; and therefore the high water will be at the rising, and low water at the setting of the moon. These principles will account for other extraordinary tides which are observed in those places whose situation exposes them to such irregularities.

CHAP. XXXIX.

ON THE PRINCIPLES OF PROJECTION, AND THE CONSTRUCTION OF GEOGRAPHICAL MAPS.

Art. 1210. THE projection of an object is its representation upon a plane, and is formed by drawing lines from the eye to the plane through every point of the object; and according to different situations of the eye, the object and the plane, the representations will be different. The projection in order to be perfect, should be a perfect representation of the object, that is, the proportion and relative situation of all the parts of the figure should be the same as in the object; but in the construction of geographical maps, this is not practicable; it being impossible to give a true representation of a spherical surface upon a plane, retaining the true proportion of the figures, magnitudes and positions of the countries, with the relative degrees of latitude and longitude. We will first show the principles of the different projections, and then apply them to our present purpose.

On the Orthographic *Projection.*

1211. If the eye be supposed to be at an indefinitely great distance, so that all the lines drawn from it to the object may be considered as parallel, and also perpendicular to the plane of projection, the projection is called *Orthographic.*

1212. The figure of a straight line AB is a straight line in the projection. For draw AE, BD perpendicular to xy the plane of the projection, and join ED, and it will represent the intersection of the plane passing through $EABD$ with the plane of projection; draw mn perpendicular to ED, and n is the projection of m; thus it appears that ED is the projection of AB. Draw AC parallel to ED, then $ACDE$ is a parallelogram, and $AC = ED$; AC may therefore represent the projection of AB. Hence, if we want to make the representation upon a plane at a distance from the body, it will be all the same if we suppose the plane to touch the body, the parallelism of the plane remaining the same. FIG. 248.

1213. The figure of the projection of a circle is an ellipse. For let ABC be a semicircle conceived to be inclined to the plane of the paper, which we will make the plane of projection; draw BE perpendicular to that plane, and ED, FIG. 249.

BD perpendicular to the diameter AC, then ED is the projection of BD (1212). By the property of the circle, $AD \times DC = BD^2$; but as the angle BDE is constant, being the inclination of the circle to the plane of projection, BD is to DE in a constant ratio, namely, that of radius to the cosine of BDE; therefore DE varies as BD, and consequently DE^2 varies as BD^2; hence, $AD \times DC$ varies as DE^2, which is the property of an ellipse; the curve AEC is therefore an ellipse. And by the last Article it appears, that the projection will be the same, at whatever distance the circle is from the plane of projection. Let O be the center, and draw OP, OQ perpendicular to AC, then OQ is the minor axis; hence, to the radius of the circle, the minor axis is the cosine of the inclination of the circle to the plane of projection. If the circle be parallel to the plane of projection, the projection will be a circle equal to it.

1214. By the property of the ellipse and circle, the area ABC : the area AEC :: BD : ED :: rad. : cos. BDE; hence, the area of the circle will be diminished by this projection in the ratio of radius : cosine of the inclination of the plane of the body to the plane of projection. And this must manifestly be true whatever be the form of the body ABC (considered as a plane), because every line in the body is to its corresponding line in the projection in that ratio. Also, the projection is not similar to the body. Hence, equal parts upon the surface of a sphere will not be projected into parts either equal or similar.

1215. If ABC be perpendicular to the plane of projection, E and D coincide, and D is the projection of B; thus the circle ABC is projected into its diameter; the arc AB is projected into its versed sine AD, and BP is projected into DO, which is equal to the sine of BP, or the cosine of AB. If $AB = 60°$, then $BP = 30°$, and $AD = DO$; these two arcs therefore, one of which is double of the other, have their projections equal. This projection is used in the construction of solar eclipses, CHAP. XXIII.

On the Stereographic *Projection.*

1216. Let an eye be situated any where upon the surface of a sphere, and from it draw a diameter, and perpendicular to this diameter draw a great circle; then if all the circles in the hemisphere opposite to the eye be projected upon that great circle by lines drawn to the eye, the projection is called *Stereographic*, and the point opposite to the eye is called the *Pole*.

FIG. 250. 1217. Let $EQPR$ be a sphere whose center is O, E the place of the eye, draw the diameter EOP, and QOR perpendicular to it, representing the plane of projection; draw ECA, EDB, and join BO. Now DO is the projection of PB; but DO is the tangent of the angle $DEO = \frac{1}{2}\ BOP$. Hence, the pro-

jection of an arc measured from the pole is equal to the tangent of half that arc.

1218. Conceive the chord AB to be the diameter of a circle described upon the surface, and let AEB represent a cone having that circle for its base. Draw BG parallel to QR; then the arc $EB = EG$, therefore the angle $EAB = EBG$ $=$ (as BG is parallel to QR) EDC; therefore as CD, AB represent two sections of the cone cutting its sides at the same angle, the sections must be similar; but the section AB is a circle; therefore the section CD is a circle. Hence, the projection of every circle is a circle.

1219. Join EG; then BG being the diameter of a circle parallel to the plane of projection, DH will be the diameter of the circle of projection, whose center is O. Hence, the projection of all circles parallel to the plane of projection will be concentric circles, the radii of which are the tangents of half the distances of the circles from the poles.

1220. The projection of the arc QPR is the straight line QOR; and the same for every great circle passing through P.

1221. Produce BO to I, join EI, and produce it to meet QR produced in K, and bisect DK in r; then considering BOI as the diameter of a circle whose plane is inclined to the plane of projection in the angle QOB, DK will be the diameter of the projection of that circle (1216). Now $DK = OK + OD =$ tan. $OEK +$ tan. $OED =$ tan. $\frac{1}{2} PI +$ tan. $\frac{1}{2} PB =$ cot. $\frac{1}{2} PB +$ tan. $\frac{1}{2} PB = \frac{2}{\sin. PB}$ $= 2$ cosec. $PB = 2$ sec. QB; therefore $\frac{1}{2} DK =$ sec. QB. Hence, the radius of projection of any great circle is the secant of the angle between the plane of the circle and the plane of projection. From these four articles it appears, that the projection of the parts of the sphere will not properly represent in magnitude and situation, the parts themselves.

1222. If E be the pole of the earth, the meridians (as they pass through P) will be projected into straight lines (1220); and the parallels to the equator will be projected into circles whose center is O (1219). And if BOI be the diameter of the ecliptic, DK is its projection (1221). This is called the *Polar* projection, and was used by PTOLEMY.

1223. If $EQPR$ be the equator, then the point on the sphere vertical to O will be the pole, and BOI will be the diameter of any meridian. And if Q be the point from which the longitude is reckoned; then the projection of the radius of that meridian will be the secant of its longitude (1221).

1224. To find the projection of the parallels of latitude, let EOP represent the plane of the equator, the representation being a straight line passing through the eye, then IG is the diameter of a parallel, the projection of which is HK $= OK - OH =$ tan. $\frac{1}{2} PI -$ tan. $\frac{1}{2} PG =$ cot. $\frac{1}{2} EI -$ tan. $\frac{1}{2} PG =$ cot. $\frac{1}{2} PG$ $-$ tan. $\frac{1}{2} PG = 2$ cot. PG. Hence, the radius of the projection of the parallels is the cot. of their latitudes. This is called an *Equatorial* projection.

1225. The stereographic projection is very convenient for practice, as all the circles are projected into circles or straight lines, which are more easily described than any other figures.

On Mercator's Projection.

FIG. 251. 1226. Let P be the pole of the earth, supposed to be a sphere, EQ the equator, PE, PR two meridians, mn a small circle parallel to ER, PC the radius of the earth, mr, nr perpendicular to it, and join EC, RC. Then, mr, nr being parallel to EC, RC respectively, the angle $mrn = ECR$; hence, by similar sectors, $ER : mn :: EC : mr$; but when the angle is given, the arc of a degree is in proportion to the radius; also, the length of a degree of the great circle ER is equal to a degree of latitude, and the length of a degree of the circle mn is a degree of longitude. Hence, a degree of latitude : a degree of longitude :: $EC : mr$ :: radius : cosine of latitude.

1227. In this projection, the sphere is projected upon a plane, and the meridians EP, RP are projected into straight and parallel lines; consequently P in the projection must be at an infinite distance from EQ. In this case, the arc mn being the same for all latitudes, the length of a degree of longitude is every where the same; to preserve therefore the proper proportion between the degrees of latitude and longitude, the degrees of latitude must increase as you go from the equator, so that they may always be to a degree of longitude in the proportion of radius : cosine of latitude.

FIG. 252. 1228. Let P be the pole, E the equator, PCQ the axis of the earth, C the center, m a place on the surface; draw mr perpendicular to PQ, and join mC, mQ. Put $Cm = r$, $Em = x$, Cr (the sine of Em the latitude of m) $= y$, and the length of Em on the projection $= z$, called the *Meridional Parts*. Then (1227) $\sqrt{r^2-y^2}$ (cos. of lat.) : $r :: \dot{x} : \dot{z} = \frac{r\dot{x}}{\sqrt{r^2-y^2}}$; but $\dot{x} = \frac{r\dot{y}}{\sqrt{r^2-y^2}}$; hence, $\dot{z} = \frac{r^2\dot{y}}{r^2-y^2} = \frac{r}{2} \times \frac{2r\dot{y}}{r^2-y^2}$, therefore $z = \frac{r}{2} \times$ h. l. $\frac{r+y}{r-y}$ + cor. $= r \times$ h. l. $\sqrt{\frac{r+y}{r-y}}$ + cor. But by plane Trigonometry, $\sqrt{r^2-y^2}$ $(mr) : r+y$ $(rQ) :: r$ (rad.) : $\frac{r \times \overline{r+y}}{\sqrt{r^2-y^2}} = r \times \sqrt{\frac{r+y}{r-y}}$ the tangent of the angle $rmQ =$ cotan. of $rQm =$ cotan. of $\frac{1}{2}\, rCm =$ cotan. of half the complement of latitude. Hence, $\sqrt{\frac{r+y}{r-y}}$ $= \frac{\text{cotan. } \frac{1}{2} \text{ comp. lat.}}{r}$; consequently $z = r \times$ h. l. $\frac{\text{cotan. } \frac{1}{2} \text{ comp. lat.}}{r}$ + cor.; but when $z = 0$, cotan. $\frac{1}{2}$ comp. lat. $= r$; therefore the last equation becomes $0 = r \times$ h. l. $\frac{r}{r}$ + cor. $= r \times$ h. l. 1 + cor. $= 0$ + cor. consequently the correction

$= 0$; hence, $z = r \times \text{h. l.} \frac{\text{cotan.} \frac{1}{2} \text{comp. lat.}}{r} = r \times \text{h. l. cotan.} \frac{1}{2} \text{comp. lat.} - r \times \text{h. l.} r$ the length of the meridian *Em* on the projection. If therefore we take the latitude $= 1°, 2°, 3° \ldots\ldots 90°$, we can construct a Table showing the length of the meridian on the projection for every degree of latitude. In like manner it may be constructed for every minute. Such a Table is called a Table of *Meridional Parts.*

1229. If we take the earth of its true figure, that of a spheroid, we may compute the meridional parts upon the same principles; but we shall not here give the investigation, as we only want to explain the nature of this projection, so far as it may be necessary to show how the charts are constructed. If we assume Sir I. NEWTON's ratio of the diameters of the earth, for 50° latitude the difference of the meridional parts on a sphere and spheroid will not be above the 60th part of the whole. It is manifest that this projection cannot give the true proportion of the parts of the earth; for the figure of the part *ERnm* on the projection would be a parallelogram. It is however very convenient for navigation, because the rumbs are all projected into straight lines; for the meridians being all straight and parallel lines, the line which cuts them all at the same angle must be a straight line, which is the property of the rumb (1231). FIG. 251.

On the Construction of Maps.

1230. A map is the representation of the whole, or part of the surface of the earth upon a plane; and to be perfect it ought, *first*, to show the true latitude and longitude of every place; *secondly*, it ought to show all countries of their proper figures and magnitudes; *thirdly*, the relative situations of the countries should be truly laid down. But all these circumstances cannot take place in any construction, as has been already observed; consequently no map upon a plane can be a true representation of the countries upon the earth's surface.

1231. A *Rumb* upon the globe is a line which cuts all the meridians at the same angle. Let *EQ* be the equator, *P* the pole, *PE*, *PR* two meridians, and *Asmv* a rumb, then the angle $Psm = Pmv$. Draw the small circle *mn* parallel to *EQ*, and conceive *PE*, *PR* to be indefinitely near each other; then we may consider the triangle *smn* to be a plane one; hence $sm : sn ::$ rad. : cos. *nsm*; but the angle *nsm* is constant for the same rumb; therefore $sm : sn$ in a constant ratio; that is, as the rumb approaches the pole, the increment of the rumb : the corresponding increment of its latitude in a constant ratio; therefore, componendo, the whole increase of the rumb : the whole corresponding increase of latitude in the same constant ratio. When therefore FIG. 253.

a ship runs upon a rumb, and varies its latitude 5° for instance, if it continue on the same rumb and describe the same space, it will have altered its latitude again 5°. In general, equal parts of the same rumb are contained between equidistant parallels of latitude.

The Orthographic *Projection of Maps.*

FIG. 254. 1232. Let FIG. 254. represent the orthographic projection of the earth upon the plane of the meridian, P, p representing the poles, and EQ the equator. Then the meridians are all projected into ellipses (1213), whose semi-minor axes Oa, Ob, Oc, Od are the cosines of the distances of the meridians from $PEpQ$; and as the cosines of the meridians near $PEpQ$ vary but very slowly, those meridians will be crowded together, and the figures of the countries very much contracted in longitude. The parallels of latitude being perpendicular to the plane of projection, will (1215) be projected into right lines in their proper proportion. This is called a *Meridional* projection.

FIG. 255. 1233. If the projection be made upon the plane of the equator $EQUA$, as in FIG. 255, the meridians will be projected into right lines (1215), and the parallels of latitude into circles concentric with $EQUA$ (1213). But these circles diminishing slowly near the equator, will be there crowded together, and the figures of the countries will there be very much contracted in latitude. The contraction of the extreme parts of the map therefore, with the objections before stated, render this projection very unfit for the construction of maps. This is called an *Equatorial* projection.

The Stereographic *Projection of Maps.*

FIG. 256. 1234. Let $PEpQ$ be a meridian upon which the projection is to be made, P, p, the poles, EQ the equator, and E the point from which the longitude is reckoned. Now the meridians are projected into circles, the radii of which are the secants of their distances from PEp (1221). Suppose therefore it were required to describe the meridian whose longitude is 60°. Now considering the radius PO as unity, the secant of 60° is 2; with an extent of compass therefore equal to 2, set one foot in P, and extend the other to EQ (produced in this case), and with that point as a center describe the circle Pdp, and it will be the true projection of that meridian. And thus all the required meridians may be drawn.

1235. To describe the circles of latitude, their radii will be the cotangents

of the latitude (1224). Let it therefore be required to describe the parallel of 60°. Take $Ea = 60°$, and with an extent of compass $= 0,577$ the cotangent of 60°, set one foot in a and extend the other to Pp (produced), and with that point as a center describe the circle acb, and it will be the projection of that parallel. In like manner may the other parallels be drawn, as many as may be required.

1236. Having laid down the meridians and parallels of latitude for this half of the earth, draw another equal circle $BQCU$ touching the former at Q, and upon that describe the meridians and parallels of latitude in like manner; and the projection of the earth will be divided into latitude and longitude; then draw the countries, laying down the places according to their respective latitudes and longitudes, and the map will be completed. This is called a *Meridional* projection; and is the method commonly made use of in the construction of maps.

1237. If the eye be at the pole P, the projection is made upon the equator $EQUA$; and here all the meridians are projected into straight lines (1222), the planes of the meridians being all perpendicular to the plane of projection; and the parallels of latitude are projected into circles (1219), the radii of which are the tangents of half the distances of the parallels from the pole. This is called an *Equatorial* projection. The rumb line ab in this projection is manifestly the logarithmic spiral; as it is also in the equatorial orthographic projection. Here the parallels of latitude are crowded together towards the poles. FIG. 257.

1238. To make a projection upon the horizon of any place, the eye is supposed to be opposite to that place. In this case, the pole is projected within the circle of projection, and its distance OP from the center O is equal to the tangent of half the complement of latitude (1217). The meridians are drawn by Art. 1221. This projection is called *Horizontal*, and is represented in FIG. 258. FIG. 258.

1239. There is another meridional projection called *Globular*, which differs from the above, only in its placing the meridians in the projection at equal distances from each other, which comes nearer to the nature of the globe; and maps are frequently thus constructed.

On MERCATOR'S *Projection of Maps.*

1240. This projection is of great use in navigation, on account of its being constructed by right lines only, and no others are necessary in its use. The most convenient way for a ship in going from one place to another is always to sail upon one point of the compass, or upon the same *rumb* (1231); and by

means of this projection you can determine immediately what rumb you are to sail upon. For let FIG. 259. represent a partial map of this construction; AB representing 4° of longitude, and AC perpendicular to it, 4° of latitude; the former degrees are equal, and the latter are increasing (1227), and are to be laid down by a Table of meridional parts (1228). Suppose a ship wants to go from a in longitude 7° and latitude 31°, to b in longitude 10° and latitude 33°; and it is required to find the rumb it must sail upon. Join ab and that is the rumb (1229). Now to determine what rumb this is, there is always in these maps, one or more points from which are drawn thirty-two straight lines representing the thirty-two points of the compass, and you may easily discover to which of these lines, or nearest to which, ab is parallel, and thus you get the point of the compass you are to sail upon. For this purpose, a parallel ruler may be very useful, laying one edge to coincide with ab, and bringing the other edge over the point from which the lines of the compass are drawn.

CHAP. XL.

ON THE USE OF INTERPOLATIONS IN ASTRONOMY.

Art. 1241. IF any quantity vary, and its value at certain intervals of time be known, it is the business of interpolation to find its value, accurately, or nearly so, at any other point of time. In Astronomy, the quantities between which we want to interpolate are of such a nature, that if they be taken at equal intervals of time, and you take their differences, and the differences of those differences, and so on, the last differences will become accurately, or nearly equal to nothing. Hence, if x represent the interval from the first time at which the value of the quantity was taken, and y be the value of the quantity corresponding, then may y be represented by $A + B \times x + C \times x \times \overline{x-1} + D \times x \times \overline{x-1} \times \overline{x-2} + \&c.$ for if we take r terms, and for x write any equidistant set of numbers, as 0, 1, 2, 3, 4, &c. the r^{th} differences of the results, or of the values of y, will become equal to nothing.

For take two terms, $A + Bx$; and for x write 0, 1, 2, 3, 4, &c. and we have these results,

$$\begin{array}{l} A \\ A+B \\ A+2B \\ A+3B \end{array}$$

The first differences of which are B, and the second differences $= 0$.

Take three terms, $A + Bx + C \times x \times \overline{x-1}$; and for x write 0, 1, 2, 3, 4, &c. and we have these results,

$$\begin{array}{l} A \\ A+B \\ A+2B+2C \\ A+3B+6C \\ A+4B+12C \end{array}$$

The first differences of which are,

$$\begin{array}{l} B \\ B+2C \\ B+4C \\ B+6C \end{array}$$

The second differences are $2C$, and the third differences $=0$.

Thus it appears, that the last differences will always become $=0$.

1242. In general therefore, let the successive values of y be a, b, c, d, &c. and let it be required to find the coefficients A, B, C, &c. First, take the successive differences thus,

$$\begin{array}{ccccccccc} a, & & b, & & c, & & d, & & \&c. \\ & b-a, & & c-b, & & d-c, & & \&c. & \\ & & c-2b+a, & & d-2c+b, & & \&c. & & \\ & & & d-3c+3b-a, & & \&c. & & & \\ & & & & \&c. \quad \&c. & & & & \end{array}$$

nd put $P=b-a$, $Q=c-2b+a$, $R=d-3c+3b-a$, &c. Now for x write 0, 1, 2, 3, 4, &c. and the successive corresponding values of y being a, b, c, d, &c. we have the following equations, $a=A$, $b=A+B$, $C=A+2B+2C$, $d=A+3B+6C+6D$, &c. Hence, $A=a$; $B=b-a=P$; $C=\frac{c-A-2B}{2}$ $=\frac{c-2b+a}{2}=\frac{Q}{2}$; $D=\frac{d-A-3B-6C}{6}=\frac{d-3c+3b-a}{2\,.\,3}=\frac{R}{2\,.\,3}$; &c. therefore y $=a+Px+\frac{Q\times x\times\overline{x-1}}{2}+\frac{R\times x\times\overline{x-1}\times\overline{x-2}}{2\,.\,3}+$ &c. where the law of continuation is manifest. Hence, if a, b, c, d, &c. be the values of a variable quantity taken at *any* successive equal intervals of time, beginning at *any* instant, and if such be their law that their last differences always become $=0$, we shall get at any intermediate time the accurate value of that quantity, because then all its intermediate values follow the same law as the values of y from the equation; but if the differences do not at last become accurately $=0$, we shall then get only an approximate value, because then the intermediate values do not follow accurately the same law, whereas the values of y found from our equation must always follow the same law, and therefore the value of y will be only an approximation to the value of the quantity at any intermediate time between those at which y was assumed accurately equal to it; the approximation however will be sufficiently accurate for all practical purposes, provided the differences become at last very small, which is the case in the application of this rule to interpolations in Astronomy. The use of interpolations is therefore to determine the place of a body, or the value of a quantity at any time, from knowing the place or value at three or four times near to the given time.

1243. But besides the use of the above equation to find the value of any term of a series from its position being given, the converse is often required, that is, having given any term to find its position or distance from the first term. In this case, we have the value of y given to find x, which will be determined from the solution of the equation, which will rise in its dimensions as it may be necessary to increase the number of terms, and this depends upon how many orders of differences you must take before they become equal to nothing.

EXAMPLE I. On March, 1783, the sun's declination at noon at Greenwich by the Nautical Ephemeris, was as follows: on the 19th N. $28'. 41''=1721''=a$; on the 20th N. $5'=300''=b$; on the 21st $S.-18'. 41''=-1121''=c$; to find the time of the equinox.

The value of c is here written negative, because the declination has passed through 0. Hence, we proceed thus,

$$\begin{array}{ccccc} 1721, & & 300, & & -1121, \\ & -1421, & & -1421. & \end{array}$$

Here, $a=1721$, $P=-1421$; hence, $y=1721-1421\times x$; now when the sun comes to the equator, y, the declination, becomes $=0$, therefore $1721-1421\times x=0$, and $x=\frac{1721}{1421}=1d.\ 5h.\ 3'.\ 53''$ the time from the 19th; hence, $20d.\ 5h.\ 3'.\ 53''$ is the time required.

If at any place we observe the sun's declination for three or four days at the equinox by the astronomical quadrant, we may thus determine the time at that place when the sun comes to the equator, without the Ephemeris.

EXAMPLE II. To find the time, from the Nautical Almanac, when the sun entered the tropic in June, 1783.

The sun enters the tropic when its longitude is three signs. Here y represents the longitude; let us therefore take three longitudes the nearest to the time, which in this case will be a sufficient number. Now on the 20th day the longitude is $2^s.\ 28^\circ.\ 55'.\ 7''$, on the 21st it is $2^s.\ 29^\circ.\ 52'.\ 21''$ and on the 22d it is $3^s.\ 0^\circ.\ 49'.\ 34''$; but it will render the operation a little more simple, that is, the numbers will be smaller, if we take from each two signs, in which case it is manifest that y begins at the beginning of the third sign, and consequently when y becomes equal to one sign the sun enters the tropic. Therefore 28°.

55'. 7" = 104107" = a, 29°. 52'. 21" = 107541" = b, 1s. 0°. 49'. 34" = 110974" = c; hence,

$$\begin{array}{ccccc} 104107, & & 107541, & & 110974, \\ & 3434, & & 3433, & \\ & & 1. & & \end{array}$$

Here $P = 3434$, and $Q = 1$, which being so very small compared with P, we may omit it; consequently $y = 104107 + 3434x$; but at the tropic, y = 1 sign = 108000"; hence, $108000 = 104107 + 3434x$, therefore $x = \frac{3893}{3434} = 1d.\ 3h.\ 12'.$ 28" the time from the 20th day, and therefore the sun enters the tropic the 21d. 3h. 12'. 28".

Example III. Given five places of a *Comet* as follows; on November 5, at 8h. 17' in Cancer 2°. 30' = 150' = a; on the 6th at 8h. 17' in 4°. 7' = 247' = b; on the 7th at 8h. 17' in 6°. 20' = 380' = c; on the 8th at 8h. 17' in 9°. 10' = 550' = d; on the 9th at 8h. 17' in 12°. 40' = 760' = e; to find its place on the 7th at 14h. 17'.

First, subtract 5d. 8h. 17' from 7d. 14h. 17' and there remains 2d. 6h. = 2,25 for the interval of time between the first observation and the given time at which the place is required; this therefore is the value of x to which we want to find the corresponding value of y; hence,

$$\begin{array}{ccccccccc} 150, & & 247, & & 380, & & 550, & & 760, \\ & 97, & & 133, & & 170, & & 210, & \\ & & 36, & & 37, & & 40, & & \\ & & & 1, & & 3, & & & \\ & & & & 2. & & & & \end{array}$$

Here $P = 97$, $Q = 36$, $R = 1$, $S = 2$; hence, $y = 150 + 97 \times 2,25 + \frac{36}{2} \times 2,25 \times 1,25 + \frac{1}{2 \cdot 3} \times 2,25 \times 1,25 \times ,25 + \frac{2}{2 \cdot 3 \cdot 4} \times 2,25 \times 1,25 \times ,25 \times -,75 = 418',96 = 6°,$ 58'. 57" the place required.

Example IV. In October, 1788, the *Moon's* declination at noon at Greenwich appears, from the Nautical Almanac, to have been as follows; on the 9th S. 12°. 42' = 762' = a; on the 10th S. 8°. 44' = 524' = b; on the 11th S. 4°. 24' = 264' = c; on the 12th N. − 0°. 10' = d; on the 13th N. − 4°. 49' = − 289' = e; to find the declination on the 10th at 6h. 30'.

Here $x = 1d.\ 6h. = 1,25$ the interval between the first given time and that at which the place is required; hence,

$$\begin{array}{ccccccc} 7470, & & 25627, & & 44370, & & 63704, \\ & 18157, & & 18743, & & 19334, & \\ & & 586, & & 591, & & \\ & & & 5. & & & \end{array}$$

Therefore $y = 7470 + 18157 \times 1,25 + \frac{586}{2} \times 1,25 \times ,25 = 8^\circ.\ 24'.\ 18''$ the place required, omitting the consideration of the last difference, as it would not affect the conclusion $\frac{1}{5}$ of a second.

1244. Because $\dot{y}$ represents the velocity with which y increases or decreases, therefore to determine that velocity, take the fluxion of both sides of the equation and you will get the relation between $\dot{y}$ and $\dot{x}$. Hence, if we substitute for $\dot{x}$ any interval of time, we shall get the quantity by which y would be increased in that time with the velocity continued uniform.

Example. Suppose it were required to find, from the last Example, the velocity of *Mercury* on the 15th at 6*h*.

Here $y = 7470 + 18157 \times x + 293 \times x \times \overline{x - 1} = 7470 + 17864 \times x + 293 \times x^2$; hence, $\dot{y} = 17864 \times \dot{x} + 586 \times x\dot{x}$; now let us suppose $\dot{x} = 1$, which answers to 24 hours, and then $\dot{y} = 17864 + 732 = 18596 = 5^\circ.\ 9'.\ 56''$ the angle that would have been described by Mercury in 24 hours with the velocity at the given time. Thus we must proceed in all other cases to find the velocity with which y increases or decreases.

1245. When y becomes a maximum or a minimum, its variation is then infinitely less than that of x, for its fluxion is then equal to nothing. Now as our equation only gives an approximate relation between the quantities required, any small variation of the law of the quantity to be interpolated from the law given by the equation, will, in this case, produce a considerable variation in the value of x, or of the time. When therefore the quantity to be interpolated becomes a maximum or a minimum, or near to it, we cannot depend upon our equation for giving the time with sufficient accuracy. For example, if we take three declinations of the sun near the solstice, and find the value of y and make its fluxion equal to nothing, we cannot be certain that we shall get the time of the solstice sufficiently accurate. Hence, the rule given by Dr. Halley for determining the time of the solstice, from describing a parabola

circumstances in respect to their differences, the reader can never be at a loss to know when they are applicable.

1247. The series $y=A+Bx+C\times x\times \overline{x-1}+D\times x\times \overline{x-1}\times \overline{x-2}+\&c.$ is of the same kind as this, $y=P+Qx+Rx^2+Sx^3+\&c.$ for by actually multiplying the factors, and collecting the coefficients of the like powers of x, and putting $P=A$ the absolute term, Q, R, S, &c. = the sum of all the coefficients of x, x^2, x^3, &c. the former series becomes $y=P+Qx+Rx^2+Sx^3+\&c.$ This we may consider as an equation to a parabolic curve whose abscissa is PW
FIG. $=x$, and ordinate $WH=y$. The interpolation therefore of the terms of this
241. series is the same as the interpolation of an ordinate of this curve, having given any number of ordinates. If two ordinates be given, we assume $y=P+Qx$, the equation being a straight line passing through the extremities; if three ordinates be given, we assume $y=P+Qx+Rx^2$; and if there be n ordinates given, we assume $y=P+Qx+Rx^2+\&c.$ to n terms; because we then have n given values of y and n corresponding given values of x; by substituting therefore in the equation successively the corresponding values of y and x, we get n equations and n unknown quantities P, Q, R, &c. from whence these quantities may be found. Thus we may describe a parabolic curve passing through n given points, that is, we can find the equation of the curve which shall pass through those points.

1248. To find the area of this curve, we have $y\dot{x} = P\dot{x} + Qx\dot{x} + Rx^2\dot{x} + Sx^3\dot{x} + \&c.$ whose fluent is $Px + \frac{1}{2}Qx^2 + \frac{1}{3}Rx^3 + \frac{1}{4}Sx^4 + \&c. =$ the area.

Let the intervals of the ordinates AP, BQ, CR, DS, &c. be unity, that is, let $x=0, 1, 2, 3, 4$, &c. and the corresponding ordinates be a, b, c, d, e, &c. Then from the equation $y=P+Qx+Rx^2+Sx^3+\&c.$ if we take $x=1$, there are two ordinates AP, BQ, and we take two terms; if we take $x=2$, there are three ordinates AP, BQ, CR, and we take three terms; &c. Hence, we have the following cases.

CASE I. For *two* ordinates.

Here $x=1$, $a=P$, $b=P+Q$, therefore $Q=b-a$; and the area $APQB=\frac{1}{2}a+\frac{1}{2}b$.

CASE II. For *three* ordinates.

$$\text{Here } x=2, \text{ and } \begin{aligned} a&=P\\ b&=P+Q+R\\ c&=P+2Q+4R \end{aligned}$$

$$\begin{array}{ll} \text{therefore} & b-a=Q+R \\ & c-b=Q+3R \\ \hline \text{hence,} & c-2b+a=2R \\ \text{and} & \frac{1}{2}c-b+\frac{1}{2}a=R \\ \hline \text{consequently} & Q=b-a-R=-\frac{3}{2}a+2b-\frac{1}{2}c \\ \hline \end{array}$$

Hence, the area $APRC=2a+\frac{1}{2}\times\overline{-\frac{3}{2}a+2b-\frac{1}{2}c}\times 4+\frac{1}{3}\times\overline{\frac{1}{2}c-b+\frac{1}{2}a}\times 8=\frac{1}{3}a+\frac{4}{3}b+\frac{1}{3}c$. This is the case in Art. 1142.

Case III. For *four* ordinates.

$$\begin{array}{lll} \text{Here } x=3, & \text{and} & a=P \\ & & b=P+Q+R+S \\ & & c=P+2Q+4R+8S \\ & & d=P+3Q+9R+27S \\ \hline \end{array}$$

$$\begin{array}{ll} \text{therefore,} & b-a=Q+R+S \\ & c-b=Q+3R+5S \\ & d-c=Q+5R+19S \\ \hline \text{hence,} & c-2b+a=2R+6S \\ & d-2c+b=2R+12S \\ \hline \text{therefore,} & d-3c+3b-a=6S \\ \text{and} & S=-\frac{1}{6}a+\frac{1}{2}b-\frac{1}{2}c+\frac{1}{6}d \\ \text{hence,} & R=\frac{1}{2}c-b+\frac{1}{2}a-3S=a-\frac{5}{2}b+2c-\frac{1}{2}d \\ & Q=b-a-R-S=-\frac{11}{6}a+3b-\frac{3}{2}c+\frac{1}{3}d \\ \hline \end{array}$$

Therefore the area $APSD=3a+\frac{1}{2}\times\overline{-\frac{11}{6}a+3b-\frac{3}{2}c+\frac{1}{3}d}\times 9+\frac{1}{3}\times\overline{a-\frac{5}{2}b+2c-\frac{1}{2}d}\times 27+\frac{1}{4}\times\overline{-\frac{1}{6}a+\frac{1}{2}b-\frac{1}{2}c+\frac{1}{6}d}\times 81=\frac{3}{8}a+\frac{9}{8}b+\frac{9}{8}c+\frac{3}{8}d.$

Thus we may proceed for any number of ordinates.

: the time of this prince, that the Greeks had
·:s of Egypt. It is therefore no wonder that
impose upon them fictitious accounts of their
:he creation according to the account of MOSES.
ud *Year* very well promoted their views; which,
sually signified the apparent annual revolution of the
meant a *revolution* in general, and was used for that of
RCH says, "the Egyptian year was a month;" he adds
ds it consisted of four months." And CENSORINUS says,
quidem, antiquissimum ferunt annum bimestrem fuisse; post
. Rege quadrimestrem factum, novissimè ARMINON ad tredecim
es quinque perduxisse." At what distinct times these several
the length of the year were made, would be an enquiry not to our
rpose; the Egyptians however, to impose upon the Greeks, would
dly take the shortest time for the year, that of a month. And this
so render the heathen chronology subject to great uncertainty.

31. The Egyptians appear to have studied Astronomy very early; but the sure of their year being for a considerable time subject to very different ngths, caused great confusion in their chronology. The Thebans, who pass-d into Egypt, are supposed to have been the first who cultivated Astronomy there. They established a year of 360 days; but it was soon found necessary to add five days more. They seem to have founded their observations upon the heliacal rising of *Sirius;* for observing the interval between two days in which Sirius thus rose, they determined the length of the year. But it was afterwards discovered that by making the year to consist of 365 days, Sirius rose later every year by six hours. They then made the year to consist of 365¼ days. The year however, so far as regarded their religious ceremonies, still consisted of 365 days. Making therefore these two years to begin together, they would not coincide again till after four times 365 years, or 1460 years. This was called the *Sothiacal* period. Mr. E. BARNARD says, that the Egyptians discovered that the stars had an annual motion of 50″. 9‴. 45⁗ in a year (*Phil. Trans.* N°. 158). According to MACROBIUS, the Egyptians made the planets revolve about the sun in the same order as we do; but it does not appear at what time the planets were discovered. He also says, that they divided the zodiac in the same manner as the Chaldeans did; and fixed the commencement at the first degree of Aries. DIODORUS SICULUS says, that the Egyptians discovered that the planets had sometimes a direct and sometimes a retrograde motion, and that they were sometimes stationary. He also asserts, that they made the sun move in a circle inclined to the equator, and in a direction contrary to the diurnal motion. The idea of dedicating the seven days of the week to the planets is also ascribed to them. DIOGENES LAERTIUS, from MANETHO, says, that

the Egyptians believed the earth to be spherical, and that the moon was eclipsed by falling into the earth's shadow. They attempted to measure the diameter of the sun, by observing the motion of the shadow of the gnomon in the time the body of the sun was ascending above the horizon. The discovery of the planets and their motions we may consider as a proof of the early arrangement of the stars into constellations; as it must be by comparing the places of the planets with the fixed stars, that their motions could be discovered.

1252. When ALEXANDER took Babylon, CALLISTHENES found that the most ancient observations made by the Chaldeans were not above 1903 years before that time, which carries them to about the time of the dispersion of mankind by the confusion of tongues. These observations are supposed to have been made in the temple of JUPITER BELUS at Babylon. M. GOUET however thinks this account is not to be depended upon, as it was first published by SIMPLICIUS in the sixth century, who took it from PORPHYRY; and HIPPARCHUS and PTOLEMY, who lived long before, knew nothing of them, though they made a very diligent search after the writings of the ancient Astronomers. They met with no observations made at Babylon before the time of NABONASSAR, who began to reign in the year 747 A. C. EPIGENES speaks of Babylonian observations for the space of 720 years. BEROSUS allows them to have been made 480 years before his time, which carries them back to 746 A. C. and this is in some measure confirmed by the oldest eclipses which are recorded by PTOLEMY, one of which is mentioned to have happened 721 years A. C. and two, 720 A. C. About this time, the Babylonians sent to HEZEKIAH to enquire about the shadow's going back on the dial of AHAZ. The period of 223 lunar months, comprehending 6585⅓ days according to the Chaldeans, make the periodic time of the moon to be 27*d.* 7*h.* 43′. 13″, and its synodic time to be 29*d.* 12*h.* 44′. 7″. They moreover determined its motion not to be uniform, but to be something more than 11° in a day and less than 12° when it moves slowest, and something more than 15° and less than 16° when it moves quickest; and the mean daily motion they fixed at 13°. 10′. 35″. It does not appear by what methods they determined these matters; but it is probable that it was from observing the path of the moon by the fixed stars; for there are stars called by the Arabs, *Minázil Al-Kamar*, or *Mansions of the Moon*, by which is meant, such stars as the moon approaches at night in the course of its revolution. They were twenty-eight in number, the latitudes and longitudes of which are given by ULUGH BEIGH. It is also asserted by PTOLEMY, that they were acquainted with the motion of the nodes and apogee of the moon; and they supposed the former made a revolution in 18*y.* 15*d.* 8*h.* which period, containing 223 complete lunations, is usually called the *Chaldean Saros.* The 223 lunations they reckoned to contain 6585*d.* 8*h.* And if we take the period of the moon as now determined, they would make 6585*d.* 7*h.* 43′, which shows how very nearly

the Chaldeans had determined the synodic revolution of the moon. This period completes the revolution in respect to the node and apogee. The knowledge of this might probably enable them to foretel eclipses, at least those of the moon. ARISTOTLE informs us, that the Chaldeans made observations on the occultations of the fixed stars and planets by the moon; from which they were led to conclude, that an eclipse of the sun was caused by the moon. And DIODORUS SICULUS speaks of their having observed comets, which they held to be lasting bodies, having revolutions like the planets, but in more extensive orbits. The same author also says, that "the southern parts of Arabia are made up of sandy plains of a prodigious extent; the travellers through which direct their course by the *Bears*, in the same manner as is done at sea." It appears therefore that the inhabitants were acquainted with some of the constellations. The Phœnicians were probably the first people who sailed by the stars. He further observes, that the Chaldeans made the annual motion of the sun oblique to the ecliptic, and contrary to the daily motion. Dialling was also first known amongst them, and long before any thing upon that subject is related by the Greeks. HERODOTUS says, that the Greeks borrowed the use of the *Pole* and *Gnomon*, and the method of dividing the day into twelve parts, from the Babylonians. The gnomon seems to have been the most ancient astronomical instrument. The Chaldeans made thirty-six constellations; twelve in the zodiac, and twenty-four without. They also made an observation on *Saturn* in the year 228 A. C. which is preserved by PTOLEMY, and it appears to be the only one which they made on the planets. The distance was measured by digits, of which 24 made a degree. They observed that the sun, moon and planets passed through the twelve signs; the sun in a year, the moon in a month, and that the planets had their particular periods. They placed the moon below all the stars and planets; made it the least of all; placed it nearest to the earth, and made the time of its revolution the least, according to DIODORUS SICULUS. Their civil year consisted of $365\frac{1}{4}$ days; and ALBATEGNIUS relates, that the Chaldeans made the sidereal year 365*d.* 6*h.* 11′. They formed some idea of the magnitude of the earth; for they discovered, it is said, that a man, walking a good rate, might follow the sun round the earth, that is, he might make the tour of the earth, in a year. Now if we allow a good rate to mean three miles in an hour, then there being 8760 hours in a year of 365 days, the circumference of the earth would be 26280 miles, which does not differ a great deal from the truth. Respecting the Chaldean Astronomers, history gives us but a very little information. BEROSUS is supposed to be the oldest, from a very absurd opinion of his respecting the phases of the moon and its eclipses. According to him, the moon is a globe having one side luminous, and the other side a sky blue. At what time he lived is uncertain.

On the Astronomy of the Chinese *and* Indians.

1253. M. Bailly states, that the first king of the Indies lived about 3553 years before our æra; this is a little more than 400 years before their astronomical epoch, which is supposed by M. le Gentil to be 3101 A.C. Their zodiac had two different divisions, one of twenty-eight, and another of twelve; and they divided their zodiac into twenty-seven constellations; they also had a moveable zodiac, to explain the precession of the equinoxes, the motion of which they stated at 54″ in a year, and the entire revolution in 24000 years. This discovery they appear to have made about the year 2250 A.C. They regulated their chronology by periods of 60 years; but in their astronomical calculations, they employed the period of 3600 years, which is six times the lunisolar period. The Brahmins were acquainted with the obliquity of the ecliptic; and they constructed Tables showing the increase of the days arising from the change of the sun's declination, for different latitudes. M. le Gentil found that according to these Tables, the obliquity of the ecliptic must have been more than 25°. They have a Table of the time which the sun employs to move through each sign of the ecliptic. The sign in which it moves slowest they make Gemini, and that in which it moves quickest is Sagittarius. The apogee of the sun was therefore less advanced by a sign when these Tables were constructed than it is now; this carries their construction back to the year 78 of our æra; at which time died Salivagena, one of their kings, who was a great encourager of Astronomy. They applied also to the sun a correction which answers to our equation of the center, being subtractive in the first six signs of anomaly, and additive in the last six; the greatest subtractive is 25′, and answers to 20° of Gemini; and the greatest additive is 11′, and answers to 20° of Sagittarius; this seems to have been the joint effect of two different corrections. The Brahmins also made use of the gnomon; and got a meridian by describing concentric circles, in the manner we do. By this they also found the latitude of the place from the length of its shadow at the day of the equinox. In the reign of d'Hoang-ti, 2697 years A. C. the Chinese had invented and constructed a sphere, with the various circles belonging to it. In a book written in the reign of Yao, about 2332 years A.C. we collect the following circumstances: 1st. Hi and Ho (two Astronomers, who were charged with composing a calendar for the people to regulate their husbandry) observed the places of the sun, moon and stars, and instructed the people with respect to the seasons. 2dly. The equality of days and nights, and the star *Niao*, determined the spring equinox. 3dly. The quality of days and nights, and the star *Hiu*, marked the autumnal equinox. 4thly. On the longest day, the star *Ho* marked the summer solstice. 5thly. On the shortest day, the star *Mao* mark-

ed the winter solstice. 6thly. One year consisted of 366 days, and three years of 365 days. And by speaking of a lunar intercalary, they must also have had a luni-solar year. The Brahmins placed the earth in the center of the world, with the seven planets revolving about it; but they do not appear to have been acquainted with its diurnal motion. The earth they placed upon a mountain of gold; they thought the stars moved, and the planets they called fishes, because they moved in the ether, as fishes do in water. These absurdities show the great antiquity of their Astronomy. M. BAILLY speaks of their Tables as being probably 5 or 6000 years old. M. le GENTIL thinks, that the Indians perceived that their Tables wanted a correction; because at the time when they calculated the mean longitude of the sun and moon, they subtracted from it a constant quantity, which was probably the correction of their epoch, and which they discovered by comparing their calculations with their observations in conjunction and opposition, the points where they wanted them to agree, for the purpose of calculating their eclipses. It is conjectured that this correction was made about the year 78, at which time their Astronomy underwent a great reform. In the year 2952 A. C. reigned FOHI, the first emperor of China; and he appears to have been the first who composed astronomical Tables, and gave the figures of the heavenly bodies. The solstices seem to have been known at that time, as that emperor every year sacrificed an animal at the time of each solstice. His successor added a feast at each equinox. Under the reign of HOANG-TI, 2697 A. C. YU-CHI observed the pole star, and the constellations about it. He also constructed a sphere, with several fixed and moveable circles. And it is said, that he made many experiments upon the weather and the air. About the same time, the circle of 60 years was established. HOANG-TI was the author of many instruments to observe the stars; and also of an instrument to find the cardinal points, without any reference to the heavens; this must have been the compass. Traces of this are found 1400 years after. The same prince instituted a society of mathematicians, and of historians. The emperor CHUENI, in the year 2513 A. C. composed an Ephemeris of the motions of five planets; and it is said, that he was raised to the empire for his knowledge in Astronomy. Under the reign of CHOU-KANG, 2169 years A. C. happened an eclipse, which is the most ancient we have any records of. From the reign of this prince to the year 776 A. C. history makes no mention of any eclipse having been observed. From the year 480 A. C. to 206 A. C. Astronomy was almost entirely neglected; the empire was divided into small states; and the society of mathematicians was destroyed. TSIN-CHI-HOANG united again the states, and formed one great empire; and in the year 246 A. C. he collected the historical and mathematical works and burned them. But the study of Astronomy revived again under LIEON-PANG in the year 206 A. C. About 300 years before our æra, the

Chinese made their year $365\frac{1}{4}$ days. The period of the moon they knew within 20″ or 30″; but they had a very inaccurate knowledge of its revolution in respect to the nodes; and in respect to that of the apogee, they seem to have been altogether unacquainted. At the same time they had a method of calculating eclipses. In the year 104 A. C. SSE-MA-TSIEN and LOHIA-HONG formed precepts for calculating the motions of the planets, and eclipses. At that time they made the obliquity of the ecliptic 23°. 39′. Towards the year 90 J. C. the emperor TCHANG-TI corrected the calendar, having found that the solstice had gone back 5°. About the year 164 J. C. TCHANG-HENG made a catalogue of 2500 stars, which is now lost. In the year 206 J. C. LIEOU-HONG and TSAY-YONG discovered that the moon's motion was not uniform, but subject to an inequality, the maximum of which was 4°. 55′. 41″; and they made the obliquity of its orbit to be about 6°. They knew the solar year to be a little less than $365\frac{1}{4}$ days; and they determined the revolution of the moon in respect to its apogee to be 27*d.* 13*h.* 16′. 50″. In the year 284, KIANG-KI determined the true place of the sun by means of eclipses of the moon. About the year 460, TSOU-CHONG determined the motion of the stars to be 1° in 46 years. LIEOU-HIAO-TSUN and LIEOU-TCHO, about the year 584, employed the first equation of the inequality of the sun. Y-HANG, who lived about the year 720, made the sun's inequality 2°. 21′. 30″. He also determined the time of *Jupiter*'s revolution; and made the latitude of *Sirius* 39°. 25′. 30″. In the year 822, SU-GANG explained the parallax of longitude, and showed its use in calculating solar eclipses. Towards the year 1280, the Chinese erected a gnomon of 40 feet; from the observations of which by CO-CHEOU-KING, they found the obliquity of the ecliptic to be 23°. 33′. 40″; and if this be corrected by refraction it becomes 23°. 34′. 36″. It is said, that this Astronomer was the first amongst them who understood any thing of spherical Trigonometry. He also invented a method of calculating solar eclipses.

1254. For our first knowledge of the modern state of Astronomy in India, we are indebted to M. la LOUBERE, who returning in 1687 from an embassy to Siam, brought with him a Siamese manuscript, containing Tables and rules for calculating the places of the sun and moon; these were put into the hands of CASSINI, to be explained. After that, two other sets of Astronomical Tables were sent to Paris from Hindostan. But for the best information of the state of the Indian Astronomy, we are obliged to M. le GENTIL, who went to India to observe the transit of *Venus* in 1769; and who, during his stay there, acquired a knowledge thereof. They appear to have no theory, but content themselves with calculations, particularly of the eclipses of the sun and moon. A Brahmin of Tirvalore instructed M. le GENTIL in the method of calculating eclipses, and communicated to him the Tables and rules for that purpose,

which he published in the *Mem. de l'Acad. Roy. des Scien.* 1772. They divide the zodiac into 27 constellations; and the ecliptic into twelve signs, of 30° each; and make the annual precession of the equinoxes to be 54". Their year begins at the beginning of their moveable zodiac. The epoch of the Tables of Siam is for the year 638 of our æra, as determined by CASSINI. The length of their sidereal year is 365*d.* 6*h.* 12'. 36", and that of the tropical is 365*d.* 5*h.* 50'. 41". These Tables have a correction of the sun's mean place, answering to our equation of the center, the maximum of which is 2°. 12'; and it is made to vary as the sine of the mean distance from the apogee, which they make 80° from the beginning of the zodiac. The motion of the moon is deduced, by certain intercalations, from a period of 19 years. The moon's apogee is supposed to have been in the beginning of the moveable zodiac 621 days after the epoch of March 21, 638, and to make a revolution in 3232 days. The first of these suppositions agrees with MAYER's Tables to less than 1°, and the second differs from them only 11*h.* 14'. 31". The Siamese rules, which calculate only for conjunctions and oppositions, give but one inequality to the moon, the maximum of which is 4°. 56', and is applied when the moon is 90° from the apogee; in other situations, the equation is as the sine of the moon's distance from the apogee. These Tables go no further.

1255. Another set of Astronomical Tables were sent from Chrisnabouram, a town in the Carnatic, about the year 1750. They are fifteen in number, and contain, besides the mean motions of the sun, moon and planets, the equations of the center of the sun and moon, and two corrections for each of the planets. They are accompanied with rules and examples. The epoch of these Tables answers to March 10, 1491, when the sun was entering the moveable zodiac. The equation of the sun's center is 2°. 10'. 30", and of the moon's 5°. 2'. 47"; and the inclination of the orbit is 4°. 30'; also the motions of the apogee and node are very nearly true. Another set of Tables were also sent about the same time, probably from Narsapour, as that place answers to the length of the day there given; these do not materially differ from the last mentioned.

1256. The Tables and methods of the Brahmins of Tirvalore differ in many respects from those above described; they suppose however the same length of the year, the same mean motions, the same inequalities of the sun and moon, and are adapted nearly to the same meridian. The epoch however goes back to 3102 A. C. The solar year is divided into twelve unequal months, each being the time the sun is moving through a sign; and in their calculations for a day, they employ the time the sun moves 1° in the ecliptic. The sidereal year consists of 365*d.* 6*h.* 12'. 30", and the tropical of 365*d.* 5*h.* 50'. 35". The Brahmins make the obliquity of the ecliptic 24°, which at the time of their

epoch differed very little from the truth. They assign two inequalities to the motions of the planets, answering very well to the annual parallax and the equation of the center. These Tables have their radical places for the year 1491, of our æra. The equation of *Saturn's* center is 7°. 39′. 44″, which agrees very well with what it ought to have been at their epoch 3102 A. C. M. de la PLACE says, "I find by my theory, that at the Indian epoch of 3102 A. C. the apparent and annual mean motion of *Saturn* was 12°. 13′. 14″, and the Indian Tables make it 12°. 13′. 13″. And that the annual and apparent mean motion of *Jupiter* at that epoch was 30°. 20′. 42″, precisely as in the Indian Astronomy." From various agreements of this kind, there is the highest degree of probability that the Indian Astronomy is as ancient as it is stated to be by the Brahmins*. In the *Phil. Trans.* 1777, the reader will find an account of the Brahmins' observatory at Benares, by Sir ROBERT BARKER. See also an account of the chronology of the Hindoos, by W. MARSDEN, Esq. in the *Phil. Trans.* 1790; and Mr. CAVENDISH on the civil year of the Hindoos, in the same work for 1792.

On the Astronomy of the Greeks *to the time of* PTOLEMY.

1257. It is agreed, that the Greeks borrowed their knowledge of Astronomy from the Egyptians and Chaldeans. PLUTARCH relates, that about the time of HESIOD, the sciences began to unfold themselves; but the progress which they made was very slow until the time of THALES, about 600 years before the Christian æra. That philosopher rendered himself famous by foretelling an eclipse of the sun; he however only predicted the year in which it would happen; and this he was probably enabled to do by the *Chaldean Saros*, a period of 223 lunations, after which, the eclipses return again nearly in the same order. He also explained the nature of eclipses; and was the first Greek who went into Egypt for improvement. He is said, by HIERONYMUS in DIOGENES LAERTIUS, to have discovered the year to consist of 365 days; this he might have got from the Egyptians; but LAERTIUS seems to give a reason to the contrary, for he says, that "he first found out the transit from the tropic to the tropic, and was the first who called the last day of the month the thirtieth." He is said by PLINY to have determined the cosmical rising of the *Pleiades* to have been 25 days after the autumnal equinox. He studied the course of the sun, and made its diameter to be the 720th part of the whole heaven, or half a

* For the proofs in support of this, see Professor PLAYFAIR'S Remarks on the Astronomy of the Brahmins, *Edinburgh Trans.* Vol. II.

[illegible] names. This cycle commenced July 16, in the year 432 A.C., [illegible] after the summer solstice, and the new moon which happened on the same day at [illegible] in the evening, was the beginning of it. This is called the Metonic cycle.

1263. Eudoxus, a scholar of Plato, flourished about 360 years A.C. When young he travelled into Egypt, and conversing with the priests, he learned many things from them relating to Astronomy. He made the solar year to consist of 365¼ days, and the synodic revolution of the moon to be 29*d*. 12*h*. 44′. 3″. Archimedes says, that he estimated the diameter of the sun to be nine times greater than that of the moon. Seneca says, that he carried into Greece the elements of the motions of the planets. He wrote a treatise on the constellations, which is lost. Vitruvius says, that Eudoxus made a sun dial upon a plane. He also constructed a sphere. The orbit of the moon he discovered to have been inclined to the ecliptic, and that the greatest latitudes did not always answer to the same place, but that they went backwards; thus he discovered the motion of the moon's nodes. He composed two works, one called the *Mirror*, the other, the *Phenomena*. In the first, he described the constellations; and in the second, he explained the times of their risings and settings. Hipparchus was in possession of these two works, but they are now lost. He advised mankind not to put any faith in the predictions of astrologers.

1264. Aristotle relates several observations which he made. He saw an eclipse of *Mars* by the moon; and an occultation of a star in *Gemini* by *Jupiter*. He observed also a comet, the tail of which extended over one third of the heavens.

1265. Callippus lived about 330 years A.C. He corrected the cycle of Meton, and formed that of 76 years. He also made a collection of observations of the rising of the stars, and joined to them meteorological remarks, for the sake of agriculture.

1266. Pytheas, who lived at Marseilles in the time of Alexander the Great, by means of a gnomon, found the length of the shadow at the summer solstice to its height as 600 to 209; from which it follows, that the obliquity of the ecliptic at that time was 23°. 50′. He says also, that the same proportion of the height of the gnomon to the length of the shadow was true at Byzance; but this could not be; the truth of the above conclusion therefore is thus rendered doubtful.

1267. Aristyllus and Timocharis lived about 300 A. C. and were the first Astronomers of the Alexandrian school. Their observations were principally confined to the stars, in order to fix their positions in the heavens; and it appears that Hipparchus made great use of them; discovering from them that

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LUTARCH however informs us, that, in his old age, he repented
not assigned to the earth its proper place. From this we may
that he was acquainted with, and approved of, the true system
by PHILOLAUS, his cotemporary. He ascribed the brightness of the
Way to the effect of a great number of very small stars. And it is said
ave been this philosopher, who first taught that the morning and evening
ar (Venus) were one and the same planet. He is also mentioned to have
taught the obliquity of the ecliptic, and the position of the tropical circles, by
the sphere. He thought the earth was a globe, and admitted that there might
be inhabitants at the antipodes.

1261. PHILOLAUS, a disciple of PYTHAGORAS, lived about 450 years A. C. He taught the true system, placing the sun in the center, and making the earth and all the planets revolve about it. He was persecuted for propagating this opinion, and obliged to fly for it; and it is remarkable, that GALILEO lost his liberty for maintaining the same. Soon after this, we find HICETAS, a Syracusan, asserting the diurnal motion of the earth.

1262. METON formed a circle of 19 years, containing 19 lunar years, and

that he made a sphere, on which were represented the motions of the sun, moon and five planets, each with their proper relative velocities. He was the first who assigned the true area of a curvilinear space. When Syracuse was taken by MARCELLUS in the year 211 A. C. he gave orders to save ARCHIMEDES; but he was killed by a soldier whilst he was drawing geometrical figures upon the ground, for not answering the questions that were put to him. MARCELLUS however rendered him that honour which was due to his memory.

1271. APOLLONIUS of Perga lived about the same time. He was the first who explained the causes of the stations and retrograde motions of the planets by epicycles (210). He is also celebrated for his Treatise on the Conic Sections; and the method of projections is also attributed to him.

1272. HIPPARCHUS, the father of Astronomy, lived between 160 and 125 years before our æra, and was born at Nice in Bithynia. He was the first person who cultivated every part of Astronomy. He began, by verifying the obliquity of the ecliptic, observed by ERATOSTHENES, and found it very correct; and this was afterwards confirmed by PTOLEMY. He fixed the latitude of Alexandria at 30°. 58′. The length of the tropical year he determined from the interval of the return of the sun to the same tropic, or the same equinox; and he conceived, that if he could get two corresponding observations after a great number of revolutions, the error would be proportionally diminished; and this is the method by which future Astronomers determined the mean motions of all the planets; a discovery of the first importance in Astronomy. He compared the observation of a solstice by ARISTARCHUS with one made by himself, at the interval of 145 years, and found that the solstice happened half a day sooner than it ought to have done, if the year consisted of 365¼ days, as the Greeks believed before him. Thus he determined the tropical year to be 365*d.* 5*h.* 55′. 12″. From his observations of the equinoxes and solstices, he found that they did not divide the year into four equal parts; and he discovered that the interval from the vernal to the autumnal equinox was 186 days, about seven days longer than the interval from the autumnal to the vernal equinox. Thus he found that the motion of the sun was not uniform. But instead of explaining this by an epicycle, which he did at first, he conceived the idea of placing the earth out of the center of the circle in which the sun was supposed to move, which would account for this irregularity. The invention therefore of the excentricity of the orbit is due to this Astronomer. Upon this principle he computed two Tables of the sun's motion; one of its mean motion, and the other of its inequalities, which is the principle of all our astronomical Tables at this time. The discovery of the inequality of the sun's motion led him to another of great importance, the inequality of days. He knew the two causes which produced this inequality; and said, their effects were not sensible in one day, but became so by accumulation. He was mistaken however in the

terity might know whether any changes had taken place in the heavens. This catalogue contains the latitude and longitude of 1022 stars, with their apparent magnitudes. PTOLEMY published these in his Almagest, with their longitudes for that time. HIPPARCHUS divided the heavens into 49 constellations; that is, 12 in the ecliptic, 21 to the north, and 16 to the south. He carried into Geography the plan which he had followed in Astronomy, and laid down the places upon the earth's surface by their latitudes and longitudes. The determination of the circumference of the earth also occupied his attention, and he added 25000 stades to the measure given by ERATOSTHENES. He appears to have collected all the eclipses of the sun and moon he could meet with, observed by the Babylonians. The works of this great Astronomer are found in PTOLEMY's Almagest.

1273. Astronomy made little or no progress from the time of HIPPARCHUS to that of PTOLEMY, who was born in the year 69 of our æra. His great work on Astronomy is entitled Μεγάλη Σύνταξις, or *Great Construction;* and the Arabs gave it the name of *Almagest*, by which it is now usually called. This work is invaluable, both as containing his own discoveries, and as preserving those of HIPPARCHUS. It was known that the moon was subject to a very considerable equation, which was fixed at 5°. 1′ at its maximum. But PTOLEMY discovered that this was true only when the apsides were in quadratures; and that when they were in syzygies it amounted to 7°. 40′; the difference 2°. 39′ he called a second inequality, which differs but a very little from what is observed at this time. He constructed an instrument to find the parallax of the moon, which he made 1°. 7′ at the zenith distance 50°, a quantity much too great. He also attempted to find the parallax of the sun, which he made 2°. 51′. He said, that when the latitude of the moon was greater than the sum of the semi-diameters of the moon and earth's shadow, then there could be no eclipse of the moon. And if the latitude of the moon were less than the sum of the semi-diameters of the sun and moon, there would be an eclipse of the sun. But to tell whether it would happen at any particular place, he found it necessary to apply the moon's parallax. When he therefore discovered that the earth would be somewhere eclipsed, to determine whether it would happen at any particular place, he computed the place of the center of the moon for several successive instants, and applied to them the respective effects of parallax, and thus he got the apparent distance of the centers of the sun and moon; from which he deduced the times of the beginning and end. The element called the *reduction to the ecliptic* he perceived and explained. It appears also, that, before the time of PTOLEMY, they reckoned the digits by the twelfth parts of the surface, and not of the diameter. The system of the planets which he embraced is well known. He explained their motions by means of an epicircle revolving upon an excentric circle; but he was not able to find their

theory of Mercury, and accounting for its various phænomena as seen from the earth.

Books the tenth and eleventh treat, in like manner, of the various phænomena of the planets Venus, Mars, Jupiter, and Saturn; and shows how the Tables have been corrected from the observations of preceding Astronomers.

The twelfth book treats of the stationary and retrograde appearance of the planets; and the thirteenth of their latitudes, the inclination of their orbits, risings, settings, &c.

On the Astronomy of the Arabs, Persians, *and* Tartars.

1274. ALMAMON, an Arabian Astronomer, lived about the year 818. He began his observations upon the obliquity of the ecliptic, which he determined to be 23°. 35′, according to GOLIUS; but in another edition of this author, it is 23°. 33′. This is related in the Elements of Astronomy, by ALFERGEN. About the same time it was determined by CALID, ABULTIBUS, SENED, and ALIS, to be 23°. 33′. 52″. We may therefore fix the obliquity of the ecliptic at that time at 23°. 34′, with the probability of its being near the truth.

1275. THEBITH, who was born about the year 836, determined the length of the sidereal year to be 365*d.* 6*h.* 9′. 11″; and made the equinoctial points to have a motion, sometimes direct, and sometimes retrograde. He found the obliquity of the ecliptic to be 23°. 33′. 30″, and concluded it to be variable.

1276. ALBATEGNIUS, who flourished amongst the Arabs about the middle of the 9th century, was the greatest Astronomer since PTOLEMY. Dr. HALLEY calls him, *Auctor pro suo sæculo admirandi acuminis, ac in administrandis observationibus exercitatissimus.* Finding that PTOLEMY's Tables of the moon and planets were defective, he constructed new and more correct Tables. He made the motion of the equinoxes to be 1° in 66 years, instead of 100 years, as it had been before supposed; and examining the obliquity of the ecliptic, he found it 23°. 35′; but if his observations be corrected for parallax and refraction, it becomes 23°. 35′. 47″. The theory of the sun also engaged his attention; he found the excentricity of the earth's orbit to be 3465, the radius being 100000. The place of the apogee he fixed at 22°. 17′ of Gemini; and PTOLEMY having placed it at 5°. 30′, he discovered that the apogee had an annual progressive motion of 59″. 4‴ in respect to the equinoxes, and (according to him) the motion of the equinoxes being 54″. 32‴, the real motion from hence is 4″. 32‴. He gave two equations to the moon, the same as those which PTOLEMY discovered. He also observed two eclipses of the moon, and two of the sun. These discoveries appear in a work of his, entitled, *De Numeris et Mo-*

tibus Stellarum; which contains also several problems upon the doctrine of the sphere.

1277. ABU-MAHMUD-AL-CHOGANDI, who lived about the year 992, with a sextant of 40 cubits radius, found the obliquity of the ecliptic 23°. 32'. 21"; the limb of this quadrant was divided into seconds.

1278. ALBATURNIUS-ABUL-RIAN, about the year 995, according to Mr. BERNARD, or about 1070 according to ABULFARAGIUS, with a quadrant of 15 cubits radius, made the obliquity of the ecliptic 23°. 35'.

1279. ARSACHEL, who lived about the year 1076, corrected the theory of the sun, by making a great number of observations in various parts of the orbit. He made the obliquity of the ecliptic 23°. 34'.

1280. ALHAZEN wrote upon the twilight, the beginning of which he made when the sun was about 19° below the horizon; and he computed the height of the atmosphere to be 51,8 miles, supposing the circumference of the earth to be 24000 miles. He wrote a Treatise on Optics in seven books, in which he explained the true principles of the refraction of a ray of light through the air, and gave a method of finding the quantity of it. He lived in the eleventh century.

1281. The use of the pendulum was known to the Arabs; but no account is given of the inventor, or at what time he lived. This very important discovery ought to have immortalised its author.

1282. In the year 1072, the Sultan MELICSHAH employed Astronomers to correct the length of the year; and OMAR CHEYAM determined the length of the tropical year to be 365*d.* 5*h.* 48'. 48", the very same quantity at which it is now fixed by M. de la LANDE. He also corrected the calendar.

1283. In the twelfth century, CHIONIADES, a great mathematician of Constantinople, obtained permission to import many books from Trebizond in Persia; from which we find that the Persians had cultivated Astronomy with great success; as their Tables of the motions of the planets, those of Mercury excepted, were very exact. M. de l'ISLE deduced from these Tables the tropical year of the Persians to be 365*d.* 5*h.* 49'. 3". 30'"; the annual motion of the apogee 5". 25'". 12""; and the sidereal year 365*d.* 6*h.* 9'. 55". 30'". The obliquity of the ecliptic in these Tables is 23°. 35'; the equation of the center of the sun 2°. 0'. 30"; and the place of the apogee, in the first year of d'IESDEGIRD, is 2s. 17°. 50'. 7".

1284. In the thirteenth century lived NASSIREDDIN, who constructed Tables from observations made at Maragh. He made the obliquity of the ecliptic 23°. 30'.

1285. AL-NODDAM, who lived soon after NASSIREDDIN, made the obliquity of the ecliptic 23°. 33'; and discovered that the obliquity was decreasing.

1286. In the fifteenth century lived ULUGH BEIGH, a prince of Tartary, and

1296. The next celebrated Astronomer was JOHN MULLER, of Konigsberg, a town in Franconia, better known by the name of REGIOMONTANUS, that word being a Latin translation of the German word *Konigsberg*. Encouraged by the reputation of PURBACH, he went to Venice at the age of fifteen, and became his pupil. Upon the death of PURBACH, he went to Rome, and made there some astronomical observations; but in 1471, he retired to Nuremberg, where he met with BERNARD WALTHER, a zealous friend to Astronomy, who was at the expence of constructing some valuable astronomical instruments, with which REGIOMONTANUS made observations. PURBACH and REGIOMONTANUS discovered the imperfections of the ancient observations, by an observation of Mars compared with two stars near it, whereby this planet was found 2° distant from the place given by the Tables. The instrument which WALTHER made was an armilla, but much more complete than any which had been before constructed. It served to observe in the plane of the ecliptic, the equator, and in the circles which are perpendicular to it. Thus they observed the latitudes and longitudes of the heavenly bodies to a considerable degree of accuracy. PURBACH and REGIOMONTANUS considered the heavens as a great dial; and that the stars would pass in succession over any meridian at the rate of 15° in an hour. This is the principle of the modern method of finding the right ascensions of the stars. REGIOMONTANUS computed an Ephemeris for 30 years forward. In the month of February, 1472, a comet appeared, on which he made observations, and it was the first that had been observed in Europe. Pope SIXTUS IV. wished to reform the calendar, and sent for REGIOMONTANUS to assist in that work; in consequence of which he went to Rome in the year 1475, and died of the plague in the year following. Other accounts however state, that he was put to death by the sons of TRAPEZUNTIUS, in revenge for his having detected errors in the translation of the Almagest by their father. SCHONER does him the honour of asserting, that he was a favourer of the system of the earth's motion.

1297. BERNARD WALTHER was born at Nuremberg in the year 1430. After the death of REGIOMONTANUS, he continued to make observations; and in the year 1484, he made use of clocks in order to measure time. The first observation he made with a clock was, to find how long Mercury rose before the sun. In an eclipse of the moon February 8, 1487, we find he marked the time by the clock. HIPPARCHUS and PTOLEMY found the longitude of the stars by comparing them with the sun, making use of the moon as an intermediate observation; but WALTHER made use of Venus instead of the moon, which was much more exact, because its motion is slower, and also on account of its parallax being so small. He made the longitude of Aldebaran 2°. 35′ of Gemini, in the year 1491. His observations were of eclipses, of the longitudes of

of the fixed stars, that it will be impossible to remove one of these bodies out of its place, without disordering the rest, and even the whole universe also." He also reasoned upon gravity; and defines it to be "a certain natural desire, given by the Supreme Being to all the parts of matter, by means of which they tend to unite under the form of a globe." Having established his system, he made observations, and compared them with the ancient ones, in order to correct the Tables. For this purpose he made himself a quadrant, and parallactic rulers, and other instruments described by PTOLEMY. He made the precession of the equinoxes to be 1° in 72 years; the obliquity of the ecliptic 23°. 28′. 14″; the excentricity of the earth's orbit 323, the radius being 10000; and the place of the earth's apogee 3ˢ. 6°. 40′. In treating of the retrogradation of the equinoxes, he observed that it had not a libratory motion, as THEBITH imagined. He remarked, that the obliquity of the ecliptic decreased, and also the excentricity of the earth's orbit, and thence concluded, that these circumstances depended upon the same cause. He made the length of the tropical year *365d. 5h.* 49′. 24″, which differing from the determinations of PTOLEMY and ALBATEGNIUS, he concluded that it was subject to change. In order to explain the irregularities of the motions of the planets, he retained the epicircles of PTOLEMY. He adopted PTOLEMY's two equations of the moon; and having observed its parallaxes, he found the greatest to be 65′. 48″, and the least 50′. 19″, and the corresponding distances $52\frac{1}{3}$ and $68\frac{1}{3}$ semidiameters of the earth; the mean distance therefore was $60\frac{1}{3}$. He attempted to get the parallax of the sun by the method used by PTOLEMY, and found it to be 3′, and thence the sun's distance 1179 semidiameters of the earth. The diameter of the sun in its apogee he made 31′. 48″, and in its perigee 33′. 54″. When the moon was in its apogee, and in conjunction or opposition, he made its diameter 30′, when in perigee 35′; when the apogee was in quadratures, the diameters he found to be 28′. 45″ and 36′. 44″. His great work on Astronomy is intitled, *Astronomia Instaurata*, and is divided into six books. The *first* contains an account of his system, and his reasons for assuming it; together with some geometrical theorems, and the doctrine of plane and spherical Trigonometry. The *second* contains the doctrine of the sphere. The *third* treats of the equinoxes, solstices, obliquity of the ecliptic, the theory of the earth's motion, and the inequality of solar days. The *fourth* treats of the motion of the moon. The *fifth* and *sixth* are upon the theory of the planets. This work was completed about the year 1530; but it was with the utmost difficulty that his friends, even in the latter part of his life, could persuade him to publish it; at length however their entreaties prevailed, and he delivered it into their hands to be published, and received a copy of it, only a few hours before he died, which happened May 23, 1543, in the seventy-first year of his age.

1301. ERASMUS REINOLD, born at Thuring in the year 1511, published se-

1507. George Joachim Rhæticus was born in Rhetia. He was a pupil of Copernicus; and in order to facilitate astronomical calculations, he began to construct a Table of sines, tangents and secants, for every ten seconds of the quadrant, but he did not live to finish the work. He died at Cassobia, in Hungary, in the year 1576.

1508. William IV. Landgrave of Hesse, distinguished himself greatly by promoting the study of Astronomy. He applied closely to this science, and attached himself to Christopher Rothman, an astronomer, and Justus Burgius, an excellent instrument maker. With this assistance, he erected an observatory on the top of his palace at Cassel, and furnished it with quadrants, sextants, and various other instruments; and with these he made a great number of observations, which Hevelius preferred to those of Tycho. From these observations he determined the latitude and longitude of 400 stars, which he inserted in a catalogue, rectifying their places to the year 1593. He died in the year 1592.

1509. Gerard Mercator, born in Flanders in the year 1512, made globes, and constructed a great many geographical maps.

1510. The next astronomer of any consequence was Tycho Brahe, born of noble parents at Knudstrop in Scania, in the year 1546. When he was only fourteen years old he was struck with astonishment at observing an eclipse of the sun to happen so very near the time it was predicted; and it seems as if this led him to the study of Astronomy. In 1563, he observed the great conjunction of the superior planets; and in tracing the courses of the planets, and comparing them with the Tables of Alphonsus and Copernicus, he saw that the Tables were subject to great errors. In announcing the great conjunction, the Tables of Alphonsus erred a month. November 11, 1572, he discovered a new star in *Cassiopea's Chair;* this star was greater and more brilliant than *Lyra* and *Procyon*, and was seen in the middle of the day; but at length its brightness declined, and it died away gradually, and disappeared in 1574. It was observed by all the Astronomers in Europe. This phænomenon excited Tycho to make a new catalogue of the fixed stars, which contained the places of 777, rectified to the beginning of the year 1600. Instead of the moon which was used to connect the sun and the stars, he made use of *Venus*, as Walther had done before him. Tycho being recommended by the Landgrave of Hesse, to Frederic II. king of Denmark, he gave him the island of Huenna, and supplied him with money to build an observatory, to furnish it with instruments, and to support himself. This Tycho very gladly accepted, and called the name of the building *Uraniburg*. It was furnished with the best instruments, consisting of quadrants, sextants, circles, armillæ, parallactic rulers, rings, astrolabes, globes, clocks, and sun-dials. These instruments were of excellent workmanship, and far more accurate than any which had been before made. Most of

the divisions were diagonal, but he had one quadrant divided according to the method of PETER NONIUS. The whole expence is said to have amounted to 200000 crowns. Here TYCHO made all his observations of the stars, comets and planets even to *Mercury*, which COPERNICUS had never been able to see. He first determined the place of a star by observing its azimuth, and the time of passing over it; but his clocks did not give the time with sufficient accuracy; he therefore determined the place, by observing its distance from two known fixed stars. In the course of the observations, TYCHO made a very important discovery, that of the refraction of the air; and this he found from comparing the height of the equator as determined from observations of the solstices, and of the circumpolar stars, for he found that they constantly differed by 4′; this he imputed to the refraction of the air. He made the horizontal refraction 34′, and at 45° altitude he made it nothing; and calculated a Table showing the refraction at all altitudes up to 45°. He constructed new Tables of the sun; and determined the precession of the equinoxes to be 1° in 71 years; he also found that the latitude of the stars, since the time of TIMOCHARIS and HIPPARCHUS, had varied; thus he discovered that the ecliptic was subject to a variation. The theory of the moon also engaged his attention, and he discovered a third equation, called the *Variation*. He also found that the motion of the nodes was not uniform; and that the inclination of the orbit was variable; the least inclination he made 4°. 58′. 30″, and the greatest 5°. 17′. 30″, which is a great proof of the goodness of his observations. He very happily represented the variation of the motion of the nodes and of the inclination, by the motion of the pole of the lunar orbit in a small circle. These discoveries relating to the moon, do him great honour. He observed a comet in the year 1577, and discovered that it had a parallax of 20′, and thence concluded that it was about three times as far from us as the moon. He conjectured that they revolved about the sun. The system which he invented we have already explained. He made the obliquity of the ecliptic 23°. 31′. 30″; and found the length of the sidereal year 365*d.* 6*h.* 9′. 26″. 45‴, and the tropical 365*d.* 5*h.* 48′. 45″, which is within 2″ or 3″ of the present determination. He found the diameter of the sun in apogee to be 30′, and in perigee 32′, and its mean distance 1150 semidiameters of the earth. Upon the death of FREDERIC II. it was represented to the young king that the treasury was exhausted, and that it was necessary to retrench the pensions; in consequence of this, TYCHO was deprived of his; upon which he removed to Copenhagen with such of his instruments as he could carry; but he there received an order to discontinue his observations. Upon this he went to Holstein, and was introduced to the emperor RODOLPHUS, who settled a pension of 3000 crowns upon him, and gave him a magnificent house; here he renewed his studies, and the famous KEPLER, who called him the HIPPARCHUS of his age, became his scholar and assistant. He died October 24, 1601, in the fifty-fifth year of his age, solacing

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9. Three more Books on the Copernican Astronomy, in 1622.
10. Rodolphine Tables, in 1627.

Besides these, he wrote several things in *Chronology*, the *Geometry of Solids*, *Trigonometry*, *Logarithms*, and *Dioptrics*.

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himself that he had not lived in vain, and that his labours would redound to the glory of GOD.

1311. LONGOMONTANUS was a pupil and assistant of TYCHO at Uraniburg. He assisted him in his catalogue of the stars, and in his theory of the moon. He afterwards went to Copenhagen, and was made professor, and acquired great reputation by his astronomical knowledge. He died in the year 1647.

1312. KEPLER was born at Wiel in Wirtemberg, the 27th of December, in the year 1570. He began to study Astronomy very early, and had a turn for seeking analogies and harmonies in nature; and having, as he thought, discovered a curious one, he published it in 1596, and sent it to TYCHO; who, although he disapproved of the work, saw so much ingenuity in it, that he sent for KEPLER to reside with him at Prague; and from the observations of TYCHO, he made his important discoveries. He made the refraction the same for all bodies at the same height, and did not agree with TYCHO, that there was no refraction above 45°; he also observed that it was different on different parts of the earth. He published a Treatise on Optical Astronomy, in which he treats of parallax, and the calculation of solar eclipses, and applies them to find the longitude on the earth's surface. He speaks of gravitation, and applies it properly to the case of the earth and moon, and to the cause of the tides (220). He directed his attention to the motion of *Mars*, and published a work entitled *De Motibus Stellæ Martis*. He first employed a circular orbit, and determined its excentricity; but by comparing some distances of *Mars* from the sun from observation, with the computed distances, he found so great a disagreement, that he concluded the orbit was not a circle; he then supposed it to be an ellipse, and found the calculated distances agreed with those deduced from observation; hence, he concluded that the planets revolved about the sun in ellipses, having the sun in the focus (217). Having determined the periodic times and mean distances of the planets, he discovered, by trial, the famous law, that the squares of their periodic times are as the cubes of their mean distances. He also found, that in the apsides the areas described by the planets in equal times were equal; and he supposed that the same was true at every other point; and thence he concluded that the planets describe about the sun equal areas in equal times. These three important discoveries are the foundation of all plane and physical Astronomy. He solved the problem now called KEPLER'S *Problem*, of cutting off from an ellipse by a line drawn to the focus, an area equal to a given area. He also announced the passage of *Mercury* over the sun in the year 1631; and the transits of *Venus* in 1631, and 1761. The works which this celebrated Astronomer published are;

1. Cosmographical Mystery, in 1596.
2. Optical Astronomy, in 1604.

3. An Account of a new Star in Sagittarius, in 1605.
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1319. GALILEO was born at Florence, in the year 1564. He discovered the laws by which bodies falling freely are accelerated, and the use of the pendulum for measuring time. Having heard of the invention of the telescope, he, by considering the principles of refraction, constructed one which magnified more than 30 times. With this he immediately discovered the spots in the sun, from which he saw that it had a rotation about its axis; he saw also various new appearances upon the face of the moon, from which he concluded that it was very rough with hills and vallies. He also found that *Venus* put on the same phases as the moon. And making his observations upon *Jupiter*, he soon discovered that it had four moons. Afterwards he thought of making use of them for finding the longitudes of places upon the surface of the earth. He also discovered, as he thought, that *Saturn* was either composed of three bodies, or that it was in the shape of an olive, as he expressed it. Upon directing his telescope to the fixed stars, he was surprised to find that, instead of being magnified, they were diminished, appearing only as points. He was a zealous defender of the Copernican system, for which he was persecuted, and cast into prison. He died in the year 1642. The principal works which this great man published, are

1. The Operations of the Compass, geometrical and military.

2. A Discourse on the Floating of Bodies upon, and their Submersion in Water.

3. Mechanics, or the Benefits derived from that Science.

4. His Balance, for finding the Proportion of Alloy, or mixed Metals.

5. His Nuncius Sideris.

6. A continuation of the last Work; containing his last Observations on Saturn, Mars, Venus, and the Sun.

7. A letter concerning the Libration of the Moon.

8. On the Solar Spots; with an Ephemeris of the Motions of Jupiter's Satellites.

9. Problems in Mathematics.

10. Mathematical Discourses.

11. A Treatise on the Mundane System.

1320. Le P. SCHEINER, the associate of GALILEO, made a great many observations upon the solar spots, which he published in a work entitled *Rosa Ursina*. He was the first who paid attention to the elliptical figure of the sun when near the horizon. He died in 1650.

1321. GODEFROI VENDELINUS, an Astronomer in Holland, published, in 1626, a Dissertation on the obliquity of the ecliptic, and established its variation.

He was the first who reduced the parallax of the sun to 15″, by the method of ARISTARCHUS.

1322. GASSENDUS observed the transit of *Mercury* over the sun on Nov. 7, 1631, and took several measures of its distance from the center of the sun.

1323. RICCIOLUS, a Jesuit, was a man indefatigable in his astronomical pursuits. He published a work entitled, *Almagestum Novum*, containing a collection of all the known observations, the methods, the determinations, the opinions and physical explanations of the phænomena. He published also his *Astronomia reformata*, and *Geographia reformata*, containing very valuable collections. He attempted to measure the earth. His death happened in 1671.

1324. PEYRESC, the protector and friend of GASSENDUS, was born in 1580. He discovered the times of the revolutions of Jupiter's satellites, but announced that they were not very accurate. He also considered their configurations, and the method of finding the longitude from them.

1325. JOHN BAPTIST MORIN, born at Villefranch in 1583, distinguished himself by his attempts to discover the longitude by means of the moon. His method was good in theory, but not practicable; on this account he could not obtain the rewards which had been offered for the discovery.

1326. SETH WARD, bishop of Salisbury, was born April 15, 1617. He was for some time of Sidney College in this University, and afterwards went to Oxford, where he was made Savilian Professor of Astronomy. In investigating the place of a planet, he supposed that the motion of a body in an ellipse was uniform about the other focus (that focus in which the sun is not); it appears however that BULLIALDUS had advanced the same eight years before; and it is said, that ALBERT CURTIUS first suggested it. It is called however WARD's *Hypothesis*. He published a Discourse concerning Comets; an Enquiry into the Principles of BULLIALDUS's philolaic Astronomy; on Trigonometry; and on Geometrical Astronomy. He died in 1689.

1327. ANDREW TACQUET was born at Anvers, and wrote some good elementary things in Astronomy. He died in 1660.

1328. THOMAS STREET, an Englishman, wrote a Treatise entitled *Astronomia Carolina*, which were in use for a long time. They first appeared in 1661; and an edition of them was published by Dr. HALLEY in 1710. He constructed the *Logistic* logarithms.

1329. AZOUT was the inventor of the micrometer with a moveable wire; and he and PICARD applied telescopes to quadrants. He also made many observations, which are recorded by MONNIER in his *Histoire Celeste*. He died in 1691.

1330. Alphonsus Borelli was born at Naples in 1608; and in the year 1666, he published a Theory of the Satellites of Jupiter.

1331. Vincent Wing, born in Rutlandshire in 1619, published a work entitled, *Astronomia Britannica*, containing many useful Tables and observations.

1332. Nicolas Mercator was born at Holstein. He published his Cosmography in 1651; his Astronomical Institutes in 1676; and his Logarithmotechnica in 1678.

1333. Bulliald us was born at London in 1605. He made many astronomical observations, and published a valuable work, entitled, *Astronomia Philolaica*. He attempted to explain the three inequalities of the moon, according to the idea of Horrox.

1334. Michel Langrenus of Anvers, mathematician to Philip IV. king of Spain, distinguished himself by his observations on the spots of the moon; these he made subservient to finding the longitude of places on the earth's surface, by observing, in a lunar eclipse, the times at which the spots entered the shadow.

1335. John Hevelius was born at Dantzic, on January 28, 1611. In 1641, he founded an observatory, and furnished it with the best instruments that could be procured. Some of them were divided into every five seconds by a division similar to that of the vernier. Of these he gave a description in his *Machina Cœlestis*. The observatory, with all the instruments and books which were in it, were destroyed by fire, on September 26, 1679. The damage was estimated at 30000 crowns. The second part of his *Machina Cœlestis* is very scarce, nearly the whole impression having been burnt. He published a great work, entitled *Selenographia*, or a description of the face of the moon and its spots, with very fine engravings. He completed the explanation of the libration of the moon, which was begun by Galileo, adding that of the longitude. Dr. Halley went to see him in 1679, and was charmed with the accuracy of his observations. He published his *Cometographia* in 1668, containing a catalogue of all the comets which had been observed, with many new observations and curious researches respecting their nature. He published a work called *Prodromus Astronomiæ, et novæ Tabulæ solares unà cum integro fixarum Catalogo*, from his own observations; his catalogue contains 1888 stars. He died January 28, 1687.

1336. Huygens was born in 1629. His first work was his *Systema Saturnium*, in which he has explained the various appearances of Saturn's ring; and in the same work he announced the discovery of a satellite of the same planet. In a work, entitled *Horologium oscillatorium*, he explained the method of applying pendulums to clocks, so that the times of all their vibrations should be equal. He was the first who investigated the center of oscillation.

1337. ROBERT HOOK was born in 1635. He invented the zenith sector, in order to discover whether the earth had any sensible annual parallax; and this led to the discovery of the aberration of light in the fixed stars. He also discovered a spot upon *Jupiter* in 1664; and made many other observations. He was the first who formed the idea of making a quadrant to take angles by reflexion.

1338. DAVID GREGORY was professor of Astronomy at Oxford. He published a System of Astronomy, in which he explained some parts of Sir I. NEWTON'S *Principia*. He died in 1708.

1339. WILLIAM WHISTON was Lucasian professor of mathematics at Cambridge. He published his Theory of the Earth in 1696; and in 1707, he published his Lectures on Astronomy.

1340. JOHN DOMINIC CASSINI was born at Perinaldo, on June 8, 1625. In the setting out of his studies, a book of Astronomy fell into his hands, which greatly attracted his attention, and led him to the pursuit of that science; and he was very early appointed professor of Astronomy at Bologna. Here he found a meridian line in the church of St. Petronia; but it not being correct, he obtained leave to rectify it. With this he determined the obliquity of the ecliptic, and the quantity of the refraction of the air. He also discovered with it, the unequal motion of the sun, and constructed new solar Tables. He resolved the problem, to find the elements of the orbit of a planet from three observations. He taught the method of calculating eclipses; and conceived a projection, which served to find the longitude of places upon the earth's surface. He composed a work upon comets, from those which he observed in 1664, and 1665. In 1665, he discovered the rotation of *Jupiter* and *Mars* about their axes, by means of their spots; and afterwards he discovered the rotation of *Venus*. He employed himself upon the theory of Jupiter's satellites, and published Tables of their motions. And from comparing the observations of their eclipses with the times as calculated from these Tables, they were found to agree much better than was expected. At the request of LEWIS XIV. he went and settled at Paris, and the Royal Observatory was entrusted to his care; and in 1671, he began to make regular observations. In the *Elemens d'Astronomie*, by his son, we find that these observations were upon the equinoxes, solstices, oppositions and conjunctions of the planets. In 1672, he determined the parallax of the sun to be $9\frac{1}{2}''$; and in the years 1671, 1672, and 1684, he discovered four of the satellites of Saturn; and further observations showed him that the fifth satellite disappeared regularly for about half a revolution, when it was to the east of Saturn; from this he concluded that it revolved about its axis; and Sir I. NEWTON further concluded, that the time of its rotation was equal to the time of its revolution about Saturn. He observed a belt upon Saturn, and a black mark upon the ring, parallel to its edge, dividing

it into two equal parts. He also gave new Tables of the satellites of Jupiter; and discovered that the duration of the eclipses of these satellites was variable, increasing in length for three years, and then decreasing in length for the same time; this proved to him that their orbits were inclined to the orbit of Jupiter. Upon examining the disc of Jupiter, he discovered that it was not a circle; and by measuring the diameters he made them differ the fifteenth part of the whole. He gave a new theory of the libration of the moon, which he explained by two movements about two different poles. In the year 1683, he discovered the zodiacal light. These great and important discoveries form an epoch in the science of Astronomy. He died September 14, 1712, leaving for his successor his son JOHN JAMES CASSINI.

1341. JOHN PICARD was born at Anjou. He and AZOUT were the first who put telescopes to quadrants. In the year 1669, he gave a measurement of the earth; and in 1670, he established the Royal Observatory, which had been committed to his care. In the *Histoire Celeste*, the reader may see his ideas for improving Astronomy. He published the *Connoissance des Temps* for 1679; and died in the year 1682.

1342. ROMER, or ROEMER, was born in Denmark in the year 1644, and came to France in 1672. He discovered that light was progressive, from the eclipses of Jupiter's satellites. In 1681, he returned to Copenhagen, and died in 1710.

1343. PHILIP De la HIRE was born at Paris in the year 1640; and in 1687, he published his astronomical Tables; he likewise made a great number of astronomical observations.

1344. JOHN FLAMSTEAD, the celebrated English Astronomer, was born at Derby August 19, 1646. In 1669, he calculated some eclipses of the fixed stars by the moon, and sent them to the Royal Society, and received the thanks of that body. In 1673, he wrote a small tract on the diameters of the planets, when at their greatest and least distances from the earth; "which I lent," says he, "to Mr. NEWTON in 1685, who has made use of it in his fourth book of his *Principia Phil. Nat. Mathemat.*" By the time he was 26 years old, he explained the true principles of the equation of time, a thing of the first importance in Astronomy. CHARLES II. having built an observatory at Greenwich, Mr. FLAMSTEAD was appointed Astronomer Royal in the year 1676, at the recommendation of Sir I. MORE. A description of the instruments with which the observatory was furnished is given in the *Prolegomena* to the third volume of the *Historia Cœlestis*, which was published in 1725. A volume of his observations were published in 1712, by Dr. HALLEY, by the order of Queen ANN; this displeased FLAMSTEAD; and he set about preparing his observations for the press, but he did not live to finish the work, and it was printed after his death. The first volume of this great work contains the observations which he

made first at Derby, and afterwards at Greenwich upon the fixed stars, planets, comets, spots of the sun, and the satellites of Jupiter, during 33 years. The second volume contains the passages of the fixed stars and planets over the meridian, with the places of the planets deduced from them. The third volume contains a prolegomena on the history of Astronomy, giving a description of the instruments used by Tycho and himself; a catalogue of the fixed stars of Ptolemy, of Ulugh Beigh, of Tycho, of the Landgrave of Hesse, of Hevelius; of the southern stars which had not been observed above our horizon; and lastly, the British catalogue of 2884 stars, with their right ascensions, north polar distances, latitudes, and longitudes, and the annual variations of the right ascensions and north polar distances. To these are added some astronomical Tables constructed by Abraham Sharp. This great work is an invaluable treasure to Astronomers. His *Atlas Cœlestis* was published in 1753. He gave new solar Tables; and a theory of the moon according to Horrox. He also published a Treatise on the doctrine of the sphere, in which he showed how to construct eclipses of the sun and moon, and occultations of the fixed stars by the moon. Dr. Halley's Tables and Sir I. Newton's theory of the moon were founded on Mr. Flamstead's observations. This great Astronomer died October 31, 1719.

1345. Sir I. Newton, the founder of physical Astronomy, was born December 25, 1642. In the year 1660, he was admitted at Trinity College, Cambridge, and in 1667, was chosen fellow of that society; and in 1669, he was elected Lucasian professor of mathematics, upon the resignation of Dr. Barrow. His great work, entitled *Philosophiæ Naturalis Principia Mathematica*, was first published in 1686; and another enlarged edition was published in 1713, with a preface by Cotes. In this work he unfolds the law of attraction, and shows how it will solve the motions and the principal phænomena of the different bodies in the system. And the same principles have been since further applied, and shown to be competent to account for all the small inequalities of the motions of the heavenly bodies. His philosophy is founded upon experiment and demonstration, and therefore its truth cannot be controverted. His Treatise on Optics alone would have immortalized him. To enumerate his various discoveries, and the extent to which his principles lead, is here unnecessary, as they have already appeared in the course of this work. His inventions also in pure mathematics are well known to have been no less important than those in philosophy; and by a union of these, the progress of science seems to be unbounded. His deep insight into nature, led him but the more to adore its Author. He spent a considerable part of his life in examining the sacred records; and that examination confirmed him in his belief of the relations contained in them. He died March 10, 1727, and was buried in Westminster Abbey.

1346. Roger Cotes was born at Burbage in Leicestershire, July 10, 1682. He was educated at Trinity College, Cambridge; and in the year 1706, was elected Plumian Professor of Astronomy and Experimental Philosophy; being the first who was appointed to that office. In the year 1713, he published a second edition of Sir I. Newton's *Principia*, and inserted all the author's improvements. To this edition he prefixed a preface, in which he explained the true method of philosophizing, showed the foundation on which the Newtonian philosophy was built, and refuted the objections which had been made to it. This extraordinary man died in 1716. After his death, Dr. Smith, Master of Trinity College, the author of a celebrated treatise on optics, and another on Harmonics, published the works which Cotes left behind him, containing his *Lectures on Hydrostatics*, and his *Harmonia Mensurarum*.

1347. James Philip Maraldi was born August 21, 1665. He determined the retrograde motion of the nodes of Jupiter, and the progressive motion of its aphelion. He also corrected the theory of Mars. In 1704 he perceived that the motion of Saturn was diminishing; and in 1714 he gave a full explanation of the phænomena of its ring. From observations on the eclipses of Jupiter's satellites, he concluded that the inclinations of their orbits were subject to a variation. This enquiry was pursued till his death, which happened in 1729. He left a nephew John Dominic Maraldi, who observed a variation in the inclination of the orbit of the third satellite, and an excentricity of the orbit of the fourth.

1348. At this time lived Mr. Pound. He measured the diameters of Jupiter, and found them to be as 12 : 13; and published new Tables of the first satellite, for the computation of its eclipses, making the equations all additive. He rectified the motions of the satellites of Saturn, and made some accurate observations on them. He also made further astronomical observations, as may be seen in the *Phil. Trans.*

1349. About the same time lived Gabriel Philip de la Hire, who examined the motions of Jupiter, and found that the progressive motion of its aphelion, as given in the Rodolphine Tables, was too slow. He also constructed a system of cross wires in the focus of the object glass of a telescope, for the observation of eclipses.

1350. Francis Bianchini, born at Verona on December 13, 1662, published a work on the rotation of *Venus*. He also made a great many observations.

1351. De Louville was born July 14, 1671, and made many astronomical observations at Paris. He was the first who applied the micrometer to the quadrant.

1352. The celebrated Astronomer, Dr. Edmund Halley, was born November 8, 1656, and entered at Oxford at the age of 17 years. Two years after,

he published a direct and geometrical method of finding the aphelia and excentricities of the orbits of the planets. At the age of 20 years he went to St. Helena to make a catalogue of the southern stars, which he published in 1679. During his stay there he observed the transit of *Mercury* over the sun's disc, and that suggested to him the idea of finding the parallax of the sun by the transit of *Venus* over its disc; this important problem he solved, and recommended to future Astronomers to put it in practice. Had he done nothing else in Astronomy, this would have immortalized him. In 1679, he made a visit to HEVELIUS, with whom he stayed and observed for some time; and returning home, he soon after set out to make a tour upon the continent with Mr. R. NELSON, his schoolfellow. In his way from Calais to Paris, he observed the remarkable comet in its ascent from the sun, which he had before observed in its descent. Upon his return, he married and settled at Islington, where he set up his astronomical instruments. In 1683, he published his *Theory of the Variation of the magnetical Compass;* in which he supposes that the earth has four magnetic poles. In 1684, he turned his thoughts to the subject of the relation between the periodic times and distances of the planets, and concluded from it that the centripetal force must vary inversely as the squares of the distances; but not being able to prove it, he applied to Mr. HOOK and Sir CHRISTOPHER WREN; they however not being able to give him satisfaction, he went to Cambridge to Mr. (afterwards Sir ISAAC) NEWTON, who soon gave him a proof of his position. Dr. HALLEY becoming acquainted with Mr. NEWTON, he persuaded him to publish his *Philosophiæ Naturalis Principia Mathematica*, and undertook the care of the publication. In 1685, he published the method of finding altitudes by the barometer; and in the next year came out his account of the trade-winds and monsoons. He also published a map, representing their directions. In the year 1687, he undertook to explain the reasons why the Mediterranean Sea is not observed to swell, notwithstanding there is no visible discharge of the prodigious quantity of water which runs into it from so many large rivers, and the constant setting in of the current from the streights. He constructed equations of three and four dimensions; and gave a rule for approximating to the roots of equations. He next undertook to publish a correct Ephemeris for 1688. In the beginning of 1691, he published Tables of the conjunctions of *Mercury* and *Venus* with the sun. The next year he produced his Tables for showing the value of annuities for life, founded on the bills of mortality; and soon after, he published his universal theorem for the foci of lenses. Wishing to make observations in order to determine the variation of the needle, he applied to king WILLIAM III. who appointed him captain of a vessel, with proper assistants. He crossed the line and proceeded as far as 52° south latitude; and in his way back he touched at St. Helena, the coast of Brazil, Cape Verd, Bar-

badoes, Madeira, the Canaries, the coast of Barbary, &c. And on his return home, he published a chart with curve lines denoting the variation of the compass. Soon after this he went out to observe the course of the tides in the British channel, with the situations of the principal headlands. Upon the death of Dr. WALLIS, he was appointed Savilian Professor of Geometry at Oxford; and, by request, he translated Apollonius from the Arabic into Latin. In 1705, he announced the return of a comet in the year 1759, which happened accordingly within about a month of the time he predicted. He had the glory of being the first who foretold an event of this kind; and it is the only one which has been predicted and the prediction fulfilled. He published a *Synopsis of the Astronomy of Comets.* In 1713, he was appointed secretary to the Royal Society. As perfecting the theory of the moon was his great object, he was now determined to complete it; and in 1715, he finished it, so far as regarded the syzygies; so that his calculation of eclipses answered to a degree of accuracy which had never been before experienced. His reputation was now so great, that upon the death of Mr. FLAMSTEAD in 1719, he was appointed the Astronomer Royal at Greenwich; in which year he published new Tables of the sun, moon and planets. This gave him an opportunity of completing the theory of the moon's motion. He therefore immediately fixed up a transit instrument, and began his observations; and though he was then in the 64th year of his age, yet he attended to observe the moon's transit for 18 years afterwards; in the first nine years of which he made 1500 observations, which he announced to the public, and showed how they tended to correct the theory of the moon. In the year 1725, he procured a mural quadrant with which he also observed. Upon the accession of GEORGE II. to the throne, his consort, Queen CAROLINE, made a visit to the Royal Observatory at Greenwich, and was much pleased with the reception she there met with; and Dr. HALLEY having formerly served as a captain in the navy, she obtained for him his half pay for that commission, which he enjoyed during his life. An offer was made him of being mathematical preceptor to the Duke of Cumberland, but he declined that honour by reason of his age, and also as it would interfere with his duty at the Observatory. In 1729 he was admitted a foreign member of the academy of sciences at Paris, in the room of Signior BIANCHINI. After his death (which happened the 14th of January, 1742), M. MAIRAN read an elogy on him before the academy, in which he speaks of the universality of his genius, as comprehending a knowledge of almost all the sciences, astronomy, geometry, algebra, optics, artillery, speculative and experimental philosophy, natural history, antiquities, philology, and criticism; abounding with ideas new, singular and useful. And concludes with observing, that he had all the qualifications necessary to recommend him to the attention of princes, and the applause of the learned. He was buried at Lee near Greenwich.

1353. JOHN JAMES CASSINI (son of JOHN DOMINIC CASSINI, before mentioned) was born at Paris February 18, 1677, and died April 15, 1756. He published a System of Astronomy, with astronomical Tables; a very valuable work. A great part of this was founded upon the observations of his father. He also published many other things in the different Memoirs. CÆSAR FRANCIS CASSINI de THURY his son died in 1784, after having made a great many useful observations in Astronomy. JOHN DOMINIC CASSINI (son of M. de THURY) is now at the observatory of Paris.

1354. BOUGUER was born at Croisic February 10, 1698. His Treatise on the figure of the earth is a valuable work. He, GODIN, and DE LA CONDAMINE went to South America to measure a degree; and in order to put the doctrine of universal attraction to the test, they found that the Cordilleras actually attracted the plumb line and drew it sensibly from its perpendicular position.

1355. MAUPERTUIS was born at St. Malo September 28, 1698, and is celebrated for his journey to Lapland in order to measure a degree of latitude. His colleagues were CLAIRAUT, CAMUS, LE MONNIER, the Abbé OUTHIER and CELSUS. MAUPERTUIS published also the Elements of Geography, and Nautical Astronomy. He died in 1759.

1356. DE LA CAILLE was born in 1713, and was one of the first Astronomers of his time. He published an Ephemeris; Tables of the sun; a Catalogue of the fixed stars; on Parallax, Refraction, and the Figure of the Earth; on Comets and Eclipses. His observations may be found in the *Memoires de l'Academie*. He went to the Cape to make observations in order to determine, in conjunction with those made in Europe, the parallax of the moon. And from these observations he also determined the quantity of refraction. His Astronomical Lessons were published in 1746. This celebrated Astronomer died in 1762.

1357. PETER HORREBOW made a great many astronomical observations, and published his *Clavis Astronomiæ*, and *Basis Astronomiæ*. He died at Copenhagen in the year 1764.

1358. JOSEPH NICHOLAS DE L'ISLE was born at Paris in 1688. By his researches, calculations and observations, he contributed much to the progress of Astronomy. He died in 1768.

1359. The celebrated English Astronomer JAMES BRADLEY was born in 1692. He immortalized himself by two of the most delicate and important discoveries that ever were made in Astronomy: the aberration of light in the heavenly bodies; and the nutation of the earth's axis; the former of which he showed to arise from the progressive motion of light and of the earth in its orbit; and the latter, from the attraction of the moon upon the protuberant parts of the earth above that of its inscribed sphere. He observed the comets which appeared in 1723, 1736, 1743, and 1757; and computed the elements of their

orbits. He constructed new Tables of *Jupiter's* satellites from his own observations and those of Mr. POUND. On the death of Dr. HALLEY in 1742, he succeeded him at the observatory at Greenwich; from which time to that of his death, he was indefatigable in observing the sun, moon, planets, and fixed stars. He settled the quantity and laws of refraction to a great degree of accuracy; and gave a very elegant rule for correcting the mean refraction from the variation of the weight and temperature of the air. In the year 1750 he procured a very fine transit instrument to be made for the observatory, by Mr. BIRD; and also a mural quadrant of brass of eight feet radius. With these instruments he continued his observations till the time of his death, which happened in 1762. The first volume of these observations was published by Dr. HORNSBY, professor of Astronomy at Oxford, who on account of his health, consigned the publication of the remaining part to Mr. ROBERTSON, Savilian professor of geometry.

1360. TOBIAS MAYER was born at Marbach in Wirtemberg, February 17, 1723. His first observations were made at Nuremberg; afterwards he went to Gottingen, where he continued to observe with very excellent instruments. His great object was to construct correct Tables of the moon; for which purpose he composed a very elegant theory, with which, and his observations, he formed new and very correct Tables of the motion of the moon. A copy of these in 1755 were sent here to the Right Honourable the Lords Commissioners of the Admiralty, putting in a claim for the reward offered for the discovery of the longitude. Dr. BRADLEY compared them with his own accurate observations, and was convinced of the excellency of the Tables. But MAYER continued correcting them till the time of his death (which happened in 1762), and left behind him a more complete set of lunar Tables, and also a very correct set of solar Tables. For these his widow received 3000*l.* He also left a catalogue of the fixed stars. A volume of his posthumous works was published in 1774; and it is to be hoped that the world will be favoured with those which remain unpublished.

1361. M. WARGENTIN, an excellent Astronomer of Sweden, published a set of Tables for computing the times of the eclipses of *Jupiter's* satellites, in the *Upsal Acts* for 1741; and since that time he has applied various corrections to them.

1362. Of those who have written upon physical Astronomy, M. CLAIRAUT, D'ALEMBERT, EULER, MAYER, FRISI, SIMPSON, LA PLACE, and M. de la GRANGE are the most eminent. By the labours of these celebrated mathematicians, the Tables of the motions of the bodies in our system have been corrected to an extreme degree of accuracy; and their names will go down to posterity, as completing that super-structure of which the GREAT NEWTON laid the foundation.

1363. Upon the death of Dr. BRADLEY, Mr. BLISS, Savilian professor of Astronomy at Oxford, succeeded him at the observatory at Greenwich, who lived there but a very little time, dying in the year 1765.

1364. M. BAILLY published at Paris the History of ancient and modern Astronomy. We are also indebted to him for a valuable Treatise on the theory of Jupiter's satellites, which was printed in the year 1766.

1365. Dr. LONG, master of Pembroke Hall, Cambridge, and Lowndian Professor of Astronomy, published a Treatise on Astronomy in five books. He also constructed a sphere of 17½ feet diameter, in which there is a floor so suspended, that the sphere has a free motion about its axis. On the concave surface the constellations are painted. The mechanism is very simple and ingenious. He died in 1770, in the 90th year of his age.

1366. In the year 1739, Mr. DONTHORNE published his *Astronomy of the Moon;* the Tables were constructed from Sir I. NEWTON'S theory; to which he added precepts for calculating eclipses.

1367. In 1741, M. le MONNIER published his *Histoire Celeste,* containing a collection of observations from the year 1666 to 1685, made by order of the king; with a preliminary discourse.

1368. M. PINGRE' published a very valuable work, entitled *Cometographie,* containing the history and theory of comets. This was printed in 1783, and 1784.

1369. To that celebrated Astronomer M. de la LANDE the world is indebted for the most important improvements in the science of Astronomy. Through so extensive a field he has left no track unbeaten; almost every part has received improvements from him; but we cannot here enter into a detail of them. His System of Astronomy is invaluable; and has tended far more to the general promotion of that science than all other works which ever appeared upon the subject. The labours of this great Astronomer will perpetuate his name.

1370. For the discovery of a seventh primary planet we are indebted to Mr. (now Dr.) HERSCHEL. By his great skill and industry in the construction of very large specula, he has made telescopes which have opened new views of the heavens, and penetrated into the depths of the universe; unfolding scenes which excite no less our wonder than our admiration. To this new planet he has discovered six satellites; and also two more belonging to *Saturn;* thus he has added nine bodies to our system! The various and interesting discoveries of this illustrious Astronomer the reader may see in the *Philosophical Transactions;* they are such as must transmit his name to the latest posterity.

1371. Dr. MASKELYNE succeeded Dr. BLISS at the observatory at Greenwich. To the abilities and indefatigable attention of this celebrated Astronomer, nautical Astronomy is altogether indebted for its present state of perfection. Of our *Nautical Almanac,* that great Astronomer M. de la LANDE thus writes:

"On a fait à Bologne, à Vienne, à Berlin, à Milan; mais le Nautical Almanac de Londres est l'ephemeride la plus parfaite qu'il y ait jamais eu." He has established the Newtonian doctrine of universal attraction upon the firmest foundation, by his experiments upon Schehallien. His regular observations of the sun, moon, planets, and fixed stars, which are every year published, are allowed to possess an unrivalled degree of accuracy; and we may consider them as the basis of future improvements of the Tables of the planetary motions. M. de la LANDE in his Astronomy (Vol. ii. p. 121. last edit.) speaking of astronomical observations, says, "Le recueil le plus moderne et le plus précieux de tous est celui de M. MASKELYNE, Astronome Royal d'Angleterre, qui commence à 1765, et qui forme déja deux volumes in-folio jusqu'à 1786. La précision de ces observations est si grande, qu'on trouve souvent la même seconde pour l'ascension droite d'une planete déduite de différentes étoiles, quoiqu'on y emploie la mesure du temps." His catalogue of fundamental stars is an invaluable treasure. These and his other various important improvements in this science entitle him to the most distinguished rank amongst Astronomers, and will render his name illustrious, as long as the science of Astronomy shall continue to be cultivated. Of this celebrated Astronomer, we shall speak more in another place.

CONCLUSION.

Art. 1372. IT has been justly observed, that the knowledge of nature is the firmest bulwark against Atheism. At a time therefore when infidelity so much prevails, and when even philosophy has been charged with promoting it, it may be proper, in order to stop the prevalence of the former, and refute the unjust charge against the latter, after having treated on the system of the universe, to take some notice of those extraordinary marks of design in its construction, which prove so clearly that it could not have owed its formation to chance, but to the contrivance of infinite WISDOM. The DEITY can only be known by his works; and the works of the creation afford a very convincing proof of a supreme BEING, who formed such vast bodies, and "gave them laws that should not be broken." If we trace not the cause from the effect, we neglect to direct our knowledge to that end, to which all our enquiries into nature ought to tend. From the works of GOD, we must seek to know him. Let us not deny the being of a supreme INTELLIGENCE, who is the cause of all things, because he is not the object of our corporeal senses; "he has not left himself without witness;" his existence and attributes are manifest from the construction of the universe, and the ends for which it was formed; but the nature of his essence surpasses the conception of our limited faculties. "We see but in part."

The obvious argument for the existence of a DEITY, who formed and governs the universe, is founded upon the uniformity of the laws which takes place in the production of similar effects; and from the simplicity of the causes which produce the various phænomena. The most common views of nature, however imperfect and of small extent, suggest the idea of the government of a GOD, and every further discovery tends to confirm that persuasion. The ancient philosophers, who scarce knew a single law by which the bodies in the system are governed, still saw the DEITY in his works; how visible therefore ought he to be to us, who are acquainted with the laws by which the whole is directed! The *same* law takes place in our system between the periodic times and distances of every body revolving about the same center. Every body describes about its respective center equal areas in equal times. Every body is spherical. Every planet, as far as our observations reach, is found to revolve about an axis; and the axis of each is observed to continue parallel to itself. Now as the circumstances which might have attended these bodies are indefinite in variety, the *uniform similarity* which is found to exist amongst them, is an irrefragable

argument of design. To produce a succession of day and night, either the sun must revolve every day about the earth, or the earth must revolve about its axis; the latter is the more simple cause, and accordingly we find that the regular return of day and night is so produced. As far also as observations have enabled us to discover, the return of day and night in the planets is produced by the operation of a similar cause. It is also found, that the axis of each planet is inclined to the plane of its orbit, by which a provision is made for a variety of seasons; and by preserving the axis always parallel to itself, summer and winter return at their stated periods. Where there are such incontestible marks of design, there must be a DESIGNER; and the unity of design through the whole system, proves it to be the work of ONE. The general laws of nature show the existence of a divine INTELLIGENCE, in a much stronger point of view, than any work of man can prove him to have acted from intention; inasmuch as the operations of the former are uniform, and subject to no variation; whereas in the latter case, we see continual alterations of plan, and deviations from established rules. And without this permanent order of things, experience could not have directed man in respect to his future operations. These fixed laws of nature, so necessary for us, is an irresistible argument that the world is the work of a wise and benevolent BEING. The laws of nature are the laws of GOD; and how far soever we may be able to trace up causes, they must terminate in his will. We see nothing in the heavens which argues imperfection; the whole creation is stamped with the marks of DIVINITY.

We can form no idea of that power called *Attraction*, by which distant bodies are made to act upon each other without any apparent connection; and yet we know that all the bodies in our system are retained in their courses by such a power. And it is a very singular instance of the unerring wisdom of the CREATOR, that the law which this power observes is such, that notwithstanding the mutual attractions of the bodies, the system will never fall into ruin, but is capable of preserving itself to all eternity. Moreover, the mutual attraction which takes place between distant bodies, could not of itself, either produce their motion about the sun, nor the rotation about their axes; it required an external impulse to operate in conjunction with it, to produce these effects; an act, which nothing but the arm of OMNIPOTENCE could accomplish. And the power which thus connects the distant bodies, operates also on the constituent particles of the same body, and preserves its figure; for without attraction the particles must have been dissipated by their rotation. An invisible power pervades the whole system, and preserves it. In the effects produced by man, we see the operation of the cause; but "the ways of the ALMIGHTY are past finding out."

The sun, that great and only fountain of light and heat, is placed in the center of the system; and whilst by its influence it retains the planets in their

orbits, it pours forth its rays and gives life to the creation. The formation of such a glorious body, and its arrangement, are circumstances which afford the clearest evidence of design.

Hence, in whatever point of view we take a survey of our system, we trace the power, wisdom, and goodness of the CREATOR. His power, in its formation; his wisdom, in the simplicity of the means to produce the ends; and his goodness in making those ends subservient to our use and enjoyment. Thus we are led by our enquiries into the structure of the universe, to the proofs of the existence and attributes of a supreme BEING, who formed and directs the whole. Arguments of this kind produce conviction which no sophistry can confound. "Every man may see it; man may behold it afar off." Let not therefore the ignorant declaim against those pursuits which direct us to a knowledge of our CREATOR, and furnish us with unanswerable arguments against the Infidel and the Atheist.

But if we carry our views up to the firmament of the fixed stars, the power of the DEITY will be still more astonishing. Let a man contemplate the starry heavens, and consider those glorious bodies only in respect to number, magnitude and distance, and it can scarcely fail to convince him of the existence of an omnipotent BEING. By the late improvements of telescopes, the starry system appears to be without bounds; and the greater part of these bodies not being visible to the naked eye, we may conclude that they were not made for our use, nor for the use of any part of our system. They are undoubtedly bodies similar to our sun, appearing so small from their immense distances; for opaque bodies at that distance could not be seen by reflected light. From the uniformity of nature, in all those parts which we have been able to examine and investigate, we may conclude, that bodies similar to our sun were created for the same cause, that of giving light and heat to the inhabitants of systems of planets surrounding them. We may therefore conceive the whole universe to be filled with created beings, enjoying the bounty of their CREATOR, and admiring his works. This benevolence of the DEITY in giving life to an almost infinite number of beings, must raise our admiration, till we are lost in contemplating his goodness. That every individual should exist under his protection, and be regularly supplied by his bountiful hand with every thing which is necessary for enjoyment, ought to make us very humble before him. And that every being in the universe should be under his care, and training up here for the further enjoyment of him hereafter, is a thought which, if duly impressed, would penetrate us with the deepest sense of gratitude to our CREATOR, and excite us to love and obedience. The disappearance of some stars may be the destruction of that system at the time appointed by the DEITY for the probation of its inhabitants; and the appearance of new stars may be the formation of new systems, for new races of beings then

called into existence to adore the works of their CREATOR. Thus we may conceive the DEITY to have been employed from all eternity, and thus continue to be employed for endless ages; forming new systems of beings to adore him; and transplanting those beings already formed into happier regions, where they may have made better opportunities of meditating on his works; and still rising in their enjoyments, go on to contemplate system after system through the boundless universe.

TABLES

FOR FACILITATING

ASTRONOMICAL CALCULATIONS.

TABLE I.

For converting Degrees, Minutes, and Seconds into Time, at the Rate of 360 Degrees for 24 Hours.

Deg. Min.	Hou. Min. Min. Sec.	Deg. Min.	Hou. Min. Min. Sec.	Sec.	Dec. of Sec.
1	0. 4	30	2. 0	1	,067
2	0. 8	40	2. 40	2	,133
3	0. 12	50	3. 20	3	,2
4	0. 16	60	4. 0	4	,266
5	0. 20	70	4. 40	5	,333
6	0. 24	80	5. 20	6	,4
7	0. 28	90	6. 0	7	,466
8	0. 32	100	6. 40	8	,533
9	0. 36	200	13. 20	9	,6
10	0. 40	300	20. 0	10	,666
20	1. 20				

TABLE II.

For converting Time into Degrees, Minutes, and Seconds, at the Rate of 24 Hours for 360 Degrees.

Hou.	Deg.	Min. Sec.	Deg. Min. Min. Sec.	Dec. of Sec.	Sec.
1	15	1	0. 15	,1	1,5
2	30	2	0. 30	,2	3,0
3	45	3	0. 45	,3	4,5
4	60	4	1. 0	,4	6,0
5	75	5	1. 15	,5	7,5
6	90	6	1. 30	,6	9,0
7	105	7	1. 45	,7	10,5
8	120	8	2. 0	,8	12,0
9	135	9	2. 15	,9	13,5
10	150	10	2. 30		
11	165	20	5. 0		
12	180	30	7. 30		
16	240	40	10. 0		
20	300	50	12. 30		

TABLE III.

Decimal Parts of an Hour.

1′	,01666	1″	,00028
2	,03333	2	,00056
3	,05	3	,00083
4	,06666	4	,00111
5	,08333	5	,00139
6	,1	6	,00167
7	,11666	7	,00194
8	,13333	8	,00222
9	,15	9	,00250
10	,16666	10	,00277
20	,33333	20	,00556
30	,5	30	,00833
40	,66666	40	,01111
50	,83333	50	,01388

TABLE IV.

For finding the Length of Circular Arcs to Radius Unity.

Deg.	Length.	Deg.	Length.	Min.	Length.	Sec.	Length.
1	0,0174533	60	1,0471976	1	0,0002909	1	0,0000048
2	0,0349066	70	1,2217305	2	0,0005818	2	0,0000097
3	0,0523599	80	1,3962634	3	0,0008727	3	0,0000145
4	0,0698132	90	1,5707963	4	0,0011636	4	0,0000194
5	0,0872665	100	1,7453293	5	0,0014544	5	0,0000242
6	0,1047198	120	2,0943951	6	0,0017453	6	0,0000291
7	0,1221730	150	2,6179939	7	0,0020362	7	0,0000339
8	0,1396263	180	3,1415927	8	0,0023271	8	0,0000388
9	0,1570796	210	3,6651914	9	0,0026180	9	0,0000436
10	0,1745329	240	4,1887902	10	0,0029089	10	0,0000485
20	0,3490659	270	4,7123890	20	0,0058178	20	0,0000970
30	0,5235988	300	5,2359878	30	0,0087266	30	0,0001454
40	0,6981317	330	5,7595865	40	0,0116355	40	0,0001939
50	0,8726646	360	6,2831853	50	0,0145444	50	0,0002424

TABLE V.

Sun's Parallax in Altitude.

Sun's Alt.	Sun's Par.
0°	8″,75
10	8, 62
20	8, 22
30	7, 58
40	6, 70
50	5, 62
55	5, 02
60	4, 37
65	3, 71
70	2, 99
75	2, 19
80	1, 52
85	0, 76
90	0, 00

TABLE VI.

The mean Right Ascensions and North-polar Distances of Thirty-six principal Fixed Stars, to th[e] Beginning of the Year 1802; with their Annual Precessions, Annual Proper Motions, and Annual Variations, from the latest Observations.

NAMES OF STARS.	Right Ascension in Signs, Degrees, &c.	Annual Precession.	Ann. proper Motion.	Annual Variation.	Right Ascension in Time.	Annual Precession.	Ann. proper Motion.	Annual Variation.
	S. D. M. S.	S.	S.	S.	S. H. M.	S.	S.	S.
γ Pegasi	0 0 45 46,8	46,13	—0,09	46,04	0 3 3,12	3,075	—0,006	3,069
α Arietis	0 29 0 34,1	50,08	+0,10	50,20	1 56 2,27	3,339	+0,007	3,347
α Ceti	1 12 59 6,3	46,84	—0,12	46,73	2 51 56,42	3,123	—0,008	3,115
Aldebaran	2 6 8 34,5	51,34	+0,03	51,39	4 24 34,30	3,423	+0,002	[illegible]
Capella	2 15 31 14,4	66,01	+0,21	66,23	5 2 4,96	4,401	+0,014	4,415
Rigel	2 16 15 24,1	43,17	—0,03	43,14	5 5 1,61	2,878	—0,002	2,876
β Tauri	2 18 26 45,8	56,70	+0,01	56,71	5 13 47,05	3,780	+0,001	3,781
α Orion	2 26 6 48,9	48,65	+0,01	48,65	5 44 27,26	3,243	+0,001	[illegible]
Sirius	3 9 6 21,8	40,21	—0,42	39,80	6 36 25,45	2,681	—0,028	[illegible]
Castor	3 20 29 7,7	57,95	—0,15	57,80	7 21 56,51	3,863	—0,010	[illegible]
Procyon	3 22 13 54,0	47,93	—0,80	47,13	7 28 55,60	3,195	—0,053	3,142
Pollux	3 23 17 40,2	56,05	—0,74	55,32	7 33 10,68	3,737	—0,049	[illegible]
α Hydræ	4 19 27 49,5	44,28	—0,09	44,19	9 17 51,30	2,952	—0,006	2,946
Regulus	4 29 27 12,0	48,41	—0,22	48,18	9 57 48,80	3,227	—0,015	3,212
β Leonis	5 24 44 13,1	46,58	—0,57	46,01	11 38 56,87	3,105	—0,038	[illegible]
β Virginis	5 25 5 41,6	46,14	+0,74	46,88	11 40 22,77	3,076	+0,049	[illegible]
Spica Virginis	6 18 41 40,5	47,22	—0,02	47,21	13 14 46,70	3,148	—0,001	[illegible]
Arcturus	7 1 39 28,8	42,18	—1,26	40,92	14 6 37,92	2,812	—0,084	2,728
1 α Libræ	7 9 56 22,2	49,54	—0,11	49,44	14 39 45,48	3,303	—0,007	3,296
2 α Libræ	7 9 59 12,0	49,56	—0,11	49,45	14 39 56,80	3,304	—0,007	3,297
α Cor. Bor.	7 21 34 36,8	37,92	+0,26	38,18	15 26 18,45	2,528	+0,017	2,545
α Serpentis	7 23 37 50,8	44,05	+0,11	44,18	15 34 31,39	2,937	+0,007	2,945
Antares	8 4 19 21,5	54,87	0,00	54,87	16 17 17,43	3,658	0,000	3,658
α Herculis	8 16 24 21,1	40,97	0,00	40,97	17 5 37,41	2,731	0,000	2,731
α Ophiuchi	8 21 26 12,1	41,58	+0,06	41,64	17 25 44,81	2,772	+0,004	2,776
α Lyræ	9 7 33 29,5	30,20	+0,23	30,40	18 30 13,97	2,013	+0,015	2,027
γ Aquilæ	9 24 12 38,0	42,80	—0,11	42,69	19 36 50,53	2,853	—0,007	2,846
α Aquilæ	9 25 16 46,9	43,40	+0,48	43,88	19 41 7,13	2,893	+0,032	2,925
β Aquilæ	9 26 23 46,1	44,21	—0,03	44,16	19 45 35,07	2,947	—0,002	2,944
1 α Capricorni	10 1 39 55,3	50,04	0,00	50,04	20 6 39,69	3,336	0,000	3,336
2 α Capricorni	10 1 45 52,2	50,04	+0,05	50,09	20 7 3,48	3,336	+0,003	3,339
α Cygni	10 8 40 13,7	30,63	—0,08	30,57	20 34 40,91	2,042	—0,005	2,038
α Aquarii	10 28 54 6,4	46,29	—0,08	46,21	21 55 36,43	3,086	—0,005	3,081
Fomalhaut	11 11 40 12,1	49,80	+0,35	50,15	22 46 40,81	3,320	+0,023	3,343
α Pegasi	11 13 43 33,5	44,64	—0,06	44,60	22 54 54,23	2,976	—0,004	2,973
α Andromedæ	11 29 32 39,1	45,97	+0,08	46,05	23 58 10,61	3,065	+0,005	3,070

TABLE VI. CONTINUED.

IES OF STARS.	Distance from the North Pole. D. M. S.	Annual Precession. S.	Annual proper Motion. S.	Annual Variation. S.	Number of Observations.	Magnitude.	Number for Annual Arguments of Aberration in N.P.D. S. D. M.	Max. of Aber. in N.P.D
ei	75 54 55,9	—20,05	— 0,15N	—20,20	7	2	7 2 25	9,1
tis	67 28 45,2	—17,54	+ 0,07S	—17,47	7	2,3	7 29 30	7,8
	86 41 36,6	—14,67	— 0,08N	—14,75	6	2	6 6 39	7,3
:harah	73 54 0,6	— 8,11	+ 0,12S	— 8,00	[illegible]	1	7 6 47	3,7
:lla	44 13 12,4	— 5,01	+ 0,44S	— 4,57	9	1	11 3 37	[illegible]
:l	98 26 19,0	— 4,76	— 0,16N	— 4,92	7	1	5 26 12	10,5
ri	61 34 23,1	— 4,02	+ 0,10S	— 3,91	9	2	10 0 25	2,5
e	82 38 27,8	— 1,36	— 0,13N	— 1,49	11	1	6 1 47	5,6
e	106 27 4,7	+ 3,17	+ 1,04S	+ 4,21	14	1	6 4 2	12,8
or	57 41 28,0	+ 7,02	+ 0,04S	+ 7,06	10	2	1 26 43	4,5
:yon	84 16 34,4	+ 7,59	+ 0,95S	+ 8,53	9	1,2	5 23 5	6,3
ux	61 30 25,7	+ 7,93	0,00	+ 7,93	8	2	2 14 38	3,9
lræ	97 46 19,3	+ 15,24	— 0,14N	+15,10	9	2	6 12 15	9,7
olus	77 4 9,5	+ 17,27	— 0,08N	+17,19	13	1	4 25 49	6,9
nis	74 19 14,6	+ 19,97	+ 0,07S	+20,04	8	1,2	4 23 16	9,0
inis	87 7 6,7	+ 19,98	+ 0,24S	+20,22	5	3	5 22 45	7,9
ε Virginis	100 7 14,6	+ 18,99	— 0,19N	+18,80	6	1	6 26 4	7,6
turus	69 46 45,4	+ 17,07	+ 1,72S	+18,79	18	1	5 1 25	12,3
.ibræ {	105 9 41,9	+ 15,37	— 0,18N	+15,19	[illegible]	6	7 11 11	6,1
	105 12 26,0	+ 15,36	— 0,15N	+15,21	6	2	7 11 11	6,1
Bor	62 36 35,5	+ 12,46	+ 0,03S	+12,49	6	2,3	5 7 19	14,8
entis	82 56 24,6	+ 11,89	— 0,19N	+11,70	[illegible]	2	5 21 28	9,8
ares	115 58 31,2	+ 8,69	— 0,26N	+ 8,43	6	1	9 1 14	3,8
culis	75 22 15,6	+ 4,71	— 0,23N	+ 4,48	10	2	5 24 27	12,3
iuchi	77 17 0,0	+ 2,99	+ 0,05S	+ 3,03	7	2	5 26 46	11,8
æ	51 29 35,3	— 2,64	— 0,27N	— 2,91	9	1	6 5 20	17,8
{	79 51 26,8	— 8,22	— 0,16N	— 8,38	6	3	6 7 37	10,9
quilæ {	81 38 33,8	— 8,56	— 0,54N	— 9,11	8	1,2	6 6 52	10,4
{	84 4 33,0	— 8,91	+ 0,35S	— 8,57	6	3,4	6 5 27	9,7
Capricorni {	103 6 22,9	—10,59	— 0,28N	—10,80	7	3	5 0 22	4,6
	103 8 41,6	—10,56	— 0,26N	—10,81	7	[illegible]	5 0 22	4,6
ni	45 25 16,4	—12,53	— 0,03N	—12,56	11	1,2	6 29 10	18,0
arii	91 16 25,1	—17,17	— 0,19N	—17,36	6	3	5 27 11	7,8
alhaut	120 39 52,7	—19,04	— 0,06N	—19,10	7	1,2	3 21 58	10,4
ai	75 51 18,2	—19,25	— 0,18N	—19,43	5	2	6 27 35	10,1
romedæ	62 0 5,8	—20,05	+ 0,06S	—19,99	6	2	7 22 54	11,6

TABLE VII. CONTINUED.

…or.	Procyon.	Pollux.	α Hydræ.	Regulus.	β Leonis.	β Virginis.	Spica Virg.
″	″	″	″	″	″	″	″
,56 16	+ 1,32 14	+ 1,49 16	+ 1,05 22	+ 0,89 27	+ 0,37 32	+ 0,34 33	− 0,20 36
,72 9	1,46 8	1,65 10	1,27 19	1,16 23	0,69 29	0,67 28	+ 0,14 33
,81 4	1,54 4	1,75 5	1,46 13	1,39 18	0,98 26	0,95 26	0,47 31
1,85	1,58	1,80	1,59	1,57	1,24	1,21	0,78
2	1	0	9	13	23	21	29
1,83 7	1,57 7	1,80 6	1,68 4	1,70 9	1,47 19	1,42 19	1,07 25
1,76	1,50	1,74	1,72	1,79	1,66	1,61	1,32
11	9	10	1	4	13	14	23
1,65 14	1,41 12	1,64 13	1,71 5	1,83 0	1,79 9	1,75 10	1,55 19
1,51 17	1,29 13	1,51 16	1,66 8	1,83 5	1,88 6	1,85 6	1,74 15
1,34 18	1,16 17	1,35 17	1,58 11	1,78 7	1,94 1	1,91 2	1,89 11
1,16	0,99	1,18	1,47	1,71	1,95	1,93	2,00
18	16	18	13	10	2	5	9
0,98 17	0,83 16	1,00 16	1,34 13	1,61 11	1,93 4	1,98 8	2,09 5
0,81 16	0,67 14	0,84 16	1,21 14	1,50 12	1,89 6	1,90 5	2,14 2
0,65	0,53	0,68	1,07	1,38	1,83	1,85	2,16
13	12	13	14	12	8	8	0
0,52 11	0,41 9	0,55 10	0,93 12	1,26 12	1,75 9	1,77 8	2,16 2
0,41 6	0,32 6	0,45 8	0,81 12	1,14 11	1,66 10	1,69 9	2,14 5
0,35	0,26	0,37	0,69	1,03	1,56	1,60	2,09
3	3	3	8	10	10	9	6
0,32 2	0,23 1	0,34 1	0,61 7	0,93 8	1,46 11	1,51 10	2,03 7
0,34 5	0,24 4	0,35 4	0,54 5	0,85 6	1,35 10	1,41 10	1,96 9
0,39	0,28	0,39	0,49	0,79	1,25	1,31	1,87
9	7	8	2	4	9	8	10
0,48 15	0,35 12	0,47 12	0,47 0	0,75 3	1,16 9	1,23 8	1,77 10
0,63 17	0,47 13	0,59 16	0,47 3	0,72 1	1,07 8	1,15 7	1,67 11
0,80	0,60	0,75	0,50	0,73	0,99	1,08	1,56
21	17	19	7	3	7	5	11
1,01 24	0,77 20	0,94 22	0,57 9	0,76 5	0,92 3	1,03 4	1,45 10
1,25 26	0,97 21	1,16 24	0,66 12	0,81 10	0,89 1	0,99 11	1,35 9
1,51	1,18	1,40	0,78	0,91	0,88	0,88	1,26
30	24	27	15	12	1	12	17
1,81 31	1,42 26	1,67 30	0,93 17	1,03 15	0,89 5	1,00 4	1,09 5
2,12 33	1,68 27	1,97 31	1,10 21	1,18 18	0,94 7	1,04 8	1,14 1
2,45	1,95	2,28	1,31	1,36	1,01	1,12	1,13
34	28	33	23	20	12	11	1
2,79 36	2,23 30	2,61 33	1,54 25	1,56 25	1,13 16	1,23 16	1,14 7
3,15 36	2,53 30	2,94 35	1,79 28	1,81 27	1,29 19	1,39 19	1,21 11
3,51	2,83	3,29	2,07	2,08	1,48	1,58	1,32
35	29	34	30	30	24	23	15
3,86 35	3,12 30	3,63 34	2,37 31	2,38 31	1,72 27	1,81 27	1,47 20
4,21 33	3,42 29	3,97 32	2,68 32	2,69 34	1,99 30	2,08 29	1,67 24
4,54	3,71	3,29	3,00	3,03	2,29	2,37	1,91
30	25	30	30	32	31	31	27
4,84 26	3,96 23	4,59 27	3,30 29	3,35 32	2,60 34	2,68 33	2,18 30
5,10 22	4,19 19	4,86 22	3,59 27	3,67 30	2,94 33	3,01 32	2,48 31
5,32 10	4,38 9	5,08 10	3,86 13	3,97 15	3,27 16	3,33 16	2,79 17
+ 5,42	+ 4,47	+ 5,18	+ 3,99	+ 4,12	+ 3,43	+ 3,49	+ 2,96

TABLE VII. CONTINUED.

Day of the Month.		Arcturus.	d	1 and 2 α Libræ.	d	α Cor. Bor.	d	α Serpentis.	d	Antares.	d	α Herculis.	d	α Ophiuchi.	d	α Lyræ.	d
		"		"		"		"		"		"		"		"	
January	0	—0,49	30	—0,68	33	—0,99	31	—0,92	29	—1,19	30	—1,24	22	—1,27	20	—1,67	11
	10	—0,19	33	0,35	32	0,68	31	—0,63	30	0,89	31	1,02	24	1,07	22	1,56	16
	20	+0,14	31	—0,03	34	0,37	32	—0,33	30	0,58	33	0,78	26	0,85	24	1,40	20
	30	0,45		+0,31		—0,05		—0,03		—0,25		0,52		0,61		1,20	
			31		32		33		31		34		28		28		25
February	9	0,76	29	0,63	31	+0,28	31	+0,28	30	+0,09	34	—0,24	29	0,33	28	0,95	27
	19	1,05		0,94		0,59		0,58		0,43		+0,05		—0,05		0,68	
			25		28		30		29		34		29		29		30
March	1	1,30	23	1,22	26	0,89	29	0,87	27	0,77	33	0,34	30	+0,24	29	0,38	32
	11	1,53	19	1,48	23	1,18	25	1,14	26	1,10	31	0,64	28	0,53	29	—0,06	33
	21	1,72	16	1,71	20	1,43	23	1,40	22	1,41	30	0,92	27	0,82	28	+0,27	33
	31	1,88		1,91		1,66		1,62		1,71		1,19		1,10		0,60	
			13		17		21		21		27		26		27		33
April	10	2,01	9	2,08	14	1,87	17	1,83	17	2,08	25	1,45	25	1,37	25	0,93	32
	20	2,10	6	2,22	11	2,04	14	2,00	16	2,33	22	1,70	22	1,62	21	1,25	30
	30	2,16		2,33		2,18		2,16		2,55		1,92		1,83		1,55	
			3		8		11		12		19		20		24		29
May	10	2,19	1	2,41	6	2.29	8	2,28	9	2,74	16	1,12	17	2,07	19	1,84	25
	20	2,20	3	2,47	2	2,37	4	2,37	7	2,90	12	1,29	14	2,26	16	2,09	22
	30	2,17		2,49		2,41		2,44		3,02		2,43		2,42		2,31	
			4		0		1		3		9		11		13		18
June	9	2,13	7	2,49	3	2,42	3	2,47	1	3,11	5	2,54	8	2,55	10	2,49	14
	19	2,06	9	2,46	5	2,39	6	2,48	3	3,16	3	2,62	4	2,65	6	2,63	9
	29	1,97		2,41		2,33		2,45		3,19		2,66		2,71		2,72	
			11		8		8		5		3		0		2		5
July	9	1,86	12	2,33	10	2,25	12	2,40	8	3,16	5	2,66	3	2,73	2	2,77	1
	19	1,74	14	2,23	11	2,13	15	2,32	11	3,11	9	2,63	8	2,71	5	2,76	6
	29	1,60		2,12		1,98		2,21		3,02		2,55		2,66		2,70	
			14		12		15		12		12		10		10		10
August	8	1,46	15	2,00	14	1,83	18	2.09	14	2,90	14	2,45	14	2,56	12	2,60	14
	18	1,31	14	1,86	13	1.65	19	1,95	15	2,76	15	2,31	15	2,44	15	2,46	18
	28	1,17		1,73		1,46		1,80		2,61		2,16		2,29		2,28	
			12		12		18		15		17		17		16		22
September	7	1.05	11	1,61	11	1,28	18	1,65	15	2,44	16	1,99	18	2,13	18	2,06	23
	17	0,94	8	1,50	9	1,10	16	1,50	13	2,28	16	1,81	18	1,95	18	1,83	25
	27	0,86		1,41		0,94		1,37		2,12		1,63		1,77		1,58	
			5		6		13		11		13		17		17		25
October	7	0,81	0	1,35	3	0,81	10	1,26	7	1,99	11	1,46	15	1,60	16	1,33	24
	17	0,81	4	1,32	2	0,71	5	1,19	4	1,88	7	1,31	12	1,44	12	1,09	23
	27	0,85		1,34		0,66		1,15		1,81		1,19		1,32		0,86	
			9		8		0		0		2		8		10		19
November	6	0,94	14	1,42	13	0,66	6	1,15	6	1,79	3	1,11	3	1,22	5	0,67	17
	16	1,08	20	1,55	17	0,72	11	1,21	12	1,82	8	1,08	1	1,17	0	0,50	12
	26	1,28		1,72		0,83		1,33		1,90		1,09		1,17		0,38	
			22		21		15		14		13		5		3		8
December	6	1,50	27	1,93	25	0,98	20	1,47	20	2,03	19	1,14	11	1,20	8	0 30	2
	16	1,77	29	2,18	29	1,18	24	1,67	23	2,22	22	1,25	15	1,28	14	0.28	1
	26	2,06	16	2,47	15	1,42	14	1,90	13	2,44	13	1,40	9	1,42	8	0,29	2
	31	+2,22		+2,62		+1,56		+2,03		+2,57		+1,49		+1,51		+0,31	

TABLE VII. CONTINUED.

Day of the Month.		α Aquilæ.	β Aquilæ.	1 and 1 α Capricor.	α Cygni.	α Aquarii.	Fomalhaut.	α Pegasi.	α Androm.
		"	"	"	"	"	"	"	"
January	0	—1,26 6	—1,25 6	—1,23 5	—1,59 5	—0,82 5	—0,67 10	—0,55 9	—0,21 14
	10	1,20 11	1,19 11	1,18 9	1,64 1	0,87 1	0,77 8	0,64 7	0,35 12
	20	1,09 14	1,08 13	1,09 13	1,63 4	0,88 1	0,85 4	0,71 5	0,47 11
	30	0,95	0,95	0,96	1,59	0,87	0,89	0,76	0,58
		17	17	16	11	3	2	2	8
February	9	0,78 20	0,78 19	0,80 19	1,48 15	0,84 9	0,91 2	0,78 1	0,66 6
	19	0,58	0,50	0,61	1,33	0,75	0,89	0,77	0,72
		22	22	21	19	10	5	3	4
March	1	0,37 24	0,37 24	0,40 24	1,14 24	0,65 13	0,84 10	0,74 8	0,76 0
	11	—0,12 26	—0,13 25	—0,16 25	0,90 28	0,52 17	0,74 14	0,66 11	0,76 5
	21	+0,14 27	+0,12 27	+0,09 28	0,62 31	0,35 19	0,60 17	0,55 14	0,71 9
	31	0,41	0,39	0,37	0,31	—0,16	0,43	0,41	0,62
		28	29	29	34	22	22	18	13
April	10	0,69 29	0,68 28	0,66 31	+0,03 35	+0,07 25	—0,21 24	0,23 21	0,49 17
	20	0,98 29	0,96 29	0,97 30	0,38 37	0,32 27	+0,03 29	—0,02 25	0,32 21
	30	1,27	1,25	1,27	0,75	0,59	0,32	+0,23	—0,11
		29	29	32	36	29	31	28	25
May	10	1,56 28	1,54 28	1,59 29	1,11 36	0,88 30	0,63 33	0,51 29	+0,14 29
	20	1,84 26	1,82 27	1,88 29	1,47 33	1,18 30	0,96 34	0,80 30	0,43 30
	30	2,10	2,09	2,17	1,80	1,48	1,30	1,10	0,73
		24	24	27	31	31	35	31	33
June	9	2,34 22	2,33 22	2,44 25	2,11 28	1,79 29	1,65 34	1,41 32	1,06 34
	19	2,56 18	2,55 18	2,69 22	2,39 24	2,08 28	1,99 34	1,73 30	1,40 33
	29	2,74	2,73	2,91	2,63	2,36	2,33	2,03	1,73
		14	15	17	19	25	[illegible]	29	34
July	9	2,88 11	2,88 11	3,08 14	2,82 14	2,61 22	2,65 33	2,32 26	2,07 31
	19	2,99 6	2,99 7	3,22 10	2,96 8	2,83 18	2,98 20	2,58 22	2,38 28
	29	3,05	3,06	3,32	3,04	3,01	3,18	2,80	2,66
		2	2	5	3	15	21	20	27
August	8	3,07 3	3,08 2	3,37 1	3,07 3	3,16 10	3,39 17	3,00 16	2,93 22
	18	3,04 6	3,06 6	3,38 4	3,04 8	3,26 6	3,56 [illegible]	3,16 11	3,15 19
	28	2,98	3,00	3,34	2,96	3,32	3,68	3,27	3,34
		10	9	7	13	2	8	8	15
September	7	2,88 13	2,91 13	3,27 10	2,83 17	3,34 2	3,76 3	3,35 3	3,49 10
	17	2,75 15	2,78 15	3,17 13	2,66 21	3,32 5	3,79 2	3,38 1	3,59 6
	27	2,60	2,63	3,04	2,45	3,27	3,77	3,37	3,65
		17	16	15	24	9	5	4	2
October	7	2,43 16	2,47 16	2,89 15	2,21 24	3,18 9	3,72 8	3,33 6	3,67 1
	17	2,27 17	2,31 16	2,74 16	1,97 26	3,09 13	3,64 11	3,27 10	3,66 5
	27	2,10	2,15	2,58	1,71	2,96	3,53	3,17	3,61
		14	15	14	26	13	12	10	8
November	6	1,96 13	2,00 13	2,44 13	1,45 24	2,83 13	3,41 14	3,07 12	3,53 9
	16	1,83 9	1,87 9	2,31 9	1,21 22	2,70 12	3,27 14	2,95 12	3,44 11
	26	1,74	1,78	2,22	0,99	2,58	3,13	2,83	3,33
		8	8	9	21	17	15	13	13
December	6	1,66 3	1,70 3	2,13 3	0,78 16	2,46 9	2,98 12	2,70 12	3,20 13
	16	1,63 1	1,67 1	2,10 1	0,62 12	2,37 8	2,86 13	2,58 [illegible]	3,07 14
	26	1,64 2	1,68 1	2,09 1	0,50 5	2,29 3	2,73 5	2,47 5	2,93 7
	31	+1,66	+ 1,69	+2,10	+0,45	+2,26	+2,68	+2,42	+2,86

TABLE VIII.

Corrections of the Right Ascensions in Time of Thirty-six principal Fixed Stars to every Tenth Degree of Longitude of the Moon's ascending Node, to serve from 1800 to 1900.

Arg. Long. of ☽'s ☊.	γ Pegasi.		α Arietis.		α Ceti.		Aldebaran.		Capella.		Rigel		β Tauri.		α Orion.		Sirius.
S. D. S.	"		"		"		"		"		"		"		"		"
0. 0. 6	−0,16+		−0,23+		−0,03+		−0,07+		−0,16+		+0,02−		−0,07+		−0,01+		−0,03+
10	0,35	19	0,43	20	0,22	19	0,28	21	0,43	27	−0,15+	17	0,30	23	0,20	19	0,19
20	0,53	18	0,62	19	0,41	19	0,49	21	0,68	25	0,33	18	0,52	22	0,40	20	0,35
		16		18		17		18		24		16		21		18	
1. 0. 7	0,69		0,80		0,58		0,67		0,92		0,49		0,73		0,58		0,50
10	0,83	14	0,94	14	0,73	15	0,84	17	1,13	21	0,64	15	0,94	21	0,74	16	0,64
20	0,94	11	1,06	12	0,87	14	0,98	14	1,30	17	0,77	13	1,07	13	0,88	14	0,75
		9		9		11		11		14		11		13		12	
2. 0. 8	1,08		1,15		0,98		1,09		1,44		0,88		1,20		1,00		0,84
10	1,08	5	1,20	5	1,05	7	1,17	8	1,53	9	0,96	8	1,29	9	1,09	9	0,91
20	1,10	2	1,21	1	1,10	5	1,21	4	1,57	4	1,00	4	1,33	4	1,13	4	0,91
		0		2		1		1		0		3		2		2	
3. 0. 9	1,10		1,19		1,11		1,22		1,57		1,03		1,35		1,15		0,96
10	1,05	5	1,13	6	1,09	2	1,18	4	1,51	6	1,01	2	1,31	4	1,13	2	0,93
20	0,98	7	1,04	9	1,04	5	1,12	6	1,42	9	0,97	4	1,24	7	1,08	5	0,89
		11		13		9		10		14		7		11		8	
4. 0. 10	0,87		0,91		0,95		1,02		1,28		0,90		1,13		1,00		0,81
10	0,73	14	0,76	15	0,83	12	0,88	14	1,09	19	0,80	10	0,98	15	0,88	12	0,71
20	0,58	15	0,59	17	0,69	14	0,72	16	0,88	21	0,67	13	0,81	17	0,74	14	0,59
		17		18		16		18		23		14		20		17	
5. 0. 11	0,41		0,41		0,53		0,54		0,65		0,53		0,61		0,57		0,45
10	0,22	19	−0,19+	22	0,35	18	0,35	19	0,39	26	0,37	16	0,40	21	0,39	18	0,30
20	−0,03 +	19	+0,03−	22	−0,16+	19	−0,14+	21	−0,11+	28	0,21	16	0,16+	24	−0,19+	20	−0,13+
6. 0. 0	+0,16 −	19	+0,23−	20	+0,03−	19	+0,07−	21	+0,16−	27	−0,02+	19	+0,07−	23	+0,01−	20	+0,03−

Arg. Long. of ☽'s ☊.	Arcturus.		1 and 2 α Libræ.		α Cor. Bor.		α Serpentis.		Antares.		α Herculis.		α Ophiuchi.		α Lyræ.		γ Aquilæ.
S. D. S.	"		"		"		"		"		"		"		"		"
0. 0. 6	+0,20 −		−0,13 +		+0,20 −		+0,05 −		−0,13 +		+0,04−		+0,02−		−0,07 +		−0,05 +
10	+0,02 −	18	0,33	20	+0,05 −	15	−0,13 +	18	0,35	22	−0,13+	17	−0,15+	17	0,19 +	12	0,22
20	−0,16 +	18	0,53	20	−0,12 +	17	0,31	18	0,57	22	0,30	17	−0,32	17	0,31	12	0,39
		17		17		16		17		20		15		16		11	
1. 0. 7	0,33		0,70		0,28		0,48		0,77		0,45		0,48		0,42		0,55
10	0,49	16	0,86	16	0,42	14	0,63	15	0,94	17	0,59	14	0,62	14	0,51	9	0,69
20	0,64	15	0,98	12	0,56	14	0,77	14	1,08	14	0,72	13	0,74	12	0,59	8	0,80
		13		11		12		11		11		10		11		7	
2. 0. 8	0,77		1,09		0,68		0,88		1,19		0,82		0,85		0,66		0,90
10	0,87	10	1,15	6	0,77	9	0,97	9	1,27	8	0,90	8	0,92	7	0,70	4	0,97
20	0,95	8	1,18	3	0,85	8	1,02	5	1,30	3	0,95	5	0,97	5	0,72	2	1,01
		5		0		5		3		0		2		2		0	
3. 0. 9	1,00		1,18		0,90		1,05		1,30		0,97		0,99		0,72		1,02
10	1,02	2	1,13	5	0,92	2	1,04	1	1,26	4	0,96	1	0,97	2	0,69	3	0,99
20	1,01	1	1,06	7	0,92	0	1,00	4	1,18	8	0,93	3	0,94	3	0,65	4	0,94
		5		11		4		7		12		7		7		6	
4. 0. 10	0,96		0,95		0,88		0,93		1,06		0,86		0,87		0,59		0,86
10	0,89	7	0,81	14	0,82	6	0,83	10	0,91	15	0,77	9	0,77	10	0,50	9	0,74
20	0,79	10	0,65	16	0,73	9	0,71	12	0,73	18	0,65	12	0,65	12	0,41	9	0,61
		12		18		10		15		19		13		14		11	
5. 0. 11	0,67		0,47		0,63		0,56		0,54		0,52		0,51		0,30		0,46
10	0,53	14	0,28	19	0,50	13	0,40	16	0,32	22	0,37	15	0,36	15	0,18	12	0,30
20	0,37	16	−0,05 +	23	0,35	15	0,22	18	0,09	23	0,17	20	0,19	17	−0,05 +	13	−0,13 +
6. 0. 0	−0,20+	17	+0,13 −	8	−0,20+	15	−0,05+	17	+0,13 −	22	−0,04 +	21	−0,02 +	17	+0,07 −	12	+0,05 −

TABLE VIII. CONTINUED.

Arg. Long. of ☽'s ☊.	Castor.	Procyon.	Pollux.	α Hydræ.	Regulus.	β Leonis.	β Virginis.	Spica Virg.
S. D. S.	″	″	″	″	″	″	″	″
0. 0. 6	+0,14—	+0,02—	+0,14—	—0,07+	+0,13 —	+0,18 —	+0,03—	—0,10 +
	23	19	23	18	20	19	19	20
10	—0,09+	—0,17+	—0,09+	0,25	—0,07 +	—0,01 +	—0,16+	0,30
	25	20	23	17	20	20	19	18
20	0,34	0,37	0,32	0,42	0,27	0,21	0,35	0,48
	22	18	23	16	20	19	17	17
1. 0. 7	0,56	0,55	0,55	0,58	0,47	0,40	0,52	0,65
	21	16	20	14	17	17	16	14
10	0,77	0,71	0,75	0,72	0,64	0,57	0,68	0,79
	19	14	17	12	15	16	14	12
20	0,96	0,85	0,92	0,84	0,79	0,73	0,82	0,91
	16	12	16	10	14	14	11	10
2. 0. 8	1,12	0,97	1,08	0,94	0,93	0,87	0,93	1,01
	12	9	12	7	11	11	9	6
10	1,24	1,06	1,20	1,01	1,04	0,98	1,02	1,07
	8	5	8	3	7	7	5	3
20	1,32	1,11	1,28	1,04	1,11	1,05	1,07	1,10
	5	3	5	1	4	5	3	0
3. 0. 9	1,37	1,14	1,33	1,05	1,15	1,10	1,10	1,10
	0	2	0	3	0	2	2	4
10	1,37	1,12	1,33	1,02	1,15	1,12	1,08	1,06
	3	4	3	6	3	2	4	6
20	1,34	1,08	1,30	0,96	1,12	1,10	1,04	1,00
	8	8	8	8	6	5	8	10
4. 0. 10	1,26	1,00	1,22	0,88	1,06	1,05	0,96	0,90
	12	12	12	12	10	9	11	13
10	1,14	0,88	1,10	0,76	0,96	0,96	0,85	0,77
	15	13	14	14	13	12	12	15
20	0,99	0,75	—0,96	0,62	0,83	0,84	0,73	0,62
	18	16	18	15	15	13	15	17
5. 0. 11	0,81	0,59	0,78	0,47	0,68	0,71	0,58	0,45
	21	18	20	17	17	17	18	18
10	0,60	0,41	0,58	0,30	0,51	0,54	0,40	0,27
	22	19	22	19	20	18	18	19
20	0,38	0,22	0,36	—0,11+	0,31	0,36	0,22	—0,08+
	24	20	22	18	18	18	19	19
6. 0. 0	—0,14 +	—0,02+	—0,14+	+0,07—	—0,13+	—0,18+	—0,03+	+0,11—

Arg Long. of ☽'s ☊.	α Aquilæ.	β Aquilæ.	1 and 2 α Capricorni.	α Cygni.	α Aquarii.	Fomalhaut.	α Pegasi.	α Androm.
S. D. S.	″	″	″	″	″	″	″	″
0. 0. 6	—0,04+	—0,03 +	+0,08—	—0,39 +	+0,01—	+0,36—	—0,16+	—0,34+
	18	18	20	12	19	21	18	18
10	0,22	0,21	—0,12+	0,51	—0,18+	0,15	0,34	0,52
	17	18	21	11	18	22	17	17
20	0,39	0,39	0,33	0,62	—0,36	—0,07+	0,51	0,69
	16	15	20	9	18	21	16	15
1. 0. 7	0,55	0,54	0,53	0,71	—0,54	0,28	0,67	0,84
	14	15	17	6	16	20	13	12
10	0,69	0,69	0,70	0,77	0,70	0,48	0,80	0,96
	12	13	15	4	13	19	11	9
20	0,81	0,82	0,85	0,81	0,83	0,67	0,91	1,05
	10	10	15	2	12	17	9	7
2. 0. 8	0,91	0,92	1,00	0,83	0,95	0,84	1,00	1,12
	7	8	9	1	8	15	5	2
10	0,98	1,00	1,09	0,82	1,03	0,99	1,05	1,14
	4	4	6	4	5	11	2	1
20	1,02	1,04	1,15	0,78	1,08	1,10	1,07	1,13
	1	1	4	5	2	8	1	4
3. 0. 9	1,03	1,05	1,19	0,73	1,10	1,18	1,06	1,09
	3	3	1	8	2	4	5	8
10	1.00	1,02	1,18	0,65	1,08	1,22	1,01	1,01
	5	5	4	10	4	1	7	10
20	0.95	0,97	1,14	0,55	1,04	1,23	0,94	0,91
	8	8	7	12	8	3	10	13
4. 0. 10	0,87	0,89	1,07	0,43	0,96	1,20	0,84	0,78
	11	11	10	13	12	7	13	17
10	0,76	0,78	0,96	0,30	0,84	1,13	0,71	0,61
	13	13	14	14	13	10	15	17
20	0,63	0,65	0,82	0,16	0,71	1,03	0,56	0,44
	15	15	16	14	15	13	16	19
5. 0. 11	0,48	0,50	0,66	—0,02+	0,56	0,90	0,40	0,25
	17	17	18	14	17	16	18	20
10	0,31	0,33	0,48	+0,12—	0,39	0,74	0,22	—0,05+
	17	18	20	15	19	19	19	20
20	—0,14+	—0,15+	—0,28 +	0,27	0,20	0,55	—0,03+	+0,15—
	18	18	20	12	21	19	19	19
6. 0. 0	+0,04—	+0,03—	+0,08 —	+0,39—	—0,01+	—0,36+	+0,16—	+0,34—

TABLE IX.

The Correction of the Obliquity of the Ecliptic for Days of the Year.

Days.	″	Days.	″
Jan. 10	−0,4	July 16	−0,6
18	−0,3	26	−0,5
25	−0,2		
31	−0,1	Aug. 1	−0,4
		8	−0,3
Feb. 8	0,0	16	−0,2
14	+0,1	21	−0,1
21	+0,2	31	0,0
March 2	+0,3	Sept. 21	+0,1
21	+0,4		
April 2	+0,3	Oct. 11	0,0
13	+0,2	19	−0,1
20	+0,1	25	−0,2
26	0,0	31	−0,3
May 2	−0,1	Nov. 6	−0,4
7	−0,2	12	−0,5
13	−0,3	17	−0,6
19	−0,4	23	−0,7
26	−0,5	30	−0,8
June 3	−0,6	Dec. 9	−0,9
25	−0,7	26	−1,0

TABLE X.

The Equation of the Obliquity of the Ecliptic.

ARGUMENT. Long. of ☽'s ☊.

Long. of ☽'s ☊	O. Sig. +	I. Sig. +	I. Sig. +	
	VI. Sig. −	VII. Sig. −	VIII. Sig. −	
0°	9″,5	8″,3	4″,8	30°
1	9, 5	8, 2	4, 6	29
2	9, 5	8, 1	4, 5	28
3	9, 5	8, 0	4, 3	27
4	9, 5	7, 9	4, 2	26
5	9, 5	7, 8	4, 0	25
6	9, 5	7, 7	3, 9	24
7	9, 5	7, 6	3, 7	23
8	9, 5	7, 5	3, 6	22
9	9, 4	7, 4	3, 4	21
10	9, 4	7, 3	3, 3	20
11	9, 4	7, 2	3, 1	19
12	9, 3	7, 1	3, 0	18
13	9, 3	7, 0	2, 8	17
14	9, 3	6, 9	2, 6	16
15	9, 2	6, 8	2, 5	15
16	9, 2	6, 6	2, 3	14
17	9, 1	6, 5	2, 1	13
18	9, 1	6, 4	2, 0	12
19	9, 0	6, 3	1, 8	11
20	9, 0	6, 1	1, 7	10
21	8, 9	6, 0	1, 5	9
22	8, 9	5, 9	1, 3	8
23	8, 8	5, 7	1, 2	7
24	8, 7	5, 6	1, 0	6
25	8, 7	5, 5	0, 8	5
26	8, 6	5, 3	0, 7	4
27	8, 5	5, 2	0, 5	3
28	8, 4	5, 1	0, 3	2
29	8, 4	4, 9	0, 2	1
30	8, 3	4, 8	0, 0	0
	− V. Sig.	− IV. Sig.	− III. Sig.	
	+ XI. Sig.	+ X. Sig.	+ IX. Sig.	Long. of ☽'s ☊

TABLE XI.

The mean astronomical Refractions, to every Ten Minutes of apparent Zenith Distances.

App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.
0°. 0'	0",00	0",17	4°. 30	4",50	0",17	9°. 0'	9",00	0",17	13°. 30'	13",65	0",18	18.° 0'	18",50	0,18
0. 10	0,17	0,16	4. 40	4,67	0,16	9. 10	9,17	0,16	13. 40	13,83	0,19	18. 10	18,68	0,19
0. 20	0,33		4. 50	4,83		9 20	9,33		13. 50	14,02		18. 20	18,87	
		0,17			0,17			0,17			0,18			0,18
0. 30	0,50	0,17	5. 0	5,00	0,17	9. 30	9,50	0,17	14. 0	14,20	0,18	18. 30	19,05	0,18
0. 40	0,67	0,16	5. 10	5,17	0,16	9. 40	9,67	0,16	14. 10	14,38	0,19	18. 40	19,23	0,19
0. 50	0,83		5. 20	5,33		9. 50	9,83		14. 20	14,57		18. 50	19,42	
		0,17			0,17			0,17			0,18			0,18
1. 0	1,00	0,17	5. 30	5,50	0,17	10. 0	10,00	0,18	14. 30	14,75	0,18	19. 0	19,60	0,18
1. 10	1,17	0,16	5. 40	5,67	0,16	10. 10	10,18	0,19	14. 40	14,93	0,19	19. 10	19,78	0,19
1. 20	1,33		5. 50	5,83		10. 20	10,37		14. 50	15,12		19. 20	19,97	
		0,17			0,17			0,18			0,18			0,18
1. 30	1,50	0,17	6. 0	6,00	0,17	10. 30	10,55	0,18	15. 0	15,30	0,17	19. 30	20,15	0,18
1. 40	1,67	0,16	6. 10	6,17	0,16	10. 40	10,73	0,19	15. 10	15,47	0,16	19. 40	20,33	0,19
1. 50	1,83		6. 20	6,33		10. 50	10,92		15. 20	15,63		19. 50	20,52	
		0,17			0,17			0,18			0,17			0,18
2. 0	2,00	0,17	6. 30	6,50	0,17	11. 0	11,10	0,17	15. 30	15,80	0,17	20. 0	20,70	0,20
2. 10	2,17	0,16	6. 40	6,67	0,16	11. 10	11,27	0,16	15. 40	15,97	0,16	20. 10	20,90	0,20
2. 20	2,33		6. 50	6,83		11. 20	11,43		15. 50	16,13		20. 20	21,10	
		0,17			0,17			0,17			0,17			0,20
2. 30	2,50	0,17	7. 0	7,00	0,17	11. 30	11,60	0,17	16. 0	16,30	0,18	20. 30	21,30	0,20
2. 40	2,67	0,16	7. 10	7,17	0,16	11. 40	11,77	0,16	16. 10	16,48	0,19	20. 40	21,50	0,20
2. 50	2,83		7. 20	7,33		11. 50	11,93		16. 20	16,67		20. 50	21,70	
		0,17			0,17			0,17			0,18			0,20
3. 0	3,00	0,17	7. 30	7,50	0,17	12. 0	12,10	0,17	16. 30	16,85	0,18	21. 0	21,90	0,18
3. 10	3,17	0,16	7. 40	7,67	0,16	12. 10	12,27	0,16	16. 40	17,03	0,19	21. 10	22,08	0,19
3. 20	3,33		7. 50	7,83		12. 20	12,43		16. 50	17,22		21. 20	22,27	
		0,17			0,17			0,17			0,18			0,18
3. 30	3,50	0,17	8. 0	8,00	0,17	12. 30	12,60	0,17	17. 0	17,40	0,18	21. 30	22,45	0,18
3. 40	3,67	0,16	8. 10	8,17	0,16	12. 40	12,77	0,16	17. 10	17,58	0,19	21. 40	22,63	0,19
3. 50	3,83		8. 20	8,33		12 50	12,93		17 20	17,77		21. 50	22,82	
		0,17			0,17			0,17			0,18			0,18
4. 0	4,00	0,17	8. 30	8,50	0,17	13. 0	13,10	0,18	17. 30	17,95	0,18	22. 0	23,00	0,20
4. 10	4,17	0,16	8. 40	8,67	0,16	13. 10	13,28	0,19	17. 40	18,13	0,19	22. 10	23,20	0,20
4. 20	4,33	0,17	8. 50	8,83	0,17	13. 20	13,47	0,18	17. 50	18,32	0,18	22. 20	23,40	0,20

TABLE XI. *Continued.*

App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.
22°. 30	23″,60		27°. 0	29 ,00		31°. 30	34″,90		36°. 0	41″,40		40°. 30	48″,65	
		0 ,20			0″,22			0″,23			0″,25			0″,28
22. 40	23, 80		27. 10	29, 22		31 40	35, 13		36. 10	41, 65		40. 40	48, 93	
		0, 20			0, 21			0, 24			0, 25			0, 29
22. 50	24, 00		27. 20	29, 43		31 50	35, 37		36. 20	41, 90		40 50	49, 22	
		0, 20			0, 22			0, 23			0, 25			0, 28
23. 0	24, 20		27. 30	29, 65		32. 0	35, 60		36. 30	42, 15		41. 0	49, 50	
		0, 20			0, 22			0, 23			0, 25			0, 30
23. 10	24, 40		27. 40	29, 87		32. 10	35, 83		36. 40	42, 40		41. 10	49, 80	
		0, 20			0, 21			0, 24			0, 25			0, 30
23. 20	24, 60		27. 50	30, 08		32. 20	36, 07		36. 50	42, 65		41. 20	50, 10	
		0, 20			0, 22			0, 23			0, 25			0, 30
23. 30	24, 80		28. 0	30, 30		32. 30	36, 30		37. 0	42, 90		41. 30	50, 40	
		0, 20			0, 22			0, 23			0, 27			0, 30
23. 40	25, 00		28. 10	30, 52		32. 40	36, 53		37. 10	43, 17		41. 40	50, 70	
		0, 20			0, 21			0, 24			0, 26			0, 30
23. 50	25, 20		28. 20	30, 73		32. 50	36, 77		37. 20	43, 43		41. 50	51, 00	
		0, 20			0, 22			0, 23			0, 27			0, 30
24. 0	25, 40		28. 30	30, 95		33. 0	37, 00		37. 30	43, 70		42. 0	51, 30	
		0, 20			0, 22			0, 23			0, 27			0, 30
24. 10	25, 60		28. 40	31, 17		33. 10	37, 23		37. 40	43, 97		42. 10	51, 60	
		0, 20			0, 21			0, 24			0, 26			0, 30
24. 20	25, 80		28. 50	31, 38		33. 20	37, 47		37. 50	44, 23		42. 20	51, 90	
		0, 20			0, 22			0, 23			0, 27			0, 30
24. 30	26, 00		29. 0	31, 60		33. 30	37, 70		38. 0	44, 50		42. 30	52, 20	
		0, 20			0, 22			0, 23			0, 27			0, 30
24. 40	26, 20		29. 10	31, 82		33. 40	37, 93		38. 10	44, 77		42. 40	52, 50	
		0, 20			0, 21			0, 2[illegible]			0, 26			0, 30
24. 50	26, 40		29. 20	32, 03		33. 50	38, 16		38 20	45, 03		42. 50	52, 80	
		0, 20			0, 22			0, 24			0, 27			0. 30
25. 0	26, 60		29. 30	32, 25		34. 0	38, 40		38 30	45, 30		43. 0	53, 10	
		0, 20			0, 22			0, 25			0, 27			0, 32
25. 10	26, 80		29 40	32, 47		34. 10	38, 65		38. 40	45, 57		43. 10	53, 42	
		0, 20			0, 21			0, 25			0, 26			0, 31
25. 20	27, 00		29 50	32, 68		34 20	38, 90		38. 50	45, 83		43. 20	53, 73	
		0, 20			0, 22			0, 25			0, 27			0, 32
25. 30	27, 20		30. 0	32, 90		34 30	39, 15		39 0	46, 10		43. 30	54, 05	
		0, 20			0, 22			0, 25			0, 28			0, 32
35. 40	27, 40		30. 10	33, 12		34. 40	39, 40		39 10	46, 38		43. 40	54, 37	
		0, 20			0, 21			0, 25			0, 29			0, 31
25. 50	27, 60		30. 20	33, 33		34. 50	39, 65		39 20	46, 67		43. 50	54, 68	
		0, 20			0, 22			0, 25			0, 28			0, 32
26 0	27, 80		30. 30	33, 55		35. 0	39, 90		39 30	46, 95		44 0	55, 00	
		0, 20			0, 22			0, 25			0, 28			0, 32
26. 10	28, 00		30. 40	33, 77		35. 10	40, 15		39. 40	47, 23		44. 10	55, 32	
		0, 20			0, 21			0, 25			0, 29			0, 31
26. 20	28, 20		30 50	33, 98		35 20	40, 40		39. 50	47, 52		44. 20	55, 63	
		0, 20			0, 22			0, 25			0, 28			0, 32
26. 30	28, 40		31. 0	34, 20		35. 30	40, 65		40. 0	47, 80		44 30	55, 95	
		0, 20			0, 23			0, 25			0, 28			0, 32
26. 40	28, 60		31. 10	34, 43		35. 40	40, 90		40 10	48, 08		44 40	56, 27	
		0, 20			0, 24			0, 25			0, 29			0, 31
26. 50	28, 80		31. 20	34, 67		35. 50	41, 15		40. 20	48, 37		44. 50	56, 58	
		0, 20			0, 23			0, 25			0, 28			0, 32

App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.
45°. 0'	0'. 56",90	0",33	49°. 30'	1'. 6",59	0",40	54°. 0'	1' 18",30	0",47	58° 30'	1'. 32",78	0",60	63°. 0	1'. 51",50	0",80
45. 10	0. 57, 23	0, 33	49. 40	1. 6, 99	0, 40	54. 10	1. 18, 77	0, 48	58. 40	1. 33, 38	0, 61	63. 10	1. 52, 30	0, 80
45. 20	0. 57, 56	0, 33	49. 50	1. 7, 39	0, 41	54. 20	1. 19, 25	0, 48	58. 50	1. 33, 99	0, 61	63. 20	1. 53, 10	0, 81
45. 30	0. 57, 89	0, 33	50. 0	1. 7, 80	0, 39	54. 30	1. 19, 73	0, 49	59. 0	1. 34, 60	0, 62	63. 30	1. 53, 91	0, 82
45. 40	0. 58, 22	0, 34	50. 10	1. 8, 19	0, 40	54. 40	1. 20, 22	0, 49	59. 10	1. 35, 22	0, 62	63. 40	1. 54, 73	0, 83
45. 50	0. 58, 56	0, 34	50. 20	1. 8, 59	0, 40	54. 50	1. 20, 71	0, 49	59. 20	1. 35, 84	0, 63	63. 50	1. 55, 56	0, 84
46. 0	0. 58, 90	0, 35	50. 30	1. 8, 99	0, 40	55. 0	1. 21, 20	0, 51	59. 30	1. 36, 47	0, 64	64. 0	1. 56, 40	0, 85
46. 10	0. 59, 25	0, 34	50. 40	1. 9, 39	0, 40	55. 10	1. 21, 71	0, 51	59. 40	1. 37, 11	0, 64	64. 10	1. 57, 25	0, 87
46. 20	0. 59, 59	0, 35	50. 50	1. 9, 79	0, 41	55. 20	1. 22, 22	0, 51	59. 50	1. 37, 75	0, 65	64. 20	1. 58, 12	0, 88
46. 30	0. 59, 94	0, 35	51. 0	1. 10, 20	0, 42	55. 30	1. 22, 73	0, 52	60. 0	1. 38, 40	0, 67	64. 30	1. 59, 00	0, 89
46. 40	1. 0, 29	0, 36	51. 10	1. 10, 62	0, 43	55. 40	1. 23, 25	0, 52	60. 10	1. 39, 07	0, 67	64. 40	1. 59, 89	0, 90
46. 50	1. 0, 65	0, 35	51. 20	1. 11, 05	0, 43	55. 50	1. 23, 77	0, 53	60. 20	1. 39, 74	0, 68	64. 50	2. 0, 79	0, 91
47. 0	1. 1, 00	0, 36	51. 30	1. 11, 48	0, 44	56. 0	1. 24, 30	0, 52	60. 30	1. 40, 42	0, 69	65. 0	2. 1, 70	0, 92
47. 10	1. 1, 36	0, 37	51. 40	1. 11, 92	0, 44	56. 10	1. 24, 82	0, 52	60. 40	1. 41, 11	0, 69	65. 10	2. 2, 62	0, 93
47. 20	1. 1, 73	0, 36	51. 50	1. 12, 36	0, 44	56. 20	1. 25, 34	0, 53	60. 50	1. 41, 80	0, 70	65. 20	2. 3, 55	0, 94
47. 30	1. 2, 09	0, 37	52. 0	1. 12, 80	0, 44	56. 30	1. 25, 87	0, 54	61. 0	1. 42, 50	0, 70	65. 30	2. 4, 49	0, 96
47. 40	1. 2, 46	0, 37	52. 10	1. 13, 24	0, 45	56. 40	1. 26, 41	0, 54	61. 10	1. 43, 20	0, 70	65. 40	2. 5, 45	0, 97
47. 50	1. 2, 83	0, 37	52. 20	1. 13, 69	0, 45	56. 50	1. 26, 95	0, 55	61. 20	1. 43, 90	0, 71	65. 50	2. 6, 42	0, 98
48. 0	1. 3, 20	0, 36	52. 30	1. 14, 14	0, 45	57. 0	1. 27, 50	0, 57	61. 30	1. 44, 61	0, 72	66. 0	2. 7, 40	1, 00
48. 10	1. 3, 56	0, 36	52. 40	1. 14, 59	0, 45	57. 10	1. 28, 07	0, 57	61. 40	1. 45, 33	0, 73	66. 10	2. 8, 40	1, 01
48. 20	1. 3, 92	0, 37	52. 50	1. 15, 04	0, 46	57. 20	1. 28, 64	0, 58	61. 50	1. 46, 06	0, 74	66. 20	2. 9, 41	1, 03
48. 30	1. 4, 29	0, 37	53. 0	1. 15, 50	0, 46	57. 30	1. 29, 22	0, 59	62. 0	1. 46, 80	0, 76	66. 30	2. 10, 44	1, 04
48. 40	1. 4, 66	0, 37	53. 10	1. 15, 96	0, 46	57. 40	1. 29, 81	0, 59	62. 10	1. 47, 56	0, 77	66. 40	2. 11, 48	1, 05
48. 50	1. 5, 03	0, 37	53. 20	1. 16, 42	0, 47	57. 50	1. 30, 40	0, 60	62. 20	1. 48, 33	0, 78	66. 50	2. 12, 53	1, 07
49 0	1. 5, 40	0, 39	53. 30	1. 16, 89	0, 47	58. 0	1. 31, 00	0, 59	62. 30	1. 49, 11	0, 79	67. 0	2. 13, 60	1, 08
49. 10	1. 5, 79	0, 40	53. 40	1. 17, 36	0, 47	58. 10	1. 31, 59	0, 59	62. 40	1. 49, 90	0, 80	67. 10	2. 14, 68	1, 10
49. 20	1. 6, 19	0, 40	53. 50	1. 17, 83	0, 47	58. 20	1. 32, 18	0, 60	62. 50	1. 50, 70	0, 80	67. 20	2. 15, 78	1, 11

TABLE XI. *Continued.*

App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.	App. Z. D.	Refraction	Diff.
67°. 30′	2′. 16″,89	1″,12	71°. 30′	2′. 48″,97	1″,62	75°. 30′	3′. 37″,60	2″,58	79°. 30′	5′. 0″,21	4″,72	83°. 30′	7′. 51″,39	11″,12
67. 40	2. 18, 01	1, 14	71. 40	2. 50, 59	1, 64	75. 40	3. 40, 18	2, 63	79. 40	5. 4, 93	4, 88	83. 40	8. 2, 51	11, 67
67. 50	2. 19, 15	1, 15	71. 50	2. 52, 23	1, 67	75. 50	3. 42, 81	2, 69	79. 50	5. 9, 81	5, 02	83. 50	8. 14, 18	12, 23
68. 0	2. 20, 30	1, 16	72. 0	2. 53, 90	1, 72	76. 0	3. 45, 50	2, 75	80. 0	5. 14, 83	5, 16	84. 0	8. 26, 41	12, 60
68. 10	2. 21, 46	1, 17	72. 10	2. 55, 62	1, 75	76. 10	3. 48, 25	2, 81	80. 10	5. 19, 99	5, 32	84. 10	8. 39, 01	13, 24
68. 20	2. 22, 63	1, 19	72. 20	2. 57, 37	1, 78	76. 20	3. 51, 06	2, 88	80. 20	5. 25, 31	5, 50	84. 20	8. 52, 25	13, 94
68. 30	2. 23, 82	1, 21	72. 30	2. 59, 15	1, 82	76. 30	3. 53, 94	2, 95	80. 30	5. 30, 81	5, 68	84. 30	9. 6, 19	14, 66
68. 40	2. 25, 03	1, 22	72. 40	3. 0, 97	1, 85	76. 40	3. 56, 89	3, 02	80. 40	5. 36, 49	5, 89	84. 40	9. 20, 85	15, 46
68. 50	2. 26, 25	1, 25	72. 50	3. 2, 82	1, 88	76. 50	3. 59, 91	3, 09	80. 50	5. 42, 38	6, 07	84. 50	9. 36, 31	16, 19
69. 0	2. 27, 50	1, 28	73. 0	3. 4, 70	1, 91	77. 0	4. 3, 00	3, 15	81. 0	5. 48, 45	6, 25	85. 0	9. 52, 50	
69. 10	2. 28, 78	1, 30	73. 10	3. 6, 61	1, 94	77. 10	4. 6, 15	3, 25	81. 10	5. 54, 70	6, 47			
69. 20	2. 30, 05	1, 32	73. 20	3. 8, 55	1, 98	77. 20	4. 9, 40	3, 31	81. 20	6. 1, 17	6, 71			
69. 30	2. 31, 40	1, 34	73. 30	3. 10, 53	2, 02	77. 30	4. 12, 71	3, 42	81. 30	6. 7, 88	6, 95			
69. 40	2. 32, 74	1, 37	73. 40	3. 12, 55	2, 06	77. 40	4. 16, 13	3, 49	81. 40	6. 14, 83	7, 22			
69. 50	2. 34, 11	1, 39	73. 50	3. 14, 61	2, 09	77. 50	4. 19, 62	3, 56	81. 50	6. 22, 05	7, 50			
70. 0	2. 35, 50	1, 41	74. 0	3. 16, 70	2, 13	78. 0	4. 23, 18	3, 69	82. 0	6. 29, 55	7, 72			
70. 10	2. 36, 91	1, 43	74. 10	3. 18, 83	2, 18	78. 10	4. 26, 87	3, 79	82. 10	6. 37, 27	8, 02			
70. 20	2. 38, 34	1, 46	74. 20	3. 21, 01	2, 23	78. 20	4. 30, 66	3, 88	82. 20	6. 45, 29	8, 35			
70. 30	2. 39, 80	1, 48	74. 30	3. 23, 24	2, 27	78. 30	4. 34, 54	3, 99	82. 30	6. 53, 64	8, 69			
70. 40	2. 41, 28	1, 50	74. 40	3. 25, 51	2, 32	78. 40	4. 38, 53	4, 11	82. 40	7. 2, 33	9, 06			
70. 50	2. 42, 78	1, 52	74. 50	3. 27, 83	2, 37	78. 50	4. 42, 64	4, 21	82. 50	7. 11, 39	9, 45			
71. 0	2. 44, 30	1, 53	75. 0	3. 30, 20	2, 41	79. 0	4. 46, 85	4, 33	83. 0	7. 20, 84	9, 74			
71. 10	2. 45, 83	1, 56	75. 10	3. 32, 61	2, 47	79. 10	4. 51, 18	4, 45	83. 10	7. 30, 58	10, 17			
71. 20	2. 47, 39	1, 58	75. 20	3. 35, 08	2, 52	79. 20	4. 55, 63	4, 58	83. 20	7. 40, 75	10, 64			

Decimal Numbers for computing the Corrections of the mean astronomical Refractions depending on the Barometer & Thermometer.

Enter the Table with the Height of the Barometer expressed in Inches and Decimals at Top, and with the Height of FAHRENHEIT's Thermometer expressed in Degrees on the Side.

Bar.	29,0	29,1	29,2	29,3	29,4	29,5	29,6	29,7	29,8	29,9	30,0
Ther.											
20°	+ ,059	+ ,063	+ ,067	+ ,070	+ ,074	+ ,077	+ ,081	+ ,085	+ ,088	+ ,092	+ ,096
21	+ ,056	+ ,060	+ ,064	+ ,067	+ ,071	+ ,075	+ ,078	+ ,082	+ ,085	+ ,089	+ ,093
22	+ ,054	+ ,057	+ ,061	+ ,064	+ ,068	+ ,072	+ ,075	+ ,079	+ ,083	+ ,086	+ ,090
23	+ ,051	+ ,054	+ ,058	+ ,062	+ ,065	+ ,069	+ ,072	+ ,076	+ ,080	+ 083	+ ,087
24	+ ,048	+ ,051	+ ,055	+ ,059	+ ,062	+ ,066	+ ,070	+ ,073	+ ,077	+ ,080	+ ,084
25	+ ,045	+ ,049	+ ,052	+ ,056	+ ,060	+ ,063	+ ,067	+ ,070	+ ,074	+ ,078	+ ,081
26	+ ,042	+ ,046	+ ,049	+ ,053	+ ,057	+ ,060	+ ,064	+ ,067	+ ,071	+ ,075	+ ,078
27	+ ,040	+ ,043	+ ,047	+ ,050	+ ,054	+ ,057	+ ,061	+ ,065	+ ,068	+ ,072	+ ,075
28	+ ,037	+ ,040	+ ,044	+ ,048	+ ,051	+ ,055	+ ,058	+ ,062	+ ,065	+ ,069	+ ,073
29	+ ,034	+ ,038	+ ,041	+ ,045	+ ,048	+ ,052	+ ,055	+ ,059	+ ,063	+ ,066	+ ,070
30	+ ,031	+ ,035	+ ,038	+ ,042	+ ,046	+ ,049	+ ,053	+ ,056	+ ,060	+ ,063	+ ,067
31	+ ,029	+ ,032	+ ,036	+ ,039	+ ,043	+ ,046	+ ,050	+ ,053	+ ,057	+ ,061	+ ,064
32	+ ,026	+ ,029	+ ,033	+ ,037	+ ,040	+ ,044	+ ,047	+ ,051	+ ,054	+ ,058	+ ,061
33	+ ,023	+ ,027	+ ,030	+ ,034	+ ,037	+ ,041	+ ,044	+ ,048	+ ,051	+ ,055	+ ,059
34	+ ,021	+ ,024	+ ,028	+ ,031	+ ,035	+ ,038	+ ,042	+ ,045	+ 049	+ ,052	+ ,056
35	+ ,018	+ ,021	+ ,025	+ ,028	+ ,032	+ ,036	+ ,039	+ ,043	+ ,046	+ ,050	+ ,053
36	+ ,015	+ ,019	+ ,022	+ ,026	+ ,029	+ ,033	+ ,036	+ ,040	+ ,043	+ ,047	+ ,050
37	+ ,013	+ ,016	+ ,020	+ ,023	+ ,027	+ ,030	+ ,034	+ ,037	+ ,041	+ ,044	+ ,048
38	+ ,010	+ ,014	+ ,017	+ ,020	+ ,024	+ ,027	+ ,031	+ ,034	+ ,038	+ ,041	+ ,045
39	+ ,007	+ ,011	+ ,014	+ ,018	+ ,021	+ ,025	+ ,028	+ ,032	+ ,035	+ ,039	+ ,042
40	+ ,005	+ ,008	+ ,012	+ ,015	+ ,019	+ ,022	+ ,026	+ ,029	+ ,033	+ ,036	+ ,040
41	+ ,002	+ ,006	+ ,009	+ ,013	+ ,016	+ ,020	+ ,023	+ ,026	+ ,030	+ ,033	+ ,037
42	,000	+ ,003	+ ,007	+ ,010	+ ,014	+ ,017	+ ,020	+ ,024	+ ,027	+ ,031	+ ,034
43	— ,003	+ ,001	+ ,004	+ ,008	+ ,011	+ ,014	+ ,018	+ ,021	+ ,025	+ ,028	+ ,032
44	— ,005	— ,002	+ ,002	+ ,005	+ ,008	+ ,012	+ ,015	+ ,019	+ ,022	+ ,026	+ ,029
45	— ,008	— ,004	— ,001	+ ,002	+ ,006	+ ,009	+ ,013	+ ,016	+ ,020	+ ,023	+ ,026
46	— ,010	— ,007	— ,004	,000	+ ,003	+ 007	+ ,010	+ ,014	+ ,017	+ ,020	+ ,024
47	— ,013	— ,009	— ,006	— ,003	+ ,001	+ ,004	+ ,008	+ ,011	+ ,014	+ ,018	+ ,021
48	— ,015	— ,012	— ,009	— ,005	— ,002	+ ,002	+ ,005	+ ,008	+ ,012	+ ,015	+ ,019
49	— ,018	— ,015	— ,011	— ,008	— ,004	— ,001	+ ,003	+ ,006	+ ,009	+ ,013	+ ,016
50	— ,020	— ,017	— ,014	— ,010	— ,007	— ,003	,000	+ ,003	+ ,007	+ ,010	+ ,014

TABLE XII. *Continued.*

Bar.	30,1	30,2	30,3	30,4	30,5	30,6	30,7	30,8	30,9	31,0
Ther.										
20°	+ ,099	+ ,103	+ ,107	+ ,110	+ ,114	+ ,118	+ ,121	+ ,125	+ ,129	+ ,132
21	+ ,096	+ ,100	+ ,104	+ ,107	+ ,111	+ ,115	+ ,118	+ ,122	+ ,126	+ ,129
22	+ ,094	+ ,097	+ ,101	+ ,104	+ ,108	+ ,112	+ ,115	+ ,119	+ ,123	+ ,126
23	+ ,091	+ ,094	+ ,098	+ ,101	+ ,105	+ ,109	+ ,112	+ ,116	+ ,120	+ ,123
24	+ ,088	+ ,091	+ ,095	+ ,098	+ ,102	+ ,106	+ ,109	+ ,113	+ ,117	+ ,120
25	+ ,085	+ ,088	+ ,092	+ ,096	+ ,099	+ ,103	+ ,106	+ ,110	+ ,114	+ ,117
26	+ ,082	+ ,085	+ ,089	+ ,093	+ ,096	+ ,100	+ ,103	+ ,107	+ ,111	+ ,114
27	+ ,079	+ ,083	+ ,086	+ ,090	+ ,093	+ ,097	+ ,100	+ ,104	+ ,108	+ ,111
28	+ ,076	+ ,080	+ ,083	+ ,087	+ ,090	+ ,094	+ ,098	+ ,101	+ ,105	+ ,108
29	+ ,073	+ ,077	+ ,080	+ ,084	+ ,088	+ ,091	+ ,095	+ ,098	+ ,102	+ ,105
30	+ ,070	+ ,074	+ ,078	+ ,081	+ ,085	+ ,088	+ ,092	+ ,095	+ ,099	+ ,102
31	+ ,068	+ ,071	+ ,075	+ ,078	+ ,082	+ ,085	+ ,089	+ ,093	+ ,096	+ ,100
32	+ ,065	+ ,068	+ ,072	+ ,075	+ ,079	+ ,083	+ ,086	+ ,090	+ ,093	+ ,097
33	+ ,062	+ ,066	+ ,069	+ ,073	+ ,076	+ ,080	+ ,083	+ ,087	+ ,090	+ ,094
34	+ ,059	+ ,063	+ ,066	+ ,070	+ ,073	+ ,077	+ ,080	+ ,084	+ ,088	+ ,091
35	+ ,057	+ ,060	+ ,064	+ ,067	+ ,071	+ ,074	+ ,078	+ ,081	+ ,085	+ ,088
36	+ ,054	+ ,057	+ ,061	+ ,064	+ ,068	+ ,071	+ ,075	+ ,078	+ ,082	+ ,085
37	+ ,051	+ ,055	+ ,058	+ ,062	+ ,065	+ ,069	+ ,072	+ ,076	+ ,079	+ ,083
38	+ ,048	+ ,052	+ ,055	+ ,059	+ ,062	+ ,066	+ ,069	+ ,073	+ ,076	+ ,080
39	+ ,046	+ ,049	+ ,053	+ ,056	+ ,060	+ ,063	+ ,067	+ ,070	+ ,074	+ ,077
40	+ ,043	+ ,046	+ ,050	+ ,053	+ ,057	+ ,060	+ ,064	+ ,067	+ ,071	+ ,074
41	+ ,040	+ ,044	+ ,047	+ ,051	+ ,054	+ ,058	+ ,061	+ ,065	+ ,068	+ ,071
42	+ ,038	+ ,041	+ ,045	+ ,048	+ ,051	+ ,055	+ ,058	+ ,062	+ ,065	+ ,069
43	+ ,035	+ ,038	+ ,042	+ ,045	+ ,049	+ ,052	+ ,056	+ ,059	+ ,063	+ ,066
44	+ ,032	+ ,036	+ ,039	+ ,043	+ ,046	+ ,050	+ ,053	+ ,056	+ ,060	+ ,063
45	+ ,030	+ ,033	+ ,037	+ ,040	+ ,044	+ ,047	+ ,050	+ ,054	+ ,057	+ ,061
46	+ ,027	+ ,031	+ ,034	+ ,037	+ ,041	+ ,044	+ ,048	+ ,051	+ ,054	+ ,058
47	+ ,025	+ ,028	+ ,031	+ ,035	+ ,038	+ ,042	+ ,045	+ ,049	+ ,052	+ ,055
48	+ ,022	+ ,025	+ ,029	+ ,032	+ ,036	+ ,039	+ ,042	+ ,046	+ ,049	+ ,053
49	+ ,019	+ ,023	+ ,026	+ ,030	+ ,033	+ ,036	+ ,040	+ ,043	+ ,047	+ ,050
50	+ ,017	+ ,020	+ ,024	+ ,027	+ ,030	+ ,034	+ ,037	+ ,041	+ ,044	+ ,047

TABLE XII. *Continued.*

Bar.	29,0	29,1	29,2	29,3	29,4	29,5	29,6	29,7	29,8	29,9	30,0
Ther.											
50°	— ,020	— ,017	— ,014	— ,010	— ,007	— ,003	,000	+ ,003	+ ,007	+ ,010	+ ,014
51	— ,023	— ,019	— ,016	— ,013	— ,009	— ,006	— ,003	+ ,001	+ ,004	+ ,008	+ ,011
52	— ,025	— ,022	— ,019	— ,015	— ,012	— ,008	— ,005	— ,002	+ ,002	+ ,005	+ ,008
53	— ,028	— ,024	— ,021	— ,018	— ,014	— ,011	— ,008	— ,004	— ,001	+ ,003	+ ,006
54	— ,030	— ,027	— ,023	— ,020	— ,017	— ,013	— ,010	— ,007	— ,003	,000	+ ,004
55	— ,032	— ,029	— ,026	— ,022	— ,019	— ,016	— ,012	— ,009	— ,006	— ,002	+ ,001
56	— ,035	— ,032	— ,028	— ,025	— ,022	— ,018	— ,015	— ,012	— ,008	— ,005	— ,002
57	— ,037	— ,034	— ,031	— ,027	— ,024	— ,021	— ,017	— ,014	— ,011	— ,007	— ,004
58	— ,040	— ,036	— ,033	— ,030	— ,026	— ,023	— ,020	— ,016	— ,013	— ,010	— ,006
59	— ,042	— ,039	— ,035	— ,032	— ,029	— ,025	— ,022	— ,019	— ,015	— ,012	— ,009
60	— ,044	— ,041	— ,038	— ,034	— ,031	— ,028	— ,024	— ,021	— ,018	— ,015	— ,011
61	— ,047	— ,043	— ,040	— ,037	— ,033	— ,030	— ,027	— ,024	— ,020	— ,017	— ,014
62	— ,049	— ,046	— ,042	— ,039	— ,036	— ,032	— ,029	— ,026	— ,023	— ,019	— ,016
63	— ,051	— ,048	— ,045	— ,041	— ,038	— ,035	— ,032	— ,028	— ,025	— ,022	— ,018
64	— ,053	— ,050	— ,047	— ,044	— ,040	— ,037	— ,034	— ,031	— ,027	— ,024	— ,021
65	— ,056	— ,052	— ,049	— ,046	— ,043	— ,039	— ,036	— ,033	— ,030	— ,026	— ,023
66	— ,058	— ,055	— ,052	— ,048	— ,045	— ,042	— ,039	— ,035	— ,032	— ,029	— ,026
67	— ,060	— ,057	— ,054	— ,051	— ,047	— ,044	— ,041	— ,038	— ,034	— ,031	— ,028
68	— ,063	— ,059	— ,056	— ,053	— ,050	— ,046	— ,043	— ,040	— ,037	— ,033	— ,030
69	— ,065	— ,062	— ,058	— ,055	— ,052	— ,049	— ,045	— ,042	— ,039	— ,036	— ,032
70	— ,067	— ,064	— ,061	— ,057	— ,054	— ,051	— ,048	— ,044	— ,041	— ,038	— ,035
71	— ,069	— ,066	— ,063	— ,060	— ,056	— ,053	— ,050	— ,047	— ,044	— ,040	— ,037
72	— ,071	— ,068	— ,065	— ,062	— ,059	— ,055	— ,052	— ,049	— ,046	— ,043	— ,039
73	— ,074	— ,070	— ,067	— ,064	— ,061	— ,058	— ,054	— ,051	— ,048	— ,045	— ,042
74	— ,076	— ,073	— ,069	— ,066	— ,063	— ,060	— ,057	— ,053	— ,050	— ,047	— ,044
75	— ,078	— ,075	— ,072	— ,068	— ,065	— ,062	— ,059	— ,056	— ,053	— ,049	— ,046
76	— ,080	— ,077	— ,074	— ,071	— ,067	— ,064	— ,061	— ,058	— ,055	— ,052	— ,048
77	— ,082	— ,079	— ,076	— ,073	— ,070	— ,066	— ,063	— ,060	— ,057	— ,054	— ,051
78	— ,084	— ,081	— ,078	— ,075	— ,072	— ,069	— ,065	— ,062	— ,059	— ,056	— ,053
79	— ,087	— ,083	— ,080	— ,077	— ,074	— ,071	— ,068	— ,065	— ,061	— ,058	— ,055
80	— ,089	— ,086	— ,082	— ,079	— ,076	— ,073	— ,070	— ,067	— ,064	— ,060	— ,057

TABLE XII. *Continued.*

Bar.	30,1	30,2	30,3	30,4	30,5	30,6	30,7	30,8	30,9	31,0
Ther.										
50°	+,017	+,020	+,024	+,027	+,030	+,034	+,037	+,041	+,044	+,047
51	+,014	+,018	+,021	+,025	+,028	+,031	+,035	+,038	+,041	+,045
52	+,012	+,015	+,019	+,022	+,025	+,029	+,032	+,035	+,039	+,042
53	+,009	+,013	+,016	+,019	+,023	+,026	+,030	+,033	+,036	+,040
54	+,007	+,010	+,014	+,017	+,020	+,024	+,027	+,030	+,034	+,037
55	+,004	+,008	+,011	+,014	+,018	+,021	+,024	+,028	+,031	+,034
56	+,002	+,005	+,009	+,012	+,015	+,019	+,022	+,025	+,029	+,032
57	—,001	+,003	+,006	+,009	+,013	+,016	+,019	+,023	+,026	+,029
58	—,003	,000	+,004	+,007	+,010	+,014	+,017	+,020	+,024	+,027
59	—,006	—,002	+,001	+,004	+,008	+,011	+,014	+,018	+,021	+,024
60	—,008	—,005	—,001	+,002	+,005	+,009	+,012	+,015	+,019	+,022
61	—,010	—,007	—,004	—,001	+,003	+,006	+,009	+,013	+,016	+,019
62	—,013	—,009	—,006	—,003	,000	+,004	+,007	+,010	+,014	+,017
63	—,015	—,012	—,009	—,005	—,002	+,001	+,005	+,008	+,011	+,014
64	—,018	—,014	—,011	—,008	—,004	—,001	+,002	+,005	+,009	+,012
65	—,020	—,017	—,013	—,010	—,007	—,004	,000	+,003	+,006	+,010
66	—,022	—,019	—,016	—,013	—,009	—,006	—,003	+,001	+,004	+,007
67	—,025	—,021	—,018	—,015	—,012	—,008	—,005	—,002	+,001	+,005
68	—,027	—,024	—,020	—,017	—,014	—,011	—,008	—,004	—,001	+,002
69	—,029	—,026	—,023	—,020	—,016	—,013	—,010	—,007	—,004	,000
70	—,032	—,028	—,025	—,022	—,019	—,015	—,012	—,009	—,006	—,003
71	—,034	—,031	—,027	—,024	—,021	—,018	—,015	—,011	—,008	—,005
72	—,036	—,033	—,030	—,027	—,023	—,020	—,017	—,014	—,011	—,007
73	—,038	—,035	—,032	—,029	—,026	—,022	—,019	—,016	—,013	—,010
74	—,041	—,038	—,034	—,031	—,028	—,025	—,022	—,018	—,015	—,012
75	—,043	—,040	—,037	—,033	—,030	—,027	—,024	—,021	—,017	—,014
76	—,045	—,042	—,039	—,036	—,032	—,029	—,026	—,023	—,020	—,017
77	—,047	—,044	—,041	—,038	—,035	—,032	—,028	—,025	—,022	—,019
78	—,050	—,047	—,043	—,040	—,037	—,034	—,031	—,028	—,024	—,021
79	—,052	—,049	—,046	—,042	—,039	—,036	—,033	—,030	—,027	—,023
80	—,054	—,051	—,048	—,045	—,042	—,038	—,035	—,032	—,029	—,026

TABLE XIII.

Augmentation of the Semi-Diameter of the Moon.

Apparent Altitude.	HORIZONTAL DIAMETER OF THE MOON.													App. Zenith Distance.
	29′ 30″	29′ 50′	30′ 10	30′ 30″	30′ 50′	31′ 10″	31′ 30′	31′ 50″	32′ 10″	32′ 30″	32′ 50″	33′ 10″	33′ 30″	
Deg.	″	″	″	″	″	″	″	″	″	″	″	″	″	Deg.
0	0, 3	0, 3	0, 3	0, 3	0, 3	0, 3	0, 3	0, 3	0, 3	0, 4	0, 4	0, 4	0, 4	90
2	1, 2	1, 3	1, 3	1, 3	1, 4	1, 4	1, 4	1, 5	1, 5	1, 5	1, 6	1, 6	1, 6	88
4	2, 2	2, 3	2, 3	2, 4	2, 4	2, 5	2, 5	2, 6	2, 6	2, 7	2, 8	2, 8	2, 9	86
6	3, 2	3, 3	3, 3	3, 4	3, 5	3, 6	3, 6	3, 7	3, 8	3, 9	4, 0	4, 0	4, 1	84
8	4, 1	4, 2	4, 3	4, 4	4, 5	4, 6	4, 7	4, 8	4, 9	5, 0	5, 2	5, 2	5, 4	82
10	5, 1	5, 2	5, 3	5, 5	5, 6	5, 7	5, 8	6, 0	6, 1	6, 2	6, 3	6, 5	6, 6	80
11	5, 6	5, 7	5, 8	6, 0	6, 1	6, 2	6, 4	6, 5	6, 7	6, 8	6, 9	7, 1	7, 2	79
12	6, 0	6, 2	6, 4	6, 5	6, 6	6, 8	6, 9	7, 1	7, 2	7, 4	7, 5	7, 7	7, 8	78
13	6, 5	6, 6	6, 8	7, 0	7, 1	7, 3	7, 5	7, 6	7, 8	7, 9	8, 1	8, 3	8, 4	77
14	7, 0	7, 1	7, 3	7, 5	7, 7	7, 8	8, 0	8, 2	8, 3	8, 5	8, 7	8, 9	9, 0	76
15	7, 4	7, 6	7, 8	8, 0	8, 2	8, 4	8, 5	8, 7	8, 9	9, 1	9, 3	9, 5	9, 6	75
16	7, 9	8, 1	8, 3	8, 5	8, 7	8, 9	9, 1	9, 3	9, 5	9, 7	9, 9	10, 1	10, 3	74
17	8, 4	8, 6	8, 8	9, 0	9, 2	9, 4	9, 6	9, 8	10, 0	10, 2	10, 4	10, 6	10, 8	73
18	8, 8	9, 1	9, 3	9, 5	9, 7	9, 9	10, 1	10, 4	10, 6	10, 8	11, 0	11, 2	11, 4	72
19	9, 3	9, 5	9, 8	10, 0	10, 2	10, 4	10, 7	10, 9	11, 1	11, 4	11, 6	11, 8	12, 0	71
20	9, 8	10, 0	10, 2	10, 5	10 7	11, 0	11, 2	11, 4	11, 7	11, 9	12, 2	12, 4	12, 6	70
21	10, 2	10, 5	10, 7	11, 0	11, 2	11, 5	11, 7	12, 0	12, 2	12, 5	12, 7	13, 0	13, 2	69
22	10, 7	10, 9	11, 2	11, 5	11, 7	12, 0	12, 2	12, 5	12, 8	13, 0	13, 3	13, 6	13, 8	68
23	11, 1	11, 4	11, 7	11, 9	12, 2	12, 5	12, 8	13, 0	13, 3	13, 6	13, 9	14, 1	14, 4	67
24	11, 6	11, 9	12, 1	12, 4	12, 7	13, 0	13, 3	13, 5	13, 8	14, 1	14, 4	14, 7	15, 0	66
25	12, 0	12, 3	12, 6	12, 9	13, 2	13, 5	13, 8	14, 1	14, 4	14, 7	14, 9	15, 2	15, 5	65
26	12, 4	12, 8	13, 0	13, 4	13, 7	14, 0	14, 3	14, 6	14, 9	15, 2	15, 5	15, 8	16, 1	64
27	12, 9	13, 2	13, 5	13, 8	14, 1	14, 5	14, 8	15, 1	15, 4	15, 7	16, 0	16, 4	16, 7	63
28	13, 3	13, 6	14, 0	14, 3	14, 6	15, 0	15, 3	15, 6	15, 9	16, 2	16, 6	16, 9	17, 2	62
29	13, 7	14, 1	14, 4	14, 7	15, 1	15, 4	15, 8	16, 1	16, 4	16, 8	17, 1	17, 4	17, 8	61
30	14, 2	14, 5	14, 9	15, 2	15, 6	15, 9	16, 2	16, 6	16, 9	17, 3	17, 6	18, 0	18, 3	60
31	14, 6	14, 9	15, 3	15, 6	16, 0	16, 4	16, 7	17, 1	17, 4	17, 8	18, 2	18, 5	18, 9	59
32	15, 0	15, 4	15, 7	16, 1	16, 5	16, 8	17, 2	17, 6	17, 9	18, 3	18, 7	19 0	19, 4	58
33	15, 4	15, 8	16, 2	16, 5	16, 9	17, 3	17, 7	18, 1	18, 4	18, 8	19, 2	19, 6	19, 9	57
34	15, 8	16, 2	16, 6	17, 0	17, 4	17, 8	18, 1	18, 5	18, 9	19, 3	19, 7	20, 1	20, 5	56
35	16, 2	16, 6	17, 0	17, 4	17, 8	18, 2	18, 6	19, 0	19, 4	19, 8	20, 2	20, 6	21, 0	55
36	16, 6	17, 0	17, 4	17, 8	18, 2	18, 6	19, 1	19, 5	19, 9	20, 3	20, 7	21, 1	21, 5	54
37	17, 0	17, 4	17, 8	18, 3	18, 7	19, 1	19, 5	19, 9	20, 3	20, 7	21, 2	21, 6	22, 0	53
38	17, 4	17, 8	18, 2	18, 7	19, 1	19, 5	20, 0	20, 4	20, 8	21, 2	21, 6	22, 1	22, 5	52
39	17, 8	18, 2	18, 6	19, 1	19, 5	20, 0	20, 4	20, 8	21, 2	21, 7	22, 1	22, 5	23, 0	51
40	18, 2	18, 6	19, 0	19, 5	19, 9	20, 4	20, 8	21, 2	21, 7	22, 2	22, 6	23, 0	23, 5	50

TABLE XIII. *Continued.*

Apparent Altitude.	HORIZONTAL DIAMETER OF THE MOON.													App. Zenith Distance.
	29′ 30″	29′ 50″	30′ 10″	30′ 30″	30′ 50″	31′ 10″	31′ 30″	31′ 50″	32′ 10″	32′ 30″	32′ 50″	33′ 10″	33′ 30″	
Deg.	″	″	″	″	″	″	″	″	″	″	″	″	″	Deg.
40	18, 2	18, 6	19. 0	19, 5	19, 9	20, 4	20, 8	21, 2	21, 7	22, 2	22, 6	23, 0	23, 5	50
41	18, 5	19, 0	19, 4	19, 9	20, 3	20, 8	21, 2	21, 7	22, 1	22, 6	23, 0	23, 5	24, 0	49
42	18, 9	19, 3	19, 8	20, 3	20, 7	21, 2	21, 7	22, 1	22, 6	23, 0	23, 5	24, 0	24, 4	48
43	19, 2	19, 7	20, 2	20, 7	21, 1	21, 6	22, 1	22, 6	23, 0	23, 5	23, 9	24, 4	24, 9	47
44	19, 6	20, 1	20, 6	21, 0	21, 5	22, 0	22, 5	23, 0	23, 4	23, 9	24, 4	24, 9	25, 4	46
45	19, 9	20, 4	20, 9	21, 4	21, 9	22, 4	22, 9	23, 4	23, 8	24, 3	24, 8	25, 3	25, 8	45
46	20, 3	20, 8	21, 3	21, 8	22, 3	22, 8	23, 3	23, 8	24, 3	24, 8	25, 3	25, 8	26, 3	44
47	20. 6	21, 1	21, 6	22, 1	22, 6	23, 1	23, 6	24, 1	24, 7	25, 2	25, 7	26, 2	26, 7	43
48	20, 9	21, 5	22, 0	22, 5	23, 0	23, 5	24. 0	24, 5	25, 1	25, 5	26, 1	26, 6	27, 1	42
49	21, 3	21, 8	22, 3	22, 8	23, 3	23, 9	24, 4	24, 9	25. 4	26, 0	26, 5	27, 0	27, 5	41
50	21, 6	22, 1	22, 6	23, 2	23, 7	24, 2	24, 8	25, 3	25, 8	26, 4	26, 9	27, 4	27, 9	40
51	21, 9	22, 4	23, 0	23, 5	24, 0	24, 6	25, 1	25, 7	26, 2	26, 7	27, 3	27, 8	28, 3	39
52	22, 2	22, 7	23, 3	23, 8	24, 4	24, 9	25, 5	26, 0	26, 5	27, 1	27, 6	28, 2	28, 7	38
53	22, 5	23, 0	23, 6	24, 1	24, 7	25, 2	25, 8	26, 3	26, 9	27, 5	28, 0	28, 6	29, 1	37
54	22, 8	23, 4	23, 9	24, 5	25, 0	25, 6	26, 1	26, 7	27, 3	27, 8	28, 4	28, 9	29, 5	36
55	23, 1	23, 6	24, 2	24, 8	25, 3	25, 9	26, 5	27, 0	27, 6	28, 2	28, 7	29, 3	29, 8	35
56	23, 4	23, 9	24, 5	25, 1	25, 6	26, 2	26, 8	27, 3	27, 9	28, 5	29, 1	29, 6	30, 2	34
57	23, 6	24, 2	24, 8	25, 3	25, 9	26, 5	27, 1	27, 7	28, 2	28, 8	29, 4	30, 0	30, 5	33
58	23, 9	24, 5	25, 1	25, 6	26. 2	26, 8	27, 4	28, 0	28, 6	29, 1	29, 7	30, 3	30, 9	32
59	24, 1	24, 7	25, 3	25, 9	26, 5	27, 1	27, 7	28, 3	28, 9	29, 5	30, 0	30, 6	31, 2	31
60	24, 4	25, 0	25, 6	26, 2	26, 8	27, 4	28, 0	28, 6	29, 2	29, 8	30, 4	31, 0	31, 6	30
62	24, 9	25, 5	26, 1	26, 7	27, 3	27, 9	28, 5	29, 1	29, 7	30, 3	30, 9	31, 6	32, 2	28
64	25, 3	25, 9	26, 5	27. 2	27, 8	28, 4	29, 0	29, 6	30, 3	30, 9	31, 5	32, 1	32, 7	26
66	25, 7	26, 3	27, 0	27, 6	28, 2	28, 9	29, 5	30, 1	30, 7	31, 4	32, 0	32, 6	33, 3	24
68	26, 1	26, 7	27, 4	28, 0	28, 7	29, 3	29, 9	30, 6	31, 2	31, 8	32, 5	33, 1	33, 7	22
70	26, 5	27, 1	27, 7	28, 4	29, 3	29, 7	30, 3	31, 0	31, 6	32, 3	32, 9	33, 6	34, 2	20
72	26, 8	27, 4	28, 1	28, 7	29, 4	30, 0	30, 7	31, 3	32, 0	32, 7	33, 3	34, 0	34, 6	18
74	27, 0	27, 7	28, 4	29, 0	29, 7	30, 4	31, 0	31, 7	32, 3	33, 0	33, 7	34, 3	35, 0	16
76	27, 3	27, 9	28, 6	29, 3	30, 0	30, 6	31, 3	32, 0	32. 6	33, 3	34, 0	34, 7	35, 3	14
78	27, 5	28, 2	28, 9	29, 5	30, 2	30, 9	31, 6	32, 2	32, 9	33, 6	34, 2	34, 9	35, 6	12
80	27, 7	28, 4	29, 1	29, 7	30, 4	31, 1	31, 8	32, 4	33, 1	33, 8	34, 5	35, 2	35, 8	10
82	27, 9	28, 5	29, 2	29, 9	30, 6	31, 3	31, 9	32, 6	33, 3	34, 0	34, 7	35, 3	36, 0	8
84	28, 0	28. 7	29, 3	30, 0	30, 7	31, 4	32, 1	32, 8	33, 4	34, 1	34, 8	35, 5	36, 2	6
86	28, 1	28, 7	29, 4	30, 1	30, 8	31, 5	32, 2	32, 9	33, 6	34, 2	34, 9	35, 6	36, 3	4
88	28, 1	28, 8	29.5	30, 2	30, 9	31, 5	32, 2	32, 9	33, 6	34, 3	35, 0	35, 7	36, 4	2
90	28. 1	28, 8	29, 5	30, 2	30, 9	31, 6	32, 3	33, 0	33, 6	34, 3	35, 0	35, 7	36, 4	0

TABLE XIV.

Mean Precession of the Equinoctial Points in Longitude, for complete Years.

Years.	Precession.	Years.	Precession.	Years.	Precession.	Years.	Precession.
1	0′. 50″,3	31	26′. 0″,8	61	0°. 51′. 11″,3	91	1°. 16′. 21″,8
2	1. 40, 7	32	26. 51, 2	62	0. 52. 1, 7	92	1. 17. 12, 2
3	2. 31, 0	33	27. 41, 5	63	0. 52. 52, 0	93	1. 18. 2, 5
4	3. 21, 4	34	28. 31, 9	64	0. 53. 42, 4	94	1. 18. 52, 9
5	4. 11, 7	35	29. 22, 2	65	0. 54. 32, 7	95	1. 19. 43, 2
6	5. 2, 1	36	30. 12, 6	66	0. 55. 23, 1	96	1. 20. 33, 6
7	5. 52, 4	37	31. 2, 9	67	0. 56. 13, 4	97	1. 21. 23, 9
8	6. 42, 8	38	31. 53, 3	68	0. 57. 3, 8	98	1. 22. 14, 3
9	7. 33, 1	39	32. 43, 6	69	0. 57. 54, 1	99	1. 23. 4, 6
10	8. 23, 5	40	33. 34, 0	70	0. 58. 44, 5	100	1. 23. 55, 0
11	9. 13, 8	41	34. 24, 3	71	0 59. 34, 8	200	2. 47. 49, 9
12	10. 4, 2	42	35. 14, 7	72	1. 0. 25, 2	300	4. 11. 44, 9
13	10. 54, 5	43	36. 5, 0	73	1. 1. 15, 5	400	5. 35. 39, 9
14	11. 44, 9	44	36. 55, 4	74	1. 2. 5, 9	500	6. 59. 34, 8
15	12. 35, 2	45	37. 45, 7	75	1. 2. 56, 2	600	8. 23. 29, 8
16	13. 25, 6	46	38. 36, 1	76	1. 3. 46, 6	700	9. 47. 24, 8
17	14. 15, 9	47	39. 26, 4	77	1. 4. 36, 9	800	11. 11. 19, 7
18	15. 6, 3	48	40. 16, 8	78	1. 5. 27, 3	900	12. 35. 14, 7
19	15. 56, 6	49	41. 7, 1	79	1. 6. 17, 6	1000	13. 59. 9, 7
20	16. 47, 0	50	41. 57, 5	80	1. 7. 8, 0	2000	27. 58. 19, 3
21	17. 37, 3	51	42. 47, 8	81	1. 7. 58, 3	3000	41. 57. 29, 0
22	18. 27, 7	52	43. 38, 2	82	1. 8. 48, 7	4000	55. 56. 38, 6
23	19. 18, 0	53	44. 28, 5	83	1. 9. 39, 0	5000	69. 55. 48, 3
24	20. 8, 4	54	45. 18, 9	84	1. 10. 29, 4		
25	20. 58, 7	55	46. 9, 2	85	1. 11. 19, 7		
26	21. 49, 1	56	46. 59, 6	86	1. 12. 10, 1		
27	22. 39, 4	57	47. 49, 9	87	1. 13. 0, 4		
28	23. 29, 8	58	48. 40, 3	88	1. 13. 50, 8		
29	24. 20, 1	59	49. 30, 6	89	1. 14. 41, 1		
30	25. 10, 5	60	50. 21, 0	90	1. 15. 31, 5		

TABLE XV.

Precession of the Equinoctial Points in Longitude to every Day in the Year, including the Solar Equation of Precession.

Days	January.	February.	March.	April.	May.	June.	July.	August.	September	October.	November	December
1	0″,6	5″,6	9″,0	12″,1	15″,5	20″,2	25″,5	30″,5	34″,4	37″,4	40″,9	45″,4
2	0, 8	5, 7	9, 1	12, 2	15, 7	20, 4	25, 6	30, 6	34, 5	37, 5	41, 0	45, 6
3	1, 0	5, 8	9, 2	12, 3	15, 8	20, 6	25, 8	30, 8	34, 6	37, 6	41, 2	45, 8
4	1, 1	6, 0	9, 3	12, 4	15, 9	20, 7	26, 0	30, 9	34, 7	37, 7	41, 3	45, 9
5	1, 3	6, 1	9, 4	12, 5	16, 1	20, 9	26, 2	31, 1	34, 8	37, 8	41, 4	46, 1
6	1, 5	6, 3	9, 5	12, 6	16, 2	21, 1	26, 3	31, 2	34, 9	37, 9	41, 6	46, 3
7	1, 6	6, 4	9, 6	12, 7	16, 3	21, 3	26, 5	31, 3	35, 0	38, 0	41, 7	46, 4
8	1, 8	6, 5	9, 7	12, 8	16, 5	21, 4	26, 7	31, 5	35, 2	38, 2	41, 9	46, 6
9	2, 0	6, 7	9, 8	12, 9	16, 6	21, 6	26, 8	31, 6	35, 3	38, 3	42, 0	46, 8
10	2, 1	6, 8	9, 9	13, 0	16, 8	21, 8	27, 0	31, 7	35, 4	38, 4	42, 1	47, 0
11	2, 3	6, 9	10, 0	13, 1	16, 9	22, 0	27, 2	31, 9	35, 5	38, 5	42, 3	47, 1
12	2, 5	7, 0	10, 1	13, 3	17, 1	22, 1	27, 4	32, 0	35, 6	38, 6	42, 4	47, 3
13	2, 6	7, 2	10, 2	13, 4	17, 2	22, 3	27, 5	32, 2	35, 7	38, 7	42, 6	47, 5
14	2, 8	7, 3	10, 3	13, 5	17, 4	22, 5	27, 7	32, 3	35, 8	38, 8	42, 7	47, 7
15	3, 0	7, 4	10, 4	13, 6	17, 5	22, 7	27, 8	32, 4	35, 9	38, 9	42, 9	47, 8
16	3, 1	7, 5	10, 5	13, 7	17, 7	22, 8	28, 0	32, 5	36, 0	39, 0	43, 0	48, 0
17	3, 3	7, 6	10, 6	13, 8	17, 8	23, 0	28, 2	32, 7	36, 1	39, 1	43, 2	48, 2
18	3, 4	7, 8	10, 7	13, 9	18, 0	23, 2	28, 3	32, 8	36, 2	39, 2	43, 3	48, 4
19	3, 6	7, 9	10, 8	14, 0	18, 1	23, 4	28, 5	32, 9	36, 3	39, 3	43, 5	48, 6
20	3, 8	8, 0	10, 9	14, 2	18, 3	23, 5	28, 7	33, 0	36, 4	39, 4	43, 6	48, 7
21	3, 9	8, 1	11, 0	14, 3	18, 4	23, 7	28, 8	33, 2	36, 5	39, 6	43, 8	48, 9
22	4, 1	8, 2	11, 1	14, 4	18, 6	23, 9	29, 0	33, 3	36, 6	39, 7	43, 9	49, 1
23	4, 2	8, 3	11, 2	14, 5	18, 8	24, 1	29, 1	33, 4	36, 7	39, 8	44, 1	49, 3
24	4, 4	8, 4	11, 3	14, 6	18, 9	24, 2	29, 3	33, 5	36, 8	39, 9	44, 3	49, 5
25	4, 5	8, 6	11, 4	14, 8	19, 1	24, 4	29, 4	33, 6	36, 9	40, 0	44, 4	49, 6
26	4, 7	8, 7	11, 5	14, 9	19, 2	24, 6	29, 6	33, 7	37, 0	40, 2	44, 6	49, 8
27	4, 8	8, 8	11, 6	15, 0	19, 4	24, 8	29, 7	33, 9	37, 1	40, 3	44, 8	50, 0
28	5, 0	8, 9	11, 7	15, 1	19, 6	24, 9	29, 9	34, 0	37, 2	40, 4	44, 9	50, 2
29	5, 1		11, 8	15, 3	19, 7	25, 1	30, 0	34, 1	37, 2	40, 5	45, 1	50, 4
30	5, 3		11, 9	15, 4	19, 9	25, 3	30, 2	34, 2	37, 3	40, 7	45, 3	50, 5
31	5, 4		12, 0		20, 1		30, 3	34, 3		40, 8		50, 7

In the months of January and February in leap-year, take out for the day preceding the given day.

TABLE XVI.

The Equation of the Equinoxes in Longitude.

ARGUMENT. Long. of ☽'s ☊.

Degrees.	O. Sig. — / VI. Sig. +	I. Sig. — / VII. Sig. +	II. Sig. — / VIII. Sig. +	
0	0′,0	8′,9	15′,5	30
1	0, 3	9, 2	15, 6	29
2	0, 6	9, 5	15, 8	28
3	0, 9	9, 7	15, 9	27
4	1, 2	10, 0	16, 1	26
5	1, 6	10, 2	16, 2	25
6	1, 9	10, 5	16, 3	24
7	2, 2	10, 8	16, 4	23
8	2, 5	11, 0	16, 6	22
9	2, 8	11, 2	16, 7	21
10	3, 1	11, 5	16, 8	20
11	3, 4	11, 7	16, 9	19
12	3, 7	12, 0	17, 0	18
13	4, 0	12, 2	17, 1	17
14	4, 3	12, 4	17, 2	16
15	4, 6	12, 6	17, 3	15
16	4, 9	12, 8	17, 3	14
17	5, 2	13, 1	17, 4	13
18	5, 5	13, 3	17, 5	12
19	5, 8	13, 5	17, 5	11
20	6, 1	13, 7	17, 6	10
21	6, 4	13, 9	17, 6	9
22	6, 7	14, 1	17, 7	8
23	7, 0	14, 3	17, 7	7
24	7, 3	14, 5	17, 8	6
25	7, 5	14, 6	17, 8	5
26	7, 8	14, 8	17, 8	4
27	8, 1	15, 0	17, 8	3
28	8, 4	15, 1	17, 9	2
29	8, 7	15, 3	17, 9	1
30	8, 9	15, 5	17, 9	0
	— V. Sig. / + XI. Sig.	— IV. Sig. / + X. Sig.	— III. Sig. / + IX. Sig.	Degrees.

TABLE XVII.

The Mean Motions of the Sun in Right Ascension in Time, *to every Day in the Year.*

Days.	January.	February.	March.	April.	May.	June.
	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.
1	0. 3. 56, 6	2. 6. 9, 8	3. 56. 33, 3	5. 58. 46, 5	7. 57. 3, 2	9. 59. 16, 4
2	0. 7. 53, 1	2. 10. 6, 3	4. 0. 29, 9	6. 2. 43, 1	8. 0. 59, 8	10. 3. 13, 0
3	0. 11. 49, 7	2. 14. 2, 9	4. 4. 26, 4	6. 6. 39, 7	8. 4. 56, 3	10. 7. 9, 5
4	0. 15. 46, 2	2. 17. 59, 4	4. 8. 23, 0	6. 10. 36, 2	8. 8. 52, 9	10. 11. 6, 1
5	0. 19. 42, 8	2. 21. 56, 0	4. 12. 19, 5	6. 14. 32, 8	8. 12. 49, 4	10. 15. 2, 6
6	0. 23. 39, 3	2. 25. 52, 5	4. 16. 16, 1	6. 18. 29, 3	8. 16. 46, 0	10. 18. 59, 2
7	0. 27. 35, 9	2. 29. 49, 1	4. 20. 12, 7	6. 22. 25, 9	8. 20. 42, 5	10. 22. 55, 7
8	0. 31. 32, 4	2. 33. 45, 7	4. 24. 9, 2	6. 26. 22, 4	8. 24. 39, 1	10. 26. 52, 3
9	0. 35. 29, 0	2. 37. 42, 2	4. 28. 5, 8	6. 30. 19, 0	8. 28. 35, 6	10. 30. 48, 9
10	0. 39. 25, 6	2. 41. 38, 8	4. 32. 2, 3	6. 34. 15, 5	8. 32. 32, 2	10. 34. 45, 4
11	0. 43. 22, 1	2. 45. 35, 3	4. 35. 58, 9	6. 38. 12, 1	8. 36. 28, 8	10. 38. 42, 0
12	0. 47. 18, 7	2. 49. 31, 9	4. 39. 55, 4	6. 42. 8, 6	8. 40. 25, 3	10. 42. 38, 5
13	0. 51. 15, 2	2. 53. 28, 4	4. 43. 52, 0	6. 46. 5, 2	8. 44. 21, 9	10. 46. 35, 1
14	0. 55. 11, 8	2. 57. 25, 0	4. 47. 48, 5	6. 50. 1, 8	8. 48. 18, 4	10. 50. 31, 6
15	0. 59. 8, 3	3. 1. 21, 5	4. 51. 45, 1	6. 53. 58, 3	8. 52. 15, 0	10. 54. 28, 2
16	1. 3. 4, 9	3. 5. 18, 1	4. 55. 41, 6	6. 57. 54, 9	8. 56. 11, 5	10. 58. 24, 7
17	1. 7. 1, 4	3. 9. 14, 7	4. 59. 38, 2	7. 1. 51, 4	9. 0. 8, 1	11. 2. 21, 3
18	1. 10. 58, 0	3. 13. 11, 2	5. 3. 34, 8	7. 5. 48, 0	9. 4. 4, 6	11. 6. 17, 9
19	1. 14. 54, 5	3. 17. 7, 8	5. 7. 31, 3	7. 9. 44, 5	9. 8. 1, 2	11. 10. 14, 4
20	1. 18. 51, 1	3. 21. 4, 3	5. 11. 27, 9	7. 13. 41, 1	9. 11. 57, 7	11. 14. 11, 0
21	1. 22. 47, 7	3. 25. 0, 9	5. 15. 24, 4	7. 17. 37, 6	9. 15. 54, 3	11. 18. 7, 5
22	1. 26. 44, 2	3. 28. 57, 4	5. 19. 21, 0	7. 21. 34, 2	9. 19. 50, 9	11. 22. 4, 1
23	1. 30. 40, 8	3. 32. 54, 0	5. 23. 17, 5	7. 25. 30, 8	9. 23. 47, 4	11. 26. 0, 6
24	1. 34. 37, 3	3. 36. 50, 5	5. 27. 14, 1	7. 29. 27, 3	9. 27. 44, 0	11. 29. 57, 2
25	1. 38. 33, 9	3. 40. 47, 1	5. 31. 10, 6	7. 33. 23, 9	9. 31. 40, 5	11. 33. 53, 7
26	1. 42. 30, 4	3. 44. 43, 6	5. 35. 7, 2	7. 37. 20, 4	9. 35. 37, 1	11. 37. 50, 3
27	1. 46. 27, 0	3. 48. 40, 2	5. 39. 3, 8	7. 41. 17, 0	9. 39. 33, 6	11. 41. 46, 9
28	1. 50. 23, 5	3. 52. 36, 8	5. 43. 0, 3	7. 45. 13, 5	9. 43. 30, 2	11. 45. 43, 4
29	1. 54. 20, 1		5. 46. 56, 9	7. 49. 10, 1	9. 47. 26, 7	11. 49. 40, 0
30	1. 58. 16, 7		5. 50. 53, 4	7. 53. 6, 6	9. 51. 23, 3	11. 53. 36, 5
31	2. 2. 13, 2		5. 54. 50, 0		9. 55. 19, 9	

In the months of January and February in leap-year, take out for the day preceding the given day.

TABLE XVII. *Continued.*

Days.	July.	August.	September.	October.	November.	December.
	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.	H. M. S.
1	11. 57. 33, 1	13. 59. 46, 3	16. 1. 59, 5	18. 0. 16, 2	20. 2. 29, 4	22. 0. 46, 0
2	12. 1. 29, 6	14. 3. 42, 8	16. 5. 56, 1	18. 4. 12, 7	20. 6. 26, 0	22. 4. 42, 6
3	12. 5. 26, 2	14. 7. 39, 4	16. 9. 52, 6	18. 8. 9, 3	20. 10. 22, 5	22. 8. 39, 2
4	12. 9. 22, 7	14. 11. 36, 0	16. 13. 49, 2	18. 12. 5, 8	20. 14. 19, 1	22. 12. 35, 7
5	12. 13. 19, 3	14. 15. 32, 5	16. 17. 45, 7	18. 16. 2, 4	20. 18. 15, 6	22. 16. 32, 3
6	12. 17. 15, 9	14. 19. 29, 1	16. 21. 42, 3	18. 19. 59, 0	20. 22. 12, 2	22. 20. 28, 8
7	12. 21. 12, 4	14. 23. 25, 6	16. 25. 38, 8	18. 23. 55, 5	20. 26. 8, 7	22. 24. 25, 4
8	12. 25. 9, 0	14. 27. 22, 2	16. 29. 35, 4	18. 27. 52, 1	20. 30. 5, 3	22. 28. 21, 9
9	12. 29. 5, 5	14. 31. 18, 7	16. 33. 32, 0	18. 31. 48, 6	20. 34. 1, 8	22. 32. 18, 5
10	12. 33. 2, 1	14. 35. 15, 3	16. 37. 28, 5	18. 35. 45, 2	20. 37. 58, 4	22. 36. 15, 0
11	12. 36. 58, 6	14. 39. 11, 8	16. 41. 25, 1	18. 39. 41, 7	20. 41. 54, 9	22. 40. 11, 6
12	12. 40. 55, 2	14. 43. 8, 4	16. 45. 21, 6	18. 43. 38, 3	20. 45. 51, 5	22. 44. 8, 2
13	12. 44. 51, 7	14. 47. 5, 0	16. 49. 18, 2	18. 47. 34, 8	20. 49. 48, 1	22. 48. 4, 7
14	12. 48. 48, 3	14. 51. 1, 5	16. 53. 14, 7	18. 51. 31, 4	20. 53. 44, 6	22. 52. 1, 3
15	12. 52. 44, 9	14. 54. 58, 1	16. 57. 11, 3	18. 55. 27, 9	20. 57. 41, 2	22. 55. 57, 8
16	12. 56. 41, 4	14. 58. 54, 6	17. 1. 7, 8	18. 59. 24, 5	21. 1. 37, 7	22. 59. 54, 4
17	13. 0. 38, 0	15. 2. 51, 2	17. 5. 4, 4	19. 3. 21, 1	21. 5. 34, 3	23. 3. 50, 9
18	13. 4. 34, 5	15. 6. 47, 7	17. 9. 1, 0	19. 7. 17, 6	21. 9. 30, 8	23. 7. 47, 5
19	13. 8. 31, 1	15. 10. 44, 3	17. 12. 57, 5	19. 11. 14, 2	21. 13. 27, 4	23. 11. 44, 0
20	13. 12. 27, 6	15. 14. 40, 8	17. 16. 54, 1	19. 15. 10, 7	21. 17. 23, 9	23. 15. 40, 6
21	13. 16. 24, 2	15. 18. 37, 4	17. 20. 50, 6	19. 19. 7, 3	21. 21. 20, 5	23. 19. 37, 1
22	13. 20. 20, 7	15. 22. 34, 0	17. 24. 47, 2	19. 23. 3, 8	21. 25. 17, 1	23. 23. 33, 7
23	13. 24. 17, 3	15. 26. 30, 5	17. 28. 43, 7	19. 27. 0, 4	21. 29. 13, 6	23. 27. 30, 3
24	13. 28. 13, 8	15. 30. 27, 1	17. 32. 40, 3	19. 30. 56, 9	21. 33. 10, 2	23. 31. 26, 8
25	13. 32. 10, 4	15. 34. 23, 6	17. 36. 36, 8	19. 34. 53, 5	21. 37. 6, 7	23. 35. 23, 4
26	13. 36. 7, 0	15. 38. 20, 2	17. 40. 33, 4	19. 38. 50, 1	21. 41. 3, 3	23. 39. 19, 9
27	13. 40. 3, 5	15. 42. 16, 7	17. 44. 29, 9	19. 42. 46, 6	21. 44. 59, 8	23. 43. 16, 5
28	13. 44. 0, 1	15. 46. 13, 3	17. 48. 26, 5	19. 46. 43, 2	21. 48. 56, 4	23. 47. 13, 0
29	13. 47. 56, 6	15. 50. 9, 8	17. 52. 23, 1	19. 50. 39, 7	21. 52. 52, 9	23. 51. 9, 6
30	13. 51. 53, 2	15. 54. 6, 4	17. 56. 19, 6	19. 54. 36, 3	21. 56. 49, 5	23. 55. 6, 1
31	13. 55. 49, 7	15. 58. 3, 0		19. 58. 32, 8		23. 59. 2, 7

TABLE XVIII.

Mean Motions of the Sun's Right Ascension in Time, to Hours and Minutes of Sidereal Time.

Sidereal Time.	Sun's mean Motion in *AR*. in Time.	Sidereal Time.	Sun's mean Motion in *AR*. in Time.	Sidereal Time.	Sun's mean Motion in *AR* in Time.
H.	M. S.	M.	S.	M.	S.
1	0. 9, 83	1	0, 16	31	5,08
2	0. 19, 66	2	0, 33	32	5,24
3	0. 29, 49	3	0, 49	33	5,41
4	0. 39, 32	4	0, 66	34	5,57
5	0. 49, 15	5	0, 82	35	5,73
6	0. 58, 98	6	0, 98	36	5, 90
7	1. 8, 81	7	1, 15	37	6, 06
8	1. 18, 64	8	1, 31	38	6, 23
9	1. 28, 47	9	1, 47	39	6, 39
10	1. 38, 30	10	1, 64	40	6, 55
11	1. 48, 13	11	1, 80	41	6, 72
12	1. 57, 95	12	1, 97	42	6, 88
13	2. 7, 78	13	2, 13	43	7, 05
14	2. 17, 61	14	2, 29	44	7, 21
15	2. 27, 44	15	2, 46	45	7, 37
16	2. 37, 27	16	2, 62	46	7, 54
17	2. 47, 10	17	2, 78	47	7, 70
18	2. 56, 93	18	2, 95	48	7, 86
19	3. 6, 76	19	3, 11	49	8, 03
20	3. 16, 59	20	3, 28	50	8, 19
21	3. 26, 42	21	3, 44	51	8, 36
22	3. 36, 25	22	3, 60	52	8, 52
23	3. 46, 08	23	3, 77	53	8, 68
24	3. 55, 91	24	3, 93	54	8, 85
		25	4, 10	55	9, 01
		26	4, 26	56	9, 17
		27	4, 42	57	9, 34
		28	4, 59	58	9, 50
		29	4, 75	59	9, 67
		30	4, 91	60	9, 83

TABLE XIX.

Equation of the Equinoxes in Right Ascension, common to all the Stars.

ARGUMENT. Long. of ☽'s ☊.

Long. of ☽'s ☊.	0. Sig. — VI. Sig. +	I. Sig. — VII. Sig. +	II. Sig. — VIII. Sig. +	
0°	0″,0	8″,2	14″,2	30°
1	0, 3	8, 4	14, 3	29
2	0, 6	8, 7	14, 5	28
3	0, 9	8, 9	14, 6	27
4	1, 1	9, 2	14, 7	26
5	1, 4	9, 4	14, 8	25
6	1, 7	9, 6	15, 0	24
7	2, 0	9, 8	15, 1	23
8	2, 3	0, 1	15, 2	22
9	2, 6	10, 3	15, 3	21
10	2, 8	10, 5	15, 4	20
11	3, 1	10, 7	15, 5	19
12	3, 4	11, 0	15, 6	18
13	3, 7	11, 2	15, 7	17
14	4, 0	11, 4	15, 7	16
15	4, 2	11, 6	15, 8	15
16	4, 5	11, 8	15, 9	14
17	4, 8	12, 0	16, 0	13
18	5, 1	12, 2	16, 0	12
19	5, 3	12, 4	16, 1	11
20	5, 6	12, 5	16, 1	10
21	5, 9	12, 7	16, 2	9
22	6, 1	12, 9	16, 2	8
23	6, 4	13, 1	16, 2	7
24	6, 7	13, 2	16, 3	6
25	6, 9	13, 4	16, 3	5
26	7, 2	13, 6	16, 3	4
27	7, 4	13, 7	16, 3	3
28	7, 7	13, 9	16, 4	2
29	7, 9	14, 0	16, 4	1
30	8, 2	14, 2	16, 4	0
	— V. Sig. + XI. Sig.	— IV. Sig. + X. Sig.	— III. Sig. + IX. Sig.	Long. of ☽'s ☊.

TABLE XX.

Equation of the Equinoxes in Right Ascension in Time, common to all the Stars.

ARGUMENT. Long. of ☽'s ☊.

Degrees.	0. Sig. —	I. Sig. —	II. Sig. —	
	VI. Sig. +	VII. Sig. +	VIII. Sig. +	
0	0",00	0",55	0",95	30
1	0, 02	0, 56	0, 95	29
2	0, 04	0, 58	0, 96	28
3	0, 06	0, 59	0, 97	27
4	0, 08	0, 61	0, 98	26
5	0, 10	0, 63	0, 99	25
6	0, 11	0, 64	1, 00	24
7	0, 13	0, 66	1, 00	23
8	0, 15	0, 67	1, 01	22
9	0, 17	0, 69	1, 02	21
10	0, 19	0, 70	1, 03	20
11	0, 21	0, 72	1, 03	19
12	0, 23	0, 73	1, 04	18
13	0, 25	0, 74	1, 04	17
14	0, 26	0, 76	1, 05	16
15	0, 28	0, 77	1, 05	15
16	0, 30	0, 79	1, 06	14
17	0, 32	0, 80	1, 06	13
18	0, 34	0, 81	1, 07	12
19	0, 36	0, 82	1, 07	11
20	0, 37	0, 84	1, 08	10
21	0, 39	0, 85	1, 08	9
22	0, 41	0, 86	1, 08	8
23	0, 43	0, 87	1, 08	7
24	0, 44	0, 88	1, 09	6
25	0, 46	0, 89	1, 09	5
26	0, 48	0, 90	1, 09	4
27	0, 50	0, 92	1, 09	3
28	0, 51	0, 93	1, 09	2
29	0, 53	0, 94	1, 09	1
30	0, 55	0, 95	1, 09	0
	— V. Sig.	— IV. Sig.	— III. Sig.	Degrees.
	+ XI. Sig.	+ X. Sig.	+ IX. Sig.	

Deviation of Stars in North Polar Distance, common to all the Stars.

Long. of moon's node.		RIGHT ASCENSION OF THE STAR.																		
		O. VI.						I. VII.						II. VIII.						III. IX.
		0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°
S.	D.	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
O. VI.	0	+0,00	—0,83	—1,66	—2,47	—3,27	—4,04	—4,78	—5,48	—6,14	—6,75	—7,32	—7,83	—8,27	—8,65	—8,98	—9,23	—9,41	—9,52	—9,55
	5	0,62	0,21	1,04	1,86	2,67	3,46	4,22	4,95	5,64	6,29	6,80	7,44	7,93	8,36	8,73	9,03	9,26	9,43	9,52
	10	1,23	+0,41	0,42	1,24	2,06	2,86	3,63	4,38	5,10	5,78	6,41	6,99	7,53	8,00	8,42	8,76	9,05	9,26	9,41
	15	1,84	1,03	+0,21	0,61	1,43	2,28	3,02	3,78	4,52	5,22	5,88	6,50	7,07	7,58	8,04	8,44	8,76	9,03	9,23
	20	2,43	1,64	0,84	+0,03	0,78	1,59	2,38	3,16	3,90	4,63	5,31	5,96	6,56	7,11	7,60	8,04	8,42	8,73	8,98
	25	3,01	2,24	1,46	0,66	0,13	0,93	1,73	2,50	3,26	4,00	4,70	5,37	5,99	6,58	7,11	7,53	8,00	8,36	8,65
I. VII.	0	3,56	2,82	2,07	1,29	+0,51	0,27	1,03	1,83	2,59	3,33	4,05	4,74	5,39	5,99	6,56	7,07	7,53	7,93	8,27
	5	4,08	3,38	2,66	1,92	1,16	+0,39	0,38	1,15	1,90	2,65	3,37	4,07	4,74	5,37	5,96	6,50	7,00	7,44	7,88
	10	4,57	3,92	3,23	2,52	1,79	1,06	+0,30	0,45	1,20	1,94	2,67	3,37	4,05	4,70	5,31	5,88	6,41	6,89	7,32
	15	5,03	4,42	3,78	3,11	2,42	1,70	0,96	+0,25	0,49	1,22	1,94	2,65	3,33	4,00	4,63	,22	5,78	6,29	6,75
	20	5,45	4,89	4,30	3,67	3,02	2,34	1,65	0,94	+0,23	0,49	1,20	1,90	2,59	3,26	3,90	4,52	5,10	5,64	6,14
	25	5,83	5,33	4,79	4,21	3,60	2,97	2,31	1,63	0,94	+0,25	0,45	1,15	1,83	2,50	3,16	3,78	4,38	4,95	5,48
II. VIII.	0	6,16	5,72	5,24	4,71	4,16	3,57	2,98	2,31	1,65	0,98	+0,30	0,38	1,05	1,73	2,38	3,02	3,63	4,22	4,78
	5	6,44	6,07	5,65	5,18	4,68	4,14	3,57	2,97	2,34	1,70	1,05	+0,39	0,27	0,93	1,59	2,23	2,87	3,46	4,04
	10	6,68	6,37	6,02	5,61	5,16	4,68	4,16	3,60	3,02	2,42	1,79	1,16	+0,51	0,13	0,78	1,43	2,06	2,67	3,27
	15	6,87	6,63	6,34	6,00	5,61	5,18	4,71	4,21	3,67	3,11	2,52	1,92	1,29	+0,66	+0,03	0,61	1,24	1,86	2,47
	20	7,01	6,83	6,61	6,34	6,02	5,65	5,24	4,79	4,30	3,78	3,23	2,66	2,07	1,46	0,84	+0,21	0,42	1,04	1,66
	25	7,09	6,99	6,83	6,63	6,37	6,07	5,72	5,33	4,89	4,42	3,92	3,38	2,82	2,24	1,64	1,03	+0,41	0,21	0,83

If the longitude of the moon's node and the right ascension of the star be both more or less than six signs, use the algebraic sign of the Table; but if one be more and the other less than six signs, change the sign of the Table.

TABLE XXI. Continued.

Long. of moon's node.		RIGHT ASCENSION OF THE STAR.																		
		O. VI.						I. VII.						II. VIII.						III.IX.
		0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°
s.	D	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
III. IX.	0	+7,11	+7,09	+7,01	+6,87	+6,68	+6,44	+6,16	+5,83	+5,45	+5,03	+4,57	+4,08	+3,56	+3,01	+2,43	+1,84	+1,23	+0,62	−0,00
	5	7,09	7,13	7,12	7,06	6,94	6,78	6,55	6,28	5,96	5,60	5,19	4,75	4,26	3,75	3,21	2,64	2,05	1,45	+0,83
	10	7,01	7,12	7,19	7,20	7,15	7,05	6,90	6,69	6,43	6,13	5,77	5,38	4,94	4,46	3,95	3,42	2,85	2,26	1,66
	15	6,87	7.06	7,20	7,28	7,30	7,27	7,19	7,05	6,85	6,61	6,31	5,97	5,58	5,14	4,67	4,17	3.63	3,06	2,47
	20	6,68	6,94	7·15	7,30	7,40	7,44	7,42	7,35	7,22	7,04	6,80	6,51	6,17	5,79	5,36	4,89	4,38	3.84	3,27
	25	6,44	6,78	7,05	7,27	7,44	7,55	7,60	7,60	7,53	7,42	7.23	7,00	6,72	6,38	6,00	5,57	5,09	4,58	4,04
IV. X.	0	6,16	6,55	6,90	7,19	7,42	7,60	7,72	7,79	7,79	7,73	7,62	7,44	7,22	6.93	6,59	6,21	5,77	5,29	4,78
	5	5,83	6,28	6,69	7,05	7,35	7,60	7,79	7,92	7,99	7,99	7,94	7,83	7,66	7,43	7.14	6,80	6,41	5,97	5,48
	10	5,45	5,96	6,43	6,85	7,22	7,53	7,79	7,99	8,12	8,20	8,21	8,15	8,04	7,86	7,64	7,34	6,99	6,59	6,14
	15	5,03	5,60	6,13	6,61	7,03	7,41	7,73	7,99	8,20	8,33	8,41	8,42	8,36	8,25	8,06	7,83	7,52	7,17	6,75
	20	4,57	5,19	5,77	6,31	6,80	7,23	7,62	7,94	8,21	8.41	8,54	8,62	8.62	8,56	8.44	8,25	8,00	7,69	7,32
	25	4,08	4,75	5.38	5,97	6,51	7,00	7,44	7,83	8,15	8,42	8,62	8,75	8,82	8,82	8,75	8,61	8,41	8.15	7,83
V. XI.	0	3,56	4.26	4 94	5,58	6,17	6,72	7,22	7,66	8,04	8,36	8,62	8,82	8,94	9,00	8,99	8,91	8,76	8,55	8,27
	5	3.01	3,75	4.46	5,14	5,79	6,38	6,93	7,43	7,86	8,25	8,56	8,82	9,00	9,12	9,16	9,14	9,05	8,89	8,65
	10	2.43	3,21	3,95	4,67	5,36	6,00	6,59	7,14	7,63	8,06	8,44	8,75	8,99	9,16	9,27	9,30	9,26	9,15	8.98
	15	1,84	2,64	3,42	4,17	4,89	5,57	6.21	6,80	7,34	7,82	8,25	8.61	8,91	9,14	9,30	9,39	9,41	9,35	9.23
	20	1,23	2,05	2,85	3,63	4,38	5,09	5.77	6.41	6,99	7,52	8,00	8.41	8.76	9,05	9,26	9,41	9.48	9,48	9 41
	25	0,62	1.45	2 26	3,06	3,84	4,58	5,29	5,97	6,59	7,17	7,69	8 15	8,55	8,89	9,15	9,35	9,48	9,53	9,52
VI. O.	0	0,00	0,83	1,66	2.47	3,27	4,04	4,78	5.48	6,14	6,75	7,32	7,83	8,27	8,65	8,98	9.23	9,41	9 52	9,55

If the longitude of the moon's node and the right ascension of the star be both more or less than six signs, use the algebraic sign of the Table; but if one be more and the other less than six signs, change the sign of the Table.

TABLE XXI. *Continued.*

Long. of moon's node.	RIGHT ASCENSION OF THE STAR.																		
	III. IX.						IV. X.						V. XI.						VI. O.
	0°	5°	10°	15	20°	25°	0°	5°	10°	15°	20°	25°	0°	5°	10°	15	20°	25°	0°
S. D.	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
O. VI. 0	—9,55	—9,52	—9,41	—9,23	—8,98	—8,65	—8,27	—7,83	—7,32	—6,75	—6,14	—5,48	—4,78	—4,04	—3,27	—2,47	—1,66	—0,83	—0,00
5	9,52	9,53	9,48	9,35	9,15	8,89	8,55	8,15	7,69	7,17	6,59	5,97	5,29	4,58	3,84	3,06	2,26	1,45	0,62
10	9,41	9,48	9,48	9,41	9,26	9,05	8,76	8,41	8,00	7,52	6,99	6,41	5,77	5,09	4,38	3,63	2,85	2,05	1,23
15	9,23	9,35	9,41	9,39	9,30	9,14	8,91	8,61	8,25	7,82	7,34	6,80	6,21	5,57	4,89	4,17	3,42	2,64	1,84
20	8,98	9,15	9,26	9,30	9,27	9,16	8,99	8,75	8,44	8,06	7,63	7,14	6,59	6,00	5,36	4,67	3,95	3,21	2,43
25	8,65	8,89	9,05	9,14	9,16	9,12	9,00	8,82	8,56	8,25	7,86	7,43	6,93	6,38	5,79	5,14	4,46	3,75	3,01
I. VII. 0	8,27	8,55	8,76	8,91	8,99	9,00	8,94	8,82	8,62	8,36	8,04	7,66	7,22	6,72	6,17	5,58	4,94	4,26	3,56
5	7,83	8,15	8,41	8,61	8,75	8,82	8,82	8,75	8,62	8,42	8,15	7,83	7,44	7,00	6,51	5,97	5,38	4,75	4,08
10	7,32	7,69	8,00	8,25	8,44	8,56	8,62	8,62	8,54	8,41	8,21	7,94	7,62	7,23	6,80	6,31	5,77	5,19	4,57
15	6,75	7,17	7,52	7,83	8,06	8,25	8,36	8,42	8,41	8,33	8,20	7,99	7,73	7,41	7,03	6,61	6,13	5,60	5,03
20	6,14	6,59	6,99	7,34	7,64	7,86	8,04	8,15	8,21	8,20	8,12	7,99	7,79	7,53	7,22	6,85	6,43	5,96	5,45
25	5,48	5,97	6,41	6,80	7,14	7,43	7,66	7,83	7,94	7,99	7,99	7,92	7,79	7,60	7,35	7,05	6,69	6,28	5,83
II. VIII. 0	4,78	5,29	5,77	6,21	6,59	6,93	7,22	7,44	7,62	7,73	7,79	7,79	7,72	7,60	7,42	7,19	6,90	6,55	6,16
5	4,04	4,58	5,09	5,57	6,00	6,38	6,72	7,00	7,23	7,42	7,53	7,60	7,60	7,55	7,44	7,27	7,05	6,78	6,44
10	3,27	3,84	4,38	4,89	5,36	5,79	6,17	6,51	6,80	7,04	7,22	7,35	7,42	7,44	7,40	7,30	7,15	6,94	6,68
15	2,47	3,06	3,63	4,17	4,67	5,14	5,58	5,97	6,31	6,61	6,85	7,05	7,19	7,27	7,30	7,28	7,20	7,06	6,87
20	1,66	2,26	2,85	3,42	3,95	4,46	4,94	5,38	5,77	6,13	6,43	6,69	6,90	7,05	7,15	7,20	7,19	7,12	7,01
25	0,83	1,45	2,05	2,64	3,21	3,75	4,26	4,75	5,19	5,60	5,96	6,28	6,55	6,78	6,94	7,06	7,12	7,13	7,09

If the longitude of the moon's node and the right ascension of the star be both more or less than six signs, use the algebraic sign of the Table; but if one be more and the other less than six signs, change the sign of the Table.

TABLE XXI. *Continued.*

Long. of moon's node.	*RIGHT ASCENSION OF THE STAR.* III. IX.						IV. X.						V. XI.						VI. O.
	0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°	5°	10°	15°	20°	25°	0°
s. D.	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"	"
III.IX. 0	—0,00	—0,62	—1,23	—1,84	—2,43	—3,01	—3,56	—4,08	—4,57	—5,03	—5,45	—5,83	—6,16	—6,44	—6,68	—6,87	—7,01	—7,09	—7,11
5	+0,83	+0,21	0,41	1,03	1,64	2,24	2,82	3,38	3,92	4,42	4,89	5,33	5,72	6,07	6,37	6,63	6,83	6,99	7,09
10	1,66	1,04	+0,42	0,21	0,84	1,46	2,07	2,66	3,23	3,78	4,30	4,79	5,24	5,65	6,02	6,34	6,61	6,83	7,01
15	2,47	1,86	1,24	+0,61	0,03	0,66	1,29	1,92	2,52	3,11	3,67	4,21	4,71	5,18	5,61	6,00	6,34	6,63	6,87
20	3,27	2,67	2,06	1,43	+0,78	+0,13	0,51	1,16	1,79	2,42	3,02	3,60	4,16	4,68	5,16	5,61	6,02	6,37	6,68
25	4,04	3,46	2,87	2,23	1,59	0,93	+0,27	0,39	1,05	1,70	2,34	2,97	3,57	4,14	4,68	5,18	5,65	6,07	6,44
IV. X. 0	4,78	4,22	3,63	3,02	2,38	1,73	1,05	+0,38	0,30	0,98	1,65	2,31	2,98	3,57	4,16	4,71	5,24	5,72	6,16
5	5,48	4,95	4,38	3,78	3,16	2,50	1,83	1,15	+0,45	0,25	0,94	1,63	2,31	2,97	3,60	4,21	4,79	5,33	5,83
10	6,14	5,64	5,10	4,52	3,90	3,26	2,59	1,90	1,20	+0,49	0,23	0,94	1,65	2,34	3,02	3,67	4,30	4,89	5,45
15	6,75	6,29	5,78	5,22	4,63	4,00	3,33	2,65	1,94	1,22	+0,49	0,25	0,98	1,70	2,42	3,11	3,78	4,42	5,03
20	7,32	6,89	6,41	5,88	5,31	4,70	4,05	3,37	2,67	1,94	1,20	+0,45	0,30	1,05	1,79	2,52	3,23	3,92	4,57
25	7,83	7,44	7,00	6,50	5,96	5,37	4,74	4,07	3,37	2,65	1,90	1,15	+0,38	0,39	1,16	1,92	2,66	3,38	4,08
V. XI. 0	8,27	7,93	7,53	7,07	6,56	5,99	5,39	4,74	4,05	3,33	2,59	1,83	1,05	+0,27	0,51	1,29	2,07	2,82	3,56
5	8,65	8,36	8,00	7,58	7,11	6,58	5,99	5,37	4,70	4,00	3,26	2,50	1,73	0,93	+0,13	0,66	1,46	2,24	3,01
10	8,98	8,73	8,42	8,04	7,60	7,11	6,56	5,96	5,31	4,63	3,90	3,16	2,38	1,59	0,78	0,03	0,84	1,64	2,43
15	9,23	9,03	8,76	8,44	8,04	7,58	7,07	6,50	5,88	5,22	4,52	3,78	3,02	2,23	1,43	+0,61	0,21	1,03	1,84
20	9,41	9,26	9,05	8,76	8,42	8,00	7,53	6,99	6,41	5,78	5,10	4,38	3,63	2,86	2,06	1,24	+0,42	0,41	1,23
25	9,52	9,43	9,26	9,03	8,73	8,36	7,93	7,44	6,89	6,29	5,64	4,95	4,22	3,46	2,67	1,86	1,04	+0,21	0,62
VI. O. 0	9,55	9,52	9,41	9,23	8,98	8,65	8,27	7,83	7,32	6,75	6,14	5,48	4,78	4,04	3,27	2,47	1,66	0,83	0,00

If the longitude of the moon's node and the right ascension of the star be both more or less than six signs, use the algebraic sign of the Table; but if one be more and the other less than six signs, change the sign of the Table.

TABLE XXII.

Time taken by Light to move over Parts of the Orbis Magnus.

Parts of Orbis Magnus.	Time.	Parts of Orbis Magnus.	Time.	Parts of Orbis Magnus.	Time.
,001	0′. 0″,5	,31	2′. 31″,0	,70	5′. 40″,9
,002	0. 1, 0	,32	2. 35, 8	,71	5. 45, 8
,003	0. 1, 5	,33	2. 40, 7	,72	5. 50, 6
,004	0. 1, 9	,34	2. 45, 6	,73	5. 55, 5
,005	0. 2, 4	,35	2. 50, 4	,74	6. 0, 4
,006	0. 2, 9	,36	2. 55, 3	,75	6. 5, 2
,007	0. 3, 4	,37	3. 0, 2	,76	6. 10, 1
,008	0. 3, 9	,38	3. 5, 1	,77	6. 15, 0
,009	0. 4, 4	,39	3. 9, 9	,78	6. 19, 9
,01	0. 4, 9	,40	3. 14, 8	,79	6. 24, 7
,02	0. 9, 7	,41	3. 19, 7	,80	6. 29, 6
,03	0. 14, 6	,42	3. 24, 5	,81	6. 34, 5
,04	0. 19, 5	,43	3. 29, 4	,82	6. 39, 3
,05	0. 24, 4	,44	3. 34, 3	,83	6. 44, 2
,06	0. 29, 2	,45	3. 39, 1	,84	6. 49, 1
,07	0. 34, 1	,46	3. 44, 0	,85	6 53, 9
,08	0. 39, 0	,47	3. 48, 9	,86	6. 58, 8
,09	0. 43, 8	,48	3. 53, 8	,87	7. 3, 7
,1	0. 48, 7	,49	3. 58, 6	,88	7. 8, 6
,11	0. 53, 6	,50	4. 3, 5	,89	7. 13, 4
,12	0. 58, 4	,51	4. 8, 4	,90	7. 18, 3
,13	1. 3, 3	,52	4. 13, 2	,91	7. 23, 2
,14	1. 8, 2	,53	4. 18, 1	,92	7. 28, 0
,15	1. 13, 0	,54	4. 23, 0	,93	7. 32, 9
,16	1. 17, 9	,55	4. 27, 8	,94	7. 37, 8
,17	1. 22, 8	,56	4. 32, 7	,95	7. 42, 6
,18	1. 27, 7	,57	4. 37, 6	,96	7. 47, 5
,19	1. 32, 5	,58	4. 42, 5	,97	7. 52, 4
,20	1. 37, 4	,59	4. 47, 3	,98	7. 57, 3
,21	1. 42, 3	,60	4. 52, 2	,99	8. 2, 1
,22	1. 47, 1	,61	4. 57, 1	1,00	8. 7, 0
,23	1. 52, 0	,62	5. 1, 9	2	16. 14, 0
,24	1. 56, 9	,63	5. 6, 8	3	24. 21, 0
,25	2. 1, 8	,64	5. 11, 7	4	32. 28, 0
,26	2. 6, 6	,65	5. 16, 5	5	40. 35, 0
,27	2. 11, 5	,66	5. 21, 4	6	48. 42, 0
,28	2. 16, 4	,67	5. 26, 3	7	56. 49, 0
,29	2. 21, 3	,68	5. 31, 2	8	64. 56, 0
,30	2. 26, 1	,69	5. 36, 0	9	73. 3, 0
				10	81. 10, 0

TABLE XXIII.

To find the Nonagesimal Degree of the Ecliptic, and its Altitude, for the Latitude of Greenwich, reduced to the Earth's Center, supposed to be 51°. 14'. 7", and the Obliquity of the Ecliptic 23°. 28'.

ARGUMENT. Right Ascension of the Meridian.

A.R. of Merid.	Nonag.	Alt. Nonag.	A.R. of Merid.	Nonag.	Alt. Nonag.	A.R. of Merid.	Nonag.	Alt. Nonag.
0°	0ˢ. 26°. 22'. 40"	44°. 20'. 25"	30°	1ˢ. 17°. 47'. 0"	53°. 48'. 10"	60	2ˢ. 8°. 49'. 10"	60°. 2'. 50"
1	0. 27. 6. 40	44. 41. 40	31	1. 18. 29. 10	54. 4. 10	61	2. 9. 31. 20	60. 11. 10
2	0. 27. 50. 40	45. 2. 50	32	1. 19. 11. 10	54. 19. 50	62	2. 10. 13. 30	60. 19. 30
3	0. 28. 34. 30	45. 23. 50	33	1. 19. 53. 10	54. 35. 20	63	2. 10. 55. 40	60. 27. 30
4	0. 29. 18. 10	45. 44. 40	34	1. 20. 35. 20	54. 50. 40	64	2. 11. 38. 0	60. 35. 0
5	1. 0. 1. 50	46. 5. 30	35	1. 21. 17. 20	55. 5. 40	65	2. 12. 20. 10	60. 42. 30
6	1. 0. 45. 10	46. 26. 10	36	1. 21. 59. 20	55. 20. 30	66	2. 13. 2. 30	60. 49. 40
7	1. 1. 28. 40	46. 46. 30	37	1. 22. 41. 20	55. 35. 0	67	2. 13. 44. 50	60. 56. 30
8	1. 2. 12. 0	47. 6. 50	38	1. 23. 23. 20	55. 49. 30	68	2. 14. 27. 10	61. 3. 0
9	1. 2. 55. 10	47. 27. 0	39	1. 24. 5. 20	56. 3. 40	69	2. 15. 9. 30	61. 9. 20
10	1. 3. 38. 30	47. 47. 0	40	1. 24. 47. 20	56. 17. 30	70	2. 15. 51. 40	61. 15. 10
11	1. 4. 21. 10	48. 6. 50	41	1. 25. 29. 30	56. 31. 10	71	2. 16. 34. 0	61. 20. 50
12	1. 5. 3. 50	48. 26. 30	42	1. 26. 11. 30	56. 44. 40	72	2. 17. 16. 30	61. 26. 20
13	1. 5. 46. 50	48. 45. 50	43	1. 26. 53. 20	56. 57. 50	73	2. 17. 58. 50	61. 31. 30
14	1. 6. 29. 40	49. 5. 10	44	1. 27. 35. 30	57. 10. 40	74	2. 18. 41. 10	61. 36. 10
15	1. 7. 12. 20	49. 24. 20	45	1. 28. 17. 30	57. 23. 20	75	2. 19. 23. 30	61. 40. 50
16	1. 7. 55. 0	49. 43. 20	46	1. 28. 59. 30	57. 35. 40	76	2. 20. 5. 50	61. 45. 10
17	1. 8. 37. 30	50. 2. 0	47	1. 29. 41. 30	57. 48. 0	77	2. 20. 48. 20	61. 49. 0
18	1. 9. 20. 0	50. 20. 30	48	2. 0. 23. 30	58. 0. 0	78	2. 21. 30. 40	61. 52. 40
19	1. 10. 2. 30	50. 39. 0	49	2. 1. 5. 40	58. 11. 40	79	2. 22. 13. 10	61. 56. 0
20	1. 10. 45. 0	50. 57. 10	50	2. 1. 47. 40	58. 23. 0	80	2. 22. 55. 30	61. 59. 10
21	1. 11. 27. 20	51. 15. 10	51	2. 2. 29. 50	58. 34. 10	81	2. 23. 38. 0	62. 2. 0
22	1. 12. 9. 30	51. 33. 0	52	2. 3. 11. 50	58. 45. 0	82	2. 24. 20. 30	62. 4. 30
23	1. 12. 51. 50	51. 50. 40	53	2. 3. 54. 0	58. 55. 40	83	2. 25. 2. 50	62. 6. 40
24	1. 13. 34. 0	52. 8. 0	54	2. 4. 36. 10	59. 6. 0	84	2. 25. 45. 20	62. 8. 30
25	1. 14. 16. 20	52. 25. 10	55	2. 5. 18. 20	59. 16. 10	85	2. 26. 27. 40	62. 10. 10
26	1. 14. 58. 30	52. 42. 10	56	2. 6. 0. 20	59. 26. 0	86	2. 27. 10. 10	62. 11. 30
27	1. 15. 40. 40	52. 59. 0	57	2. 6. 42. 30	59. 35. 40	87	2. 27. 52. 40	62. 12. 30
28	1. 16. 22. 50	53. 15. 40	58	2. 7. 24. 40	59. 45. 0	88	2. 28. 35. 10	62. 13. 20
29	1. 17. 4. 50	53. 32. 0	59	2. 8. 6. 50	59. 54. 0	89	2. 29. 17. 30	62. 13. 50
30	1. 17. 47. 0	53. 48. 10	60	2. 8. 49. 10	60. 2. 50	90	3. 0. 0. 0	62. 13. 53

TABLE XXIII. *Continued.*

ARGUMENT. Right Ascension of the Meridian.

AR. of Merid.	Nonag.	Alt. Nonag.	AR. of Merid.	Nonag.	Alt. Nonag.	AR. of Merid.	Nonag.	Alt. Non g.
90°	3ˢ. 0°. 0′. 0″	62°. 13′. 53″	120°	3ˢ. 21°. 10′. 50″	60°. 2′. 50″	150°	4ˢ. 12°. 13′. 0″	53°. 48′. 10″
91	3. 0. 42. 30	62. 13. 50	121	3. 21. 53. 10	59. 54. 0	151	4. 12. 55. 10	53. 32. 0
92	3. 1. 24. 50	62. 13. 20	122	3. 22. 35. 20	59. 45. 0	152	4 13. 37. 10	53. 15. 40
93	3. 2. 7. 20	62. 12. 30	123	3. 23. 17. 30	59. 35. 40	153	4. 14. 19. 20	52. 59. 0
94	3. 2. 49. 50	62. 11. 30	124	3. 23. 59. 40	59. 26. 0	154	4. 15. 1. 30	52. 42. 10
95	3. 3. 32. 20	62. 10. 10	125	3. 24. 41. 40	59. 16. 10	155	4. 15. 43. 40	52. 25. 10
96	3. 4. 14. 40	62. 8. 30	126	3. 25. 23. 50	59. 6. 0	156	4. 16. 26. 0	52. 8. 0
97	3. 4. 57. 10	62. 6. 40	127	3. 26. 6. 0	58. 55. 40	157	4. 17. 8. 10	51. 50. 40
98	3. 5. 39. 30	62. 4. 30	128	3. 26. 48. 10	58. 45. 0	158	4. 17. 50 30	51. 33. 0
99	3. 6. 22. 0	62. 2. 0	129	3. 27. 30. 10	58. 34. 10	159	4. 18. 32. 40	51. 15. 10
100	3. 7. 4. 30	61. 59. 10	130	3. 28. 12. 20	58. 23. 0	160	4. 19. 15. 0	50. 57. 10
101	3. 7. 46. 50	61. 56. 0	131	3. 28. 54. 20	58. 11. 40	161	4. 19. 57. 30	50. 39. 0
102	3. 8. 29. 20	61. 52. 40	132	3. 29. 36. 30	58. 0. 0	162	4. 20. 40. 0	50. 20. 30
103	3. 9. 11. 40	61. 49. 0	133	4. 0. 18. 30	57. 48. 0	163	4. 21. 22. 30	50. 2. 0
104	3. 9. 54. 10	61. 45. 10	134	4. 1. 0. 30	57. 35. 40	164	4. 22. 5. 0	49. 43. 20
105	3. 10. 36. 30	61. 40. 50	135	4. 1. 42. 30	57. 23. 20	165	4. 22. 47. 40	49. 24. 20
106	3. 11. 18. 50	61. 36. 10	136	4. 2. 24. 30	57. 10. 40	166	4. 23. 30. 20	49. 5. 10
107	3. 12. 1. 10	61. 31. 30	137	4. 3. 6. 40	56. 57. 50	167	4. 24. 13. 10	48. 45. 50
108	3. 12. 43. 30	61. 26. 20	138	4. 3. 48. 30	56. 44. 40	168	4. 24. 56. 10	48. 26. 30
109	3. 13. 26. 0	61. 20. 50	139	4. 4. 30. 30	56. 31. 10	169	4. 25. 38. 50	48. 6. 50
110	3. 14. 8. 20	61. 15. 10	140	4. 5. 12. 40	56. 17. 30	170	4. 26. 21. 30	47. 47. 0
111	3. 14. 50. 30	61. 9. 20	141	4. 5. 54. 40	56. 3. 40	171	4. 27. 4. 50	47. 27. 0
112	3. 15. 32. 50	61. 3. 0	142	4. 6. 36. 40	55. 49. 30	172	4. 27. 48. 0	47. 6. 50
113	3. 16. 15. 10	60. 56. 30	143	4. 7. 18. 40	55. 35. 0	173	4. 28. 31. 20	46. 46. 30
114	3. 16. 57. 30	60. 49. 40	144	4. 8. 0. 40	55. 20. 30	174	4. 29. 14. 50	46. 26. 10
115	3. 17. 39. 50	60. 42. 30	145	4. 8. 42. 40	55. 5. 40	175	4. 29. 58. 10	46. 5. 30
116	3. 18. 22. 0	60. 35. 0	146	4. 9. 24. 40	54. 50. 40	176	5. 0. 41. 50	45. 44. 40
117	3. 19. 4. 20	60. 27. 30	147	4. 10. 6. 50	54. 35. 20	177	5. 1. 25. 30	45. 23. 50
118	3. 19. 46. 30	60. 19. 30	148	4. 10. 48. 50	54. 19. 50	178	5. 2. 9. 20	45. 2. 50
119	3. 20. 28. 40	60. 11. 10	149	4. 11. 30. 50	54. 4. 10	179	5. 2. 53. 20	44. 41. 40
120	3. 21. 10. 50	60. 2. 50	150	4. 12. 13. 0	53. 48. 10	180	5. 3. 37. 20	44. 20. 25

TABLE XXIII. *Continued.*

ARGUMENT. Right Ascension of the Meridian.

AR. of Merid.	Nonag.	Alt. Nonag.	AR. of Merid.	Nonag.	Alt. Nonag.	AR. of Merid.	Nonag.	Alt. Nonag.
180°	5ˢ. 3°. 37′. 20″	44°. 20′. 25″	210°	5ˢ. 27°. 32′. 10″	32°. 52′. 10″	240°	7ˢ 0°. 50′. 20″	21°. 23. 0′
181	5. 4. 21. 40	43. 58. 50	211	5. 28. 26. 10	32. 28. 20	241	7. 2. 17. 20	21. 2. 40
182	5. 5. 6. 10	43. 37. 10	212	5. 29. 20. 10	32. 4. 30	242	7. 3. 46. 30	20. 42. 40
183	5. 5. 50. 30	43. 15. 30	213	6. 0. 15. 10	31. 40. 30	243	7. 5. 17. 50	20. 22. 50
184	5. 6. 35. 20	42. 53. 40	214	6. 1. 10. 30	31. 16. 40	244	7. 6. 51. 0	20. 3. 40
185	5. 7. 20. 10	42. 31. 40	215	6. 2. 6. 50	30. 52. 50	245	7. 8. 26. 30	19. 44. 40
186	5. 8. 5. 20	42. 9. 30	216	6. 3. 3. 40	30. 29. 0	246	7. 10. 4. 0	19. 26. 10
187	5. 8. 50. 40	41. 47. 20	217	6. 4. 1. 10	30. 5. 10	247	7. 11. 43. 50	19. 8. 0
188	5. 9. 36. 10	41. 24. 50	218	6. 4. 59. 40	29. 41. 10	248	7. 13. 25. 40	18. 50. 30
189	5. 10. 21. 50	41. 2. 30	219	6. 5. 58. 40	29. 17. 20	249	7. 15. 10. 0	18. 33. 20
190	5. 11. 7. 50	40. 39. 50	220	6. 6. 58. 20	28. 53. 50	250	7. 16. 56. 50	18. 16. 40
191	5. 11. 53. 50	40. 17. 20	221	6. 7. 59. 10	28. 30. 0	251	7. 18. 45. 50	18. 0. 50
192	5. 12. 40. 20	39. 54. 30	222	6. 9. 0. 50	28. 6. 20	252	7. 20. 37. 20	17. 45. 20
193	5. 13. 27. 0	39. 31. 40	223	6. 10. 3. 20	27. 42. 50	253	7. 22. 31. 20	17. 30. 20
194	5. 14. 13. 50	39. 8. 40	224	6. 11. 6. 40	27. 19. 20	254	7. 24. 27. 30	17. 16. 20
195	5. 15. 1. 10	38. 45. 40	225	6. 12. 11. 20	26. 56. 0	255	7. 26. 26. 30	17. 2. 40
196	5. 15. 48. 30	38. 22. 30	226	6. 13. 16. 40	26. 32. 40	256	7. 28. 27. 40	16. 49. 50
197	5. 16. 36. 30	37. 59. 30	227	6. 14. 23. 20	26. 9. 30	257	8. 0. 31. 10	16. 37. 40
198	5. 17. 24. 30	37. 36. 0	228	6. 15. 31. 10	25. 46. 20	258	8. 2. 36. 50	16. 26. 30
199	5. 18. 12. 50	37. 12. 40	229	6. 16. 39. 50	25. 23. 30	259	8. 4. 44. 50	16. 15. 50
200	5. 19. 1. 50	36. 49. 10	230	6. 17. 50. 0	25. 0. 40	260	8. 6. 55. 0	16. 6. 10
201	5. 19. 51. 0	36. 25. 40	231	6. 19. 1. 20	24. 38. 0	261	8. 9. 7. 20	15. 57. 0
202	5. 20. 40. 20	36. 2. 10	232	6. 20. 14. 0	24. 15. 30	262	8. 11. 21. 20	15. 49. 0
203	5. 21. 30. 20	35. 38. 40	233	6. 21. 28. 10	23. 53. 10	263	8. 13. 37. 0	15. 41. 50
204	5. 22. 20. 40	35. 15. 0	234	6. 22. 43. 30	23. 31. 0	264	8. 15. 54. 20	15. 35. 30
205	5. 23. 11. 20	34. 51. 20	235	6. 24. 0. 30	23. 9. 0	265	8. 18. 13. 10	15. 30. 10
206	5. 24. 2. 30	34. 27. 30	236	6. 25. 19. 10	22. 47. 20	266	8. 20. 33. 10	15. 25. 40
207	5. 24. 54. 10	34. 3. 40	237	6. 26. 39. 20	22. 25. 50	267	8. 22. 54. 0	15. 22. 30
208	5. 25. 46. 20	33. 40. 0	238	6. 28. 1. 20	22. 4. 30	268	8. 25. 15. 30	15. 19. 50
209	5. 26. 39. 0	33. 16. 0	239	6. 29. 25. 0	21. 43. 30	269	8. 27. 37. 40	15. 18. 20
210	5. 27. 32. 10	32. 52. 10	240	7. 0. 50. 20	21. 23. 0	270	9. 0. 0. 0	15. 17. 53

Argument. Right Ascension of the Meridian.

A.R. of Merid.	Nonag.	Alt. Nonag.	A.R. of Merid.	Nonag.	Alt. Nonag.	A.R. of Merid.	Nonag.	Alt. Nonag.
270°	9ˢ. 0°. 0′. 0″	15°. 17′. 53″	300°	10ˢ. 29°. 9′. 40″	21°. 23′. 0″	330°	0ˢ. 2°. 27′. 50″	32°. 52′. 10″
271	9. 2. 22. 20	15. 18. 20	301	11. 0. 35. 0	21. 43. 30	331	0. 3. 21. 0	33. 16. 0
272	9. 4. 44. 30	15. 19. 50	302	11. 1. 58. 40	22. 4. 30	332	0. 4. 13. 40	33. 40. 0
273	9. 7. 6. 0	15. 22. 30	303	11. 3. 20. 40	22. 25. 50	333	0. 5. 5. 50	34. 3. 40
274	9. 9. 26. 50	15. 25. 40	304	11. 4. 40. 50	22. 47. 20	334	0. 5. 57. 30	34. 27. 30
275	9. 11. 46. 50	15. 30. 10	305	11. 5. 59. 30	23. 9. 0	335	0. 6. 48. 40	34. 51. 20
276	9. 14. 5. 40	15. 35. 30	306	11. 7. 16. 30	23. 31. 0	336	0. 7. 39. 20	35. 15. 0
277	9. 16. 23. 0	15. 41. 50	307	11. 8. 31. 50	23. 53. 10	337	0. 8. 29. 40	35. 38. 40
278	9. 18. 38. 40	15. 49. 0	308	11. 9. 46. 0	24. 15. 30	338	0. 9. 19. 40	36. 2. 10
279	9. 20. 52. 40	15. 57. 0	309	11. 10. 58. 40	24. 38. 0	339	0. 10. 9. 0	36. 25. 40
280	9. 23. 5. 0	16. 6. 10	310	11. 12. 10. 0	25. 0. 40	340	0. 10. 58. 10	36. 49. 10
281	9. 25. 15. 10	16. 15. 50	311	11. 13. 20. 10	25. 23. 30	341	0. 11. 47. 10	37. 12. 40
282	9. 27. 23. 10	16. 26. 30	312	11. 14. 28. 50	25. 46. 20	342	0. 12. 35. 30	37. 36. 0
283	9. 29. 28, 50	16. 37. 40	313	11. 15. 36. 40	26. 9. 30	343	0. 13. 23. 30	37. 59. 30
284	10. 1. 32. 20	16. 49. 50	314	11. 16. 43. 20	26. 32. 40	344	0. 14. 11. 50	38. 22. 30
285	10. 3. 33. 20	17. 2. 40	315	11. 17. 48. 40	26. 56. 0	345	0. 14. 58. 50	38. 45. 40
286	10. 5. 32. 30	17. 16. 20	316	11. 18. 53. 20	27. 19. 20	346	0. 15. 46. 10	39. 8. 40
287	10. 7. 28. 40	17. 30. 20	317	11. 19. 56. 40	27. 42. 50	347	0. 16. 33. 0	39. 31. 40
288	10. 9. 22. 40	17. 45. 20	318	11. 20. 59. 10	28. 6. 20	348	0. 17. 19. 40	39. 54. 30
289	10. 11. 14. 10	18. 0. 50	319	11. 22. 0. 50	28. 30. 0	349	0. 18. 6. 10	40. 17. 20
290	10. 13. 3. 10	18. 16. 40	320	11. 23. 1. 40	28. 53. 50	350	0. 18. 52. 10	40. 39. 50
291	10. 14. 50. 0	18. 33. 20	321	11. 24. 1. 20	29. 17. 20	351	0. 19. 38. 10	41. 2. 30
292	10. 16. 34. 20	18. 50. 30	322	11. 25. 0. 20	29. 41. 10	352	0. 20. 23. 50	41. 24. 50
293	10. 18. 16. 10	19. 8. 0	323	11. 25. 58. 50	30. 5. 10	353	0. 21. 9. 20	41. 47. 20
294	10. 19. 56. 0	19. 26. 10	324	11. 26. 56. 20	30. 29. 0	354	0. 21. 54. 40	42. 9. 30
295	10. 21. 33. 30	19. 44. 40	325	11. 27. 53. 10	30. 52. 50	355	0. 22. 39. 50	42. 31. 40
296	10. 23. 9. 0	20. 3. 40	326	11. 28. 49. 30	31. 16. 40	356	0. 23. 24. 40	42. 53. 40
297	10. 24. 42. 10	20. 22. 50	327	11. 29. 44. 50	31. 40. 30	357	0. 24. 9. 30	43. 15. 30
298	10. 26. 13. 30	20. 42. 40	328	0. 0. 39. 50	32. 4. 30	358	0. 24. 53. 50	43. 37. 10
299	10. 27. 42. 40	21. 2. 40	329	0. 1. 33. 50	32. 28. 20	359	0. 25. 38. 20	43. 58. 50
300	10. 29. 9. 40	21. 23. 0	330	0. 2. 27. 50	32. 52. 10	360	0. 26. 22. 40	44. 20. 25

TABLE XXV.

The Angle between the Ecliptic and parallel to the Equator, to the Obliquity of the Ecliptic 23°. 28′; with the Variation for 10″ Variation of the Obliquity.

Sun's Declinat.	Angle of Ecliptic and parallel to Equator.	Diff.	Var. for 10″ Var. of Obl. of Ecliptic.	Sun's Declinat.	Angle of Ecliptic and parallel to Equator.	Diff.	Var. for 10″ Var. of Obl. of Ecliptic.
0°. 0	23°. 28′. 0′	0′. 18″	10′,0	16°. 40′	16°. 45′. 38″	20′. 18″	14″,4
0. 30	23. 27. 42	0. 55	10, 0	17. 0	16. 25. 20	21. 12	14, 7
1. 0	23. 26. 47	1. 30	10, 0	17. 20	16. 4. 8	22. 10	15, 0
1. 30	23. 25. 17		10, 0	17. 40	15. 41. 58		15, 4
		2. 7				23. 12	
2. 0	23. 23. 10	2. 44	10, 1	18. 0	15. 18. 46	24. 18	15, 8
2. 30	23. 20. 26	3. 21	10, 1	18. 20	14. 54. 28	25. 31	16, 3
3. 0	23. 17. 5	3. 57	10, 1	18. 40	14. 28. 57	26. 50	16, 8
3. 30	23. 13. 8		10, 1	19. 0	14. 2. 7		17, 3
		4. 36				21. 5	
4. 0	23. 8. 32	5. 14	10, 1	19. 15	13. 41. 2	21. 58	17, 8
4. 30	23. 3. 18	5. 52	10, 2	19. 30	13. 19. 4	22. 57	18, 3
5. 0	22. 57. 26	6. 31	10, 2	19. 45	12. 56. 7	23. 59	18, 9
5. 30	22. 50. 55		10, 3	20. 0	12. 32. 8		19, 5
		7. 12				16. 38	
6. 0	22. 43. 43	7. 52	10, 3	20. 10	12. 15. 30	17. 11	20, 5
6. 30	22. 35. 51	8. 34	10, 4	20. 20	11. 58. 19	17. 47	20, 4
7. 0	22. 27. 17	9. 16	10, 5	20. 30	11. 40. 32	18. 26	21, 0
7. 30	22. 18. 1		10, 6	20. 40	11. 22. 6		21, 5
		9 59				19. 7	
8. 0	22. 8. 2	10. 45	10, 6	20. 50	11. 2. 59	19. 53	22, 2
8. 30	21. 57. 17	11. 31	10, 8	21. 0	10. 43. 6	10. 14	22, 9
9. 0	21. 45. 46	12. 18	10, 9	21. 5	10. 32. 52	10. 28	23, 3
9. 30	21. 33. 28		11, 0	-21. 10	10. 22. 24		23, 7
		13. 7				10. 41	
10. 0	21. 20. 21	13. 58	11, 1	21. 15	10. 11. 43	10. 57	24, 1
10. 30	21. 6. 23	14. 51	11, 2	21. 20	10. 0. 46	11. 11	24, 6
11. 0	20. 51. 32	15. 46	11, 4	21. 25	9. 49. 35	11. 27	25, 0
11. 30	20. 35. 46		11, 5	21. 30	9. 38. 8		25, 5
		16. 44				11. 45	
12. 0	20. 19. 2	17. 44	11, 7	21. 35	9. 26. 23	12. 4	26, 0
12. 30	20. 1. 18	18. 48	11, 9	21. 40	9. 14. 19	12. 22	26, 6
13. 0	19. 42. 30	19. 53	12, 1	21. 45	9. 1. 57	12. 44	27, 3
13. 30	19. 22. 37		12, 3	21. 50	8. 49. 13		27, 9
		21. 5				13. 6	
14. 0	19. 1. 32	22. 20	12, 6	21. 55	8. 36. 7	13. 30	28, 6
14. 30	18. 39. 12	23. 40	12, 8	22. 0	8. 22. 37	13. 57	29, 4
15. 0	18. 15. 32	16. 34	13, 1	22. 5	8. 8. 40	14. 25	30, 3
15. 20	17. 58. 58		13, 3	22. 10	7. 54. 15		31, 2
		17. 14				14. 57	
15. 40	17. 41. 44	17. 56	13, 5	22. 15	7. 39. 18	15. 31	32, 2
16. 0	17. 23. 48	18. 41	13, 8	22. 20	7. 23. 47	16. 9	33, 4
16. 20	17. 5. 7	19. 29	14, 1	22. 25	7. 7. 38	16. 52	34, 7
16. 40	16. 45. 38		14, 4	22. 30	6. 50. 46		36, 1

TABLE XXVI.

The Angle of Position of any Point of the Ecliptic, according to the Obliquity of the Ecliptic 23°. 28′, *with the Variation for the Variation of the Obliquity by One Minute.*

ARGUMENT. Longitude of the Point of the Ecliptic.

	Sig. O. +	Sig. VI. —		Sig. I. +	Sig. VII. —		Sig. II. +	Sig. VIII. —		
	Ang. of Position.	Difference.	Variat.	Ang. of Position.	Difference.	Variat.	Ang. of Position.	Difference.	Variat.	
0°	23°. 28′. 0″		60″,0	20°. 36′. 15′,2		54′,1	12°. 14′. 48′,0		34″,0	30°
1	23. 27. 48,5	0′. 11″,5	60,0	20. 24. 39,4	11′. 35″,8	53,7	11. 53. 7,2	21′. 40′,8	33,1	29
2	23. 27. 14,1	0. 34,4	59,9	20. 12. 41,2	11. 58,2	53,3	11. 31. 10,3	21. 56,9	32,1	28
3	23. 26. 16,7	0. 57,4	59,9	20. 0. 20,8	12. 20,4	52,8	11. 8. 57,6	22. 12,7	31,1	27
4	23. 24. 56,4	1. 20,3	59,8	19. 47. 38,1	12. 42,7	52,3	10. 46. 29,6	22. 28,0	30,1	26
5	23. 23. 13,1	1. 43,3	59,8	19. 34. 33,4	13. 4,7	51,8	10. 23. 46,6	22. 43,0	29,1	25
6	23. 21. 6,9	2. 6,2	59,7	19. 21. 6,6	13. 26,8	51,3	10. 0. 49,2	22. 57,4	28,1	24
7	23. 18. 37,7	2. 29,2	59,6	19. 7. 18,0	13. 48,6	50,8	9. 37. 37,7	23. 11,5	27,0	23
8	23. 15. 45,6	2. 52,1	59,5	18. 53. 7,4	14. 10,6	50,3	9. 14. 12,6	23. 25,1	26,0	22
9	23. 12. 30,6	3. 15,0	59,4	18. 38. 35,2	14. 32,2	49,7	8. 50. 34,4	23. 38,2	24,9	21
10	23. 8. 52,6	3. 38,0	59,3	18. 23. 41,4	14. 53,8	49,2	8. 26. 43,7	23. 50,7	23,8	20
11	23. 4. 51,7	4. 0,9	59,2	18. 8. 26,2	15. 15,2	48,6	8. 2. 40,8	24. 2,9	22,7	19
12	23. 0. 27,9	4. 23,8	59,1	17. 52. 49,6	15. 36,6	48,0	7. 38. 26,3	24. 14,5	21,6	18
13	22. 55. 41,1	4. 46,8	58,9	17. 36. 51,9	15. 57,7	47,3	7. 14. 0,8	24. 25,5	20,5	17
14	22. 40. 31,4	5. 9,7	58,8	17. 20. 33,2	16. 18,7	46,7	6. 49. 24,8	24. 36,0	19,4	16
15	22. 44. 58,9	5. 32,5	58,6	17. 3. 53,6	16. 39,6	46,1	6. 24. 38,6	24. 46,0	18,2	15
16	22. 39. 3,4	5. 55,5	58,4	16. 46. 53,4	17. 0,2	45,4	5. 59. 43,5	24. 55,3	17,1	14
17	22. 32. 45,1	6. 18,3	58,1	16. 29. 32,7	17. 20,7	44,7	5. 34. 39,3	25. 4,2	15,9	13
18	22. 26. 3,9	6. 41,2	57,9	16. 11. 51,7	17. 41,0	43,9	5. 9. 26,0	25. 12,4	14,7	12
19	22. 18. 59,9	7. 4,0	57,6	15. 53. 50,7	18. 1,0	43,2	4. 44. 6,8	25. 20,1	13,5	11
20	22. 11. 33,0	7. 26,9	57,3	15. 35. 29,8	18. 20,9	42,5	4. 18. 39,8	25. 27,0	12,3	10
21	22. 3. 48,4	7. 49,6	57,1	15. 16. 49,3	18. 40,5	41,7	3. 53. 6,8	25. 33,5	11,1	9
22	21. 55. 30,9	8. 12,5	56,8	14. 57. 49,5	18. 59,8	40,9	3. 27. 27,0	25. 39,3	9,9	8
23	21. 46. 55,8	8. 35,1	56,6	14. 38. 30,6	19. 18,9	40,1	3. 1. 42,5	25. 44,5	8,7	7
24	21. 37. 57,8	8. 58,0	56,3	14. 18. 52,9	19. 37,7	39,3	2. 35. 53,4	25. 49,1	7,4	6
25	21. 28. 37,2	9. 20,6	56,0	13. 58. 56,6	19. 56,3	38,5	2. 10. 0,5	25. 52,9	6,2	5
26	21. 18. 54,0	9. 43,2	55,6	13. 38. 42,0	20. 14,6	37,6	1. 44. 4,3	25. 56,2	5,0	4
27	21. 8. 48,1	10. 5,9	55,2	13. 18. 9,6	20. 32,4	36,7	1. 18. 5,5	25. 58,8	3,7	3
28	20. 58. 19,6	10. 28,5	54,9	12. 57. 19,5	20. 50,1	35,8	0. 52. 4,8	26. 0,7	2,5	2
29	20. 47. 28,7	10. 50,9	54,5	12. 36. 12,2	21. 7,3	34,9	0. 26. 2,7	26. 2,1	1,2	1
30	20. 36. 15,2	11. 13,5	54,1	12. 14. [illegible]	21. 24,2	34,0	0. 0. 0,0	26. 2,7	0,0	0
	Sig. V. —	Sig. XI. +		Sig. IV. —	Sig. X. +		Sig. III. —	Sig. IX. +		

TABLE XXVII.

The Angle of Position of any Point of the Ecliptic, according to the Obliquity of the Ecliptic 23°. 28', with the Variation for the Variation of the Obliquity by One Minute.

ARGUMENT. Right Ascension of the Point of the Ecliptic.

	Sig. O. +		Sig. VI —	Sig. I. +		Sig VII —	Sig. II +		Sig VIII. —	
	Ang. of Position.	Difference.	Variat.	Ang. of Position.	Difference.	Variat.	Ang. of Position.	Difference.	Variat.	
0°	23°. 28 0,0	0'. 13,7	60',0	20 10' 24",7	12' 54',6	50",8	11.29. 8',2	21.12",4	28",0	30
1	23. 27 46,3	0. 40,9	0 0	19. 57. 30,1	13. 16,3	50,2	11. 7. 52,8	21. 23,2	27,1	29
2	23. 27. 5,4	1. 8,1	59,9	19. 44. 13,8	13. 37,4	49,5	10. 46. 29,6	21. 33,6	26,2	28
3	23. 25. 57,3	1. 35,4	59,8	19. 30. 36,4	13. 59,2	48,9	10. 24. 56,0	21. 43,7	25,3	27
4	23 24. 21,9	2. 2,6	59,8	19. 16. 57,2	14. 19,6	48,4	10 3. 12,3	21. 53,4	24,4	26
5	23. 22. 19,9	2 29,6	59,7	19. 2. 17,6	14 40,0	47,6	9. 41. 18,9	22 2,6	23,5	25
6	23. 19. 49,7	2. 56,7	59,6	18. 47. 37,6	15. 0,1	47,0	9. 19. 16,3	22. 11,6	22,6	24
7	23. 16 53,0	3 23,6	59,4	18. 32. 37,5	15. 19,8	46,3	8. 57. 4,7	22. 20,1	21,7	23
8	23 13. 29,4	3. 50,6	59,2	18. 17. 17,7	15. 39,1	45,6	8. 34. 44,6	22. 28,2	20,8	22
9	23. 9. 3[illegible],8	4. 17,3	59,0	18. 1. 38,6	15 58,2	44,9	8. 12. 16,4	22. 36,1	19,9	21
10	23. 5. 21,5	4. 48,9	58,8	17. 45. 40,4	16. 16,9	44,3	7. 49. 40,3	22. 43,4	19,0	20
11	23. 0. 37,6	5. 10,4	58,6	17. 29. 23,5	16 35,2	43,6	7 26. 56,9	22. 50,4	18,1	19
12	22 55 27,2	5. 36,8	58,4	17. 12. 48,3	16. 53,1	42,9	7. 4. 6,5	22. 57,2	17,1	18
13	22. 49 50,4	6. 2,9	58,1	16. 55. 55,2	17 10,8	42,1	6. 41. 9,3	23. 3,4	16,2	17
14	22. 43. 47,5	6. 29,0	57,9	16. 38. 44,4	17 27,9	41,4	6. 18. 5,9	23. 9,3	15,2	16
15	22. 37. 18,5	6. 54,9	57,6	16. 21. 16,5	17. 44,9	40,5	5. 54. 56,6	23. 14,8	14,2	15
16	22. 30. 23,6	7. 20,5	57,3	16 3. 31,6	18. 1,2	39,6	5. 31. 41,8	23. 20,0	13,3	14
17	22 23 3,1	7 45,8	57,0	15 45 30,4	18. 17,4	38,8	5. 8. 21,8	23. 24. 8	12,3	13
18	22. 15 17,3	8. 11,5	56,7	15 27. 13,0	18. 33,2	38,1	4. 44. 57,0	23. 29,3	11,4	12
19	22 7. 5,8	8. 36,3	56,2	15 8. 39,8	18. 48,4	37,3	4. 21. 27,7	23. 33,3	10,4	11
20	21. 58. 29,5	9. 1,0	55,8	14. 49 51,4	19 3,5	36,4	3. 57. 54,4	23. 36,9	9,5	10
21	21. 49. 28,5	9. 25,7	55,3	14. 30 47,9	19. 18,0	35,6	3. 34. 17,5	23. 40,3	8,5	9
22	21. 40. 2,8	9. 50,0	54,8	14 11 29,9	19. 32,3	34,8	3. 10. 37,2	23. 43,2	7,6	8
23	21 30 12,8	10. 14,0	54,4	13 51 57,6	19. 46,2	34,1	2. 46. 54,0	23. 45,8	6,6	7
24	21. 19. 58,8	10. 37,8	53,9	13 32 11,4	19. 59,5	33,2	2. 23. 8,2	23 48,0	5,7	6
25	21. 9 21,0	11. 1,4	53,4	13. 12 11,9	20. 12,7	32,3	1 59 20,2	23. 49,9	4 7	5
26	20. 58. 19,6	11 24,6	52,9	12 51. 59,2	20. 25,4	31,5	1 35. 30,3	23 51,2	3,8	4
27	20. 46. 55,0	11. 47,5	52,8	12 31 33,8	20. 37,7	30,6	1. 11. 39,1	23 52,5	2,9	3
28	20. 35. 7,5	12. 10,2	51,7	12 10. 56,1	20. 49,7	29,8	0. 47. 46,6	23. 53,1	1,9	2
29	20 22. 57,3	12. 32,6	51,2	11 50 6,4	21. 1,2	28,9	0. 23. 53,5	23. 53,5	1,0	1
30	20. 11. 24,7		50,5	11 29. 5,2		28,0	0. 0 0,0		0,0	0
	Sig V. —		Sig. XI +	Sig. IV —		Sig. X +.	Sig. III. —		Sig XI. +	

TABLE XXVIII.

Augmentation of the Angle of Position of a Point in the Ecliptic, for the Latitude of the Zodiacal Stars.

Ecliptic Angle of Position.	LATITUDE OF THE STAR.												
	0°. 30′	1°. 0′	1°. 30′	2°. 0′	2°. 30′	3°. 0′	3°. 30′	4°. 0′	4° 30′	5° 0′	5°. 30′	6°. 0′	6°. 30′
°	″	″	″	″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″	′ ″
0	0	0	0	0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0	0. 0
1	0,1	0,5	1,2	2,2	0. 3,4	0. 4,9	0. 6,6	0. 8,8	0. 11,1	0. 13,7	0. 16,6	0. 19,8	0. 23,3
2	0,2	1,1	2,4	4,4	0. 6,8	0. 9,9	0. 13,4	0. 17,6	0. 22,2	0. 27,5	0. 33,3	0. 39,7	0. 46,6
3	0,4	1,6	3,7	6,6	0. 10,3	0. 14,8	0. 20,2	0. 26,4	0. 33,4	0. 41,3	0. 50,0	0. 59,6	1. 9,9
4	0,5	2,2	4,9	8,8	0. 13,7	0. 19,8	0. 26,9	0. 35,2	0. 44,6	0. 55,1	1. 6,7	1. 19,5	1. 33,3
5	0,7	2,8	6,2	11,0	0. 17,2	0. 24,7	0. 33,8	0. 44,0	0. 55,8	1. 9,0	1. 23,4	1. 39,4	1. 56,7
6	0,8	3,3	7,4	13,2	0. 20,6	0. 29,7	0. 40,5	0. 52,9	1. 7,0	1. 22,8	1. 40,2	1. 59,4	2. 20,2
7	1,0	3,8	8,6	15,4	0. 24,1	0. 34,7	0. 47,3	1. 1,8	1. 18,3	1. 36,7	1. 57,1	2. 19,5	2. 43,8
8	1,1	4,4	9,9	17,6	0 27,6	0. 39,7	0. 54,2	1. 10,7	1. 29,6	1. 50,7	2. 14,0	2. 39,7	3. 7,5
9	1,2	5,0	11,2	19,8	0. 31,0	0. 44,8	1. 1,0	1. 19,8	1. 41,0	2. 4,8	2. 31,1	3. 0,0	3. 31,3
10	1,4	5,5	12,4	22,1	0. 34,6	0. 49,9	1. 7,9	1. 28,8	1. 52,4	2. 19,0	2. 48,2	3. 20,3	3. 55,3
11	1,5	6,1	13,7	24,4	0. 38,2	0. 55,0	1. 14,9	1. 37,9	2. 3,9	2. 33,1	3. 5,4	3. 40,9	4. 19,4
12	1,7	6,7	15,0	26,7	0. 41,7	1. 0,1	1. 21,9	1. 47,0	2. 15,5	2. 47,4	3. 22,8	4. 1,5	4. 43,6
13	1,8	7,2	16,3	29,0	0. 45,3	1. 5,3	1. 29,0	1. 56,3	2. 27,2	3. 1,9	3. 40,2	4. 22,3	5. 8,1
14	1,9	7,8	17,6	31,4	0. 48,9	1. 10,5	1. 36,1	2. 5,5	2. 39,0	3. 16,4	3. 57,8	4. 43,3	5. 32,8
15	2,1	8,4	18,9	33,7	0. 52,5	1. 15,8	1. 43,2	2. 14,9	2. 50,9	3. 31,1	4. 15,6	5. 4,5	5. 57,6
16	2,2	9,0	20,3	36,1	0. 56,3	1. 21,1	1. 50,5	2. 24,4	3. 2,9	3. 45,9	4. 33,6	5. 25,7	6. 22,7
17	2,4	9,6	21,6	38,4	1. 0,0	1. 26,5	1. 57,8	2. 34,0	3. 15,0	4. 0,9	4. 51,7	5. 47,4	6. 48,1
18	2,5	10,2	22,9	40,9	1. 3,8	1. 32,0	2. 5,3	2 43,6	3. 27,2	4. 16,0	5. 10,0	6. 9,2	7. 13,9
19	2,7	10,8	24,3	43,3	1. 7,6	1. 37,4	2. 12,7	2. 53,4	3. 39,6	4. 31,3	5. 28,5	6. 31,3	7 39,6
20	2,9	11,4	25,7	45,8	1. 11,5	1. 43,0	2. 20,3	3. 8,3	3. 52,2	4. 46,8	5. 47,3	6. 53,6	8. 5,9
21	3,0	12,0	27,1	48,2	1. 15,4	1. 48,7	2. 27,9	3. 13,3	4. 4,9	5. 2,5	6. 6,3	7. 16,3	8. 32,5
22	3,1	12,7	28,5	50,7	1. 19,3	1. 54,4	2. 35,7	3. 23,5	4. 17,7	5. 18,4	6. 25,5	7. 39,2	8. 59,4
23	3,3	13,3	30,0	53,3	1. 23,4	2. 0,0	2. 43,6	3. 33,7	4. 30,8	5. 34,5	6. 45,1	8. 2,5	9. 26,9
23 28	3,4	13,7	30,7	54,5	1. 25,3	2. 2,9	2. 47,4	3. 38,7	4. 36,9	5. 42,1	6. 54,1	8. 13,4	9. 39,6

Epochs of the mean Longitude of the Moon's Ascending Node.

O. S.	S. D. M.	O. S.	S. D. M.	N. S.	S. D. M.	N. S.	S. D. M.	N. S.	S. D. M.	N. S.	S. D. M.
1690	0. 0. 15	1722	3. 11. 19	1753	7. 12. 17	1785	10. 23. 20	1817	2. 4. 27	1849	5. 15. 31
1691	11. 10. 55	1723	2. 21. 59	1754	6. 22. 57	1786	10. 4. 1	1818	1. 15. 7	1850	4. 26. 11
1692	10. 21. 32	1724	2. 2. 36	1755	6. 3. 37	1787	9. 14. 41	1819	0. 25. 48	1851	4. 6. 51
1693	10. 2. 13	1725	1. 13. 16	1756	5. 14. 14	1788	8. 25. 18	1820	0. 6. 25	1852	3. 17. 28
1694	9. 12. 53	1726	0. 23. 56	1757	4. 24. 55	1789	8. 5. 58	1821	11. 17. 5	1853	2. 28. 8
1695	8. 23. 33	1727	0. 4. 37	1758	4. 5. 35	1790	7. 16. 39	1822	10. 27. 45	1854	2. 8. 48
1696	8. 4. 10	1728	11. 15. 14	1759	3. 16. 15	1791	6. 27. 19	1823	10. 8. 26	1855	1. 19. 29
1697	7. 14. 51	1729	10. 25. 54	1760	2. 26. 52	1792	6. 7. 56	1824	9. 19. 8	1856	1. 0. 6
1698	6. 25. 31	1730	10. 6. 34	1761	2. 7. 33	1793	5. 18. 36	1825	8. 29. 43	1857	0. 10. 46
1699	6. 6. 11	1731	9. 17. 15	1762	1. 18. 13	1794	4. 29. 17	1826	8. 10 23	1858	11. 21. 26
1700	5. 16. 48	1732	8. 27. 52	1763	0. 28. 53	1795	4. 9. 57	1827	7. 21. 4	1859	11. 2. 7
1701	4. 27. 29	1733	8. 8. 32	1764	0. 9. 30	1796	3. 20. 34	1828	7. 1. 41	1860	10 12. 44
1702	4. 8. 9	1734	7. 19. 12	1765	11. 20. 11	1797	3. 1. 14	1829	6. 12. 21	1861	9. 23 24
1703	3. 18. 49	1735	6. 29. 53	1766	11. 0. 51	1798	2. 11. 55	1830	6. 23. 1	1862	9. 4. 4
1704	2. 29. 26	1736	6. 10. 30	1767	10. 11. 31	1799	1. 22. 35	1831	5. 3. 42	1863	8. 14. 45
1705	2. 10. 6	1737	5. 21. 10	1768	9. 22. 8	1800	1. 3. 16	1832	4. 14. 19	1864	7. 25. 22
1706	1. 20. 47	1738	5. 1. 50	1769	9. 2. 49	1801	0. 13. 55	1833	3. 24. 59	1865	7. 6. 2
1707	1. 1. 27	1739	4. 12. 31	1770	8. 13. 29	1802	11. 24. 36	1834	3. 5. 39	1866	6. 16. 42
1708	0. 12. 4	1740	3. 23. 8	1771	7. 24. 9	1803	11. 5. 16	1835	2. 16. 20	1867	5. 27. 23
1709	11. 22. 44	1741	3. 3. 48	1772	7. 4. 46	1804	10. 15. 53	1836	1. 26. 57	1868	5. 8. 0
1710	11. 3. 25	1742	2. 14. 28	1773	6. 15. 27	1805	9. 26. 33	1837	1. 7. 37	1869	4. 18. 40
1711	10. 14. 5	1743	1. 25. 9	1774	5. 26. 7	1806	9. 7. 14	1838	0. 18. 17	1870	3. 29. 20
1712	9. 24. 42	1744	1. 5. 46	1775	5. 6. 47	1807	8. 17. 54	1839	11. 28. 58	1871	3. 10. 1
1713	9. 5. 22	1745	0. 16. 26	1776	4. 17. 24	1808	7. 28. 31	1840	11. 9. 35	1872	2. 20. 38
1714	8. 16. 3	1746	11. 27. 6	1777	3. 28. 5	1809	7. 9 11	1841	10. 20. 15	1873	2. 1. 18
1715	7. 26. 43	1747	11. 7. 47	1778	3. 8. 45	1810	6. 19. 52	1842	10. 0. 55	1874	1. 11. 58
1716	7. 7. 20	1748	10. 18. 24	1779	2. 19. 25	1811	6. 0. 32	1843	9. 11. 35	1875	0. 22. 39
1717	6. 18. 0	1749	9. 29. 4	1780	2. 0. 2	1812	5. 11. 9	1844	8. 22. 13	1876	0. 3. 16
1718	5. 28. 41	1750	9. 9. 44	1781	1. 10. 42	1813	4. 21. 48	1845	8. 2. 53	1877	11. 13. 56
1719	5. 9. 21	1751	8. 20. 24	1782	0. 21. 23	1814	4. 2. 30	1846	7. 13. 33	1878	10. 24. 36
1720	4. 19. 58	1752	8. 1. 2	1783	0. 2. 3	1815	3. 13. 10	1847	6. 24. 13	1879	10. 5. 17
1721	4. 0. 38	NS 1752	8. 1. 37	1784	11. 12. 40	1816	2. 23. 47	1848	6. 4. 51	1880	9 15. 54

TABLE XXX.

Mean Retrograde Motion of the Moon's Node, to every Day in the Year.

Subtract from the Longitude of the Epoch.

Days.	January.	February.	March.	April.	May.	June.	July.	August.	Septemb.	October.	Novemb.	Decemb.
1	0°. 5′	1°. 42′	3°. 11′	4°. 49′	6°. 24′	8°. 3′	9°. 38′	11°. 17′	12°. 55′	14°. 31′	16°. 9′	17°. 44′
2	0. 6	1. 45	3. 14	4. 52	6. 28	8. 6	9. 41	11. 20	12. 58	14. 34	16. 12	17. 48
3	0 10	1 48	3 17	4. 55	6. 31	8. 9	9. 45	11. 23	13. 2	14. 37	16. 15	17. 51
4	0. 13	1. 51	3. 20	4. 59	6. 34	8. 12	9. 48	11. 26	13. 5	14. 40	16. 19	17. 54
5	0. 16	1 54	3. 23	5 2	6. 37	8. 16	9. 51	11. 29	13. 8	14. 43	16. 22	17. 57
6	0. 19	1. 58	3. 27	5 5	6. 40	8. 19	9 54	11. 33	13. 11	14. 46	16. 25	18. 0
7	0. 22	2. 1	3. 30	5. 8	6 44	8. 22	9. 57	11 36	13. 14	14. 50	16. 28	18. 3
8	0. 25	2. 4	3. 33	5. 11	6. 47	8. 25	10. 1	11. 39	13. 17	14. 53	16. 31	18. 7
9	0. 29	2. 7	3. 36	5. 15	6. 50	8. 28	10. 4	11. 42	13. 21	14. 56	16. 34	18. 10
10	0. 32	2 10	3. 39	5. 18	6. 53	8. 32	10. 7	11. 45	13. 24	14. 59	16. 38	18. 13
11	0. 35	2. 13	3. 42	5. 21	6. 56	8. 35	10. 10	11. 49	13. 27	15. 2	16. 41	18. 16
12	0. 38	2. 17	3. 46	5. 24	6. 59	8. 38	10. 13	11. 52	13. 30	15. 6	16. 44	18. 19
13	0. 41	2. 20	3. 49	5. 27	7. 3	8. 41	10. 16	11. 55	13. 33	15. 9	16. 47	18. 23
14	0. 44	2. 23	3. 52	5. 30	7. 6	8. 44	10. 20	11. 58	13. 37	15. 12	16. 50	18. 26
15	0. 48	2. 26	3. 55	5. 34	7. 9	8. 47	10. 23	12. 1	13. 40	15. 15	16. 54	18 29
16	0. 51	2 29	3. 58	5. 37	7. 12	8. 51	10. 26	12. 4	13. 43	15. 18	16. 57	18. 32
17	0. 54	2. 33	4. 1	5. 40	7. 15	8. 54	10. 29	12. 8	13. 46	15. 21	17. 0	18. 35
18	0. 57	2. 36	4. 5	5. 43	7. 18	8. 57	10. 32	12. 11	13 49	15. 25	17. 3	18. 38
19	1. 0	2. 39	4. 8	5. 46	7. 22	9. 0	10. 36	12. 14	13. 52	15. 28	17. 6	18. 42
20	1. 4	2. 42	4. 11	5. 50	7. 25	9. 3	10. 39	12. 17	13. 56	15 31	17. 9	18. 45
21	1. 7	2. 45	4. 14	5. 53	7. 28	9. 7	10. 42	12. 20	13. 59	15. 34	17 13	18. 48
22	1. 10	2. 48	4. 17	5. 56	7. 31	9. 10	10. 45	12. 23	14. 2	15. 37	17. 16	18. 51
23	1. 13	2. 52	4. 21	5. 59	7. 34	9. 13	10. 48	12. 27	14. 5	15. 40	17. 19	18. 54
24	1. 16	2. 55	4. 24	6. 2	7. 38	9. 16	10. 51	12. 30	14. 8	15. 44	17. 22	18. 57
25	1. 19	2. 58	4. 27	6. 5	7. 41	9. 19	10. 55	12. 33	14 11	15. 47	17. 25	19. 1
26	1. 23	3. 1	4. 30	6. 9	7. 44	9. 22	10. 58	12. 36	14. 15	15. 50	17. 28	19. 4
27	1 26	3. 4	4. 33	6. 12	7. 47	9. 26	11. 1	12. 39	14. 18	15. 53	17. 32	19. 7
28	1. 29	3. 7	4. 36	6. 15	7 50	9. 29	11. 4	12. 43	14. 21	15. 56	17. 35	19. 10
29	1. 32		4. 40	6. 18	7. 53	9. 32	11. 7	12. 46	14. 24	16. 0	17. 38	19. 13
30	1. 35		4 43	6. 21	7. 57	9. 35	11. 10	12. 49	14. 27	16. 3	17. 41	19. 17
31	1. 38		4. 46		8. 0		11. 14	12. 52		16. 6		19. 20

In the months of January and February in leap-year, take out for the day preceding the given day.

TABLE XXXI.

owing the Decrease of the Diameters of the Sun or Moon, which are inclined to the Horizon; supposing the apparent Diameter to be 30′.

Inclination to Horizon.	ALTITUDE OF THE SUN OR MOON.							
	10°	11°	12°	13°	14°	16°	18°	20°
°	″	″	″	″	″	″	″	″
0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
3	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
6	0,2	0,1	0,1	0,1	0,1	0,1	0,0	0,0
9	0,4	0,3	0,3	0,2	0,2	0,2	0,1	0,1
12	0,7	0,6	0,5	0,4	0,4	0,3	0,2	0,2
15	1,0	0,9	0,7	0,6	0,6	0,4	0,3	0,3
18	1,5	1,3	1,1	0,9	0,8	0,6	0,5	0,4
21	2,0	1,7	1,4	1,2	1,1	0,8	0,7	0,5
24	2,6	2,2	1,8	1,6	1,4	1,1	0,9	0,7
27	3,2	2,7	2,3	2,0	1,7	1,3	1,1	0,9
30	3,9	3,3	2,8	2,4	2,1	1,6	1,3	1,1
33	4,6	3,9	3,3	2,8	2,5	1,9	1,5	1,3
36	5,4	4,5	3,8	3,3	2,9	2,2	1,8	1,5
39	6,2	5,2	4,3	3,7	3,3	2,6	2,0	1,7
42	7,0	5,9	4,9	4,2	3,7	2,9	2,3	1,9
45	7,7	6,5	5,4	4,7	4,2	3,2	2,6	2,1
48	8,5	7,1	6,0	5,2	4,6	3,5	2,8	2,3
51	9,3	7,8	6,6	5,7	5,0	3,9	3,1	2,5
54	10,1	8,5	7,2	6,2	5,4	4,2	3,4	2,7
57	10,8	9,1	7,7	6,6	5,8	4,5	3,6	2,9
60	11,5	9,7	8,1	7,0	6,2	4,8	3,9	3,1
63	12,2	10,3	8,6	7,4	6,6	5,1	4,1	3,3
66	12,9	10,8	9,1	7,8	6,9	5,3	4,3	3,5
69	13,4	11,3	9,6	8,2	7,2	5,6	4,5	3,7
72	13,9	11,7	9,9	8,5	7,5	5,8	4,7	3,8
75	14,4	12,1	10,2	8,8	7,7	6,0	4,8	3,9
78	14,7	12,4	10,5	9,1	7,9	6,1	4,9	4,0
81	15,0	12,6	10,6	9,3	8,1	6,2	5,0	4,1
84	15,2	12,7	10,7	9,3	8,2	6,3	5,1	4,2
87	15,3	12,8	10,8	9,4	8,2	6,4	5,1	4,2
90	15,3	12,8	10,9	9,4	8,3	6,4	5,1	4,2

TABLE XXXI. *Continued.*

Inclination to Horizon.	ALTITUDE OF THE SUN OR MOON.							
	22°	26°	30°	[illegible]	40°	46°	52°	70°
°	"	"	"	"	"	"	"	"
0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
3	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
6	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
9	0,1	0,1	0,0	0,0	0,0	0,0	0,0	0,0
12	0,1	0,1	0,1	0,1	0,0	0,0	0,0	0,0
15	0,2	0,2	0,1	0,1	0,1	0,1	0,1	0,0
18	0,3	0,2	0,2	0,2	0,1	0,1	0,1	0,0
21	0,5	0,3	0,3	0,2	0,2	0,1	0,1	0,1
24	0,6	0,4	0,3	0,3	0,2	0,2	0,1	0,1
27	0,7	0,5	0,4	0,3	0,2	0,2	0,2	0,1
30	0,9	0,6	0,5	0,4	0,3	0,2	0,2	0,1
33	1,0	0,8	0,6	0,5	0,3	0,3	0,2	0,2
36	1,2	0,9	0,7	0,5	0,4	0,3	0,3	0,2
39	1,4	1,0	0,8	0,6	0,5	0,4	0,3	0,2
42	1,6	1,1	0,9	0,7	0,5	0,4	0,4	0,3
45	1,8	1,3	1,0	0,8	0,6	0,5	0,4	0,3
48	2,0	1,4	1,1	0,9	0,7	0,5	0,4	0,3
51	2,2	1,6	1,2	1,0	0,7	0,6	0,5	0,3
54	2,3	1,7	1,3	1,0	0,8	0,6	0,5	0,4
57	2,5	1,8	1,4	1,1	0,8	0,7	0,6	0,4
60	2,6	1,9	1,5	1,2	0,9	0,7	0,6	0,4
63	2,8	2,0	1,6	1,3	0,9	0,8	0,6	0,4
66	2,9	2,1	1,7	1,3	1,0	0,8	0,7	0,5
69	3,1	2,2	1,7	1,4	1,0	0,8	0,7	0,5
72	3,2	2,3	1,8	1,4	1,1	0,9	0,7	0,5
75	3,3	2,4	1,8	1,5	1,1	0,9	0,7	0,5
78	3,4	2,5	1,9	1,5	1,1	0,9	0,8	0,5
81	3,4	2,5	1,9	1,5	1,2	0,9	0,8	0,5
84	3,5	2,6	1,9	1,6	1,2	0,9	0,8	0,6
87	3,5	2,6	2,0	1,6	1,2	1,0	0,8	0,6
90	3,5	2,6	2,0	1,6	1,2	1,0	0,8	0,6

TABLE XXXII.

For reducing Sidereal to Mean Solar Time.

Hours.	Min. Sec.	Minutes.	Sec.	Seconds.	Sec.
1	0. 9, 83	1	0, 16	1	0, 00
2	0. 19, 66	2	0, 33	2	0, 01
3	0. 29, 49	3	0, 49	3	0, 01
4	0. 39, 32	4	0, 66	4	0, 01
5	0. 49, 15	5	0, 82	5	0, 01
6	0. 58, 98	6	0, 98	6	0, 02
7	1. 8, 81	7	1, 15	7	0, 02
8	1. 18, 64	8	1, 31	8	0, 02
9	1. 28, 47	9	1, 47	9	0, 02
10	1. 38, 30	10	1, 64	10	0, 03
11	1. 48, 13	11	1, 80	11	0, 03
12	1. 57, 96	12	1, 97	12	0, 03
13	2. 7, 78	13	2, 13	13	0, 04
14	2. 17, 61	14	2, 29	14	0, 04
15	2. 27, 44	15	2, 46	15	0, 04
16	2. 37, 27	16	2, 62	16	0, 04
17	2. 47, 10	17	2, 78	17	0, 05
18	2. 56, 93	18	2, 95	18	0, 05
19	3. 6, 76	18	3, 11	19	0, 05
20	3. 16, 59	20	3, 28	20	0, 05
21	3. 26, 42	30	4, 91	30	0, 08
22	3. 36, 25	40	6, 55	40	0, 11
23	3. 46, 08	50	8, 19	50	0, 14
24	3. 55, 91	60	9, 83	60	0, 16

TABLE XXXIII.

For converting Mean Solar into Sidereal Time.

Hours.	Min. Sec.	Minutes.	Sec.	Seconds.	Sec.
1	0. 9, 86	1	0,16	1	0,00
2	0. 19, 71	2	0,33	2	0,01
3	0. 29, 57	3	0,49	3	0,01
4	0. 39, 43	4	0,66	4	0,01
5	0. 49, 28	5	0,82	5	0,01
6	0. 59, 14	6	0,99	6	0,02
7	1. 8, 99	7	1,15	7	0,02
8	1. 18, 85	8	1,31	8	0,02
9	1. 28, 71	9	1,48	9	0,02
10	1. 38, 56	10	1,64	10	0,03
11	1. 48, 42	11	1,82	11	0,03
12	1. 58, 28	12	1,97	12	0,03
13	2. 8, 13	13	2,14	13	0,04
14	2. 17, 99	14	2,30	14	0,04
15	2. 27, 85	15	2,46	15	0,04
16	2. 37, 70	16	2,63	16	0,04
17	2. 47, 56	17	2,79	17	0,05
18	2. 57, 42	18	2,96	18	0,05
19	3. 7, 27	19	3,12	19	0,05
20	3. 17, 13	20	3,28	20	0,05
21	3. 26, 98	30	4,93	30	0,08
22	3. 36, 84	40	6,57	40	0,11
23	3. 46, 70	50	8,21	50	0,14
24	3. 56, 55	60	9,86	60	0,16

ARGUMENT. Sun's mean Anomaly.

	Sig. O.		Sig. I.		Sig. II.		
Deg.	Sun's hor. mot.	Sun's semid.	Sun's hor. mot.	Sun's semid.	Sun's hor. mot.	Sun's semid.	Deg.
0	2'. 22",99	15'. 45",50	2'. 23",60	15'. 47",53	2'. 25",32	15'. 53",17	30
1	2. 22, 99	15. 45, 50	2. 23, 64	15. 47, 66	2. 25, 39	15. 53, 41	29
2	2. 22, 99	15. 45, 51	2. 23, 68	15. 47, 80	2. 25, 46	15. 53, 65	28
3	2. 22, 99	15. 45, 52	2. 23, 73	15. 47, 94	2. 25, 53	15. 53, 89	27
4	2. 23, 00	15. 45, 53	2. 23, 77	15. 48, 09	2. 25, 61	15. 54, 14	26
5	2. 23, 00	15. 45, 55	2. 23, 82	15. 48, 24	2. 25, 68	15. 54, 38	25
6	2. 23, 01	15. 45, 58	2. 23, 86	15. 48, 40	2. 25, 76	15. 54, 63	24
7	2. 23, 02	15. 45, 61	2. 23, 91	15. 48, 56	2. 25, 84	15. 54, 88	23
8	2. 23, 03	15. 45, 64	2. 23, 96	15. 48, 72	2. 25, 91	15. 55, 13	22
9	2. 23, 04	15. 45, 68	2. 24, 01	15. 48, 89	2. 25, 99	15. 55, 39	21
10	2. 23, 05	15. 45, 73	2. 24, 06	15. 49, 06	2. 26, 07	15. 55, 65	20
11	2. 23, 07	15. 45, 78	2. 24, 12	15. 49, 23	2. 26, 15	15. 55, 91	19
12	2. 23, 09	15. 45, 83	2. 24, 17	15. 49, 41	2. 26, 23	15. 56, 17	18
13	2. 23, 10	15. 45, 88	2. 24, 23	15. 49, 59	2. 26, 31	15. 56, 44	17
14	2. 23, 12	15. 45, 95	2. 24, 28	15. 49, 77	2. 26, 39	15. 56, 70	16
15	2. 23, 14	15. 46, 01	2. 24, 34	15. 49, 96	2. 26, 47	15. 56, 96	15
16	2. 23. 16	15. 46, 08	2. 24, 40	15. 50, 15	2. 26, 56	15. 57, 23	14
17	2. 23, 18	15. 46, 16	2. 24, 46	15. 50, 35	2. 26, 64	15. 57, 50	13
18	2. 23, 21	15. 46, 24	2. 24, 52	15. 50, 55	2. 26, 72	15. 57, 77	12
19	2. 23, 23	15. 46, 32	2. 24, 58	15. 50, 75	2. 26, 80	15. 58, 05	11
20	2. 23, 25	15. 46, 41	2. 24, 64	15. 50, 96	2. 26, 89	15. 58, 32	10
21	2. 23, 29	15. 46, 50	2. 24, 70	15. 51, 16	2. 26, 97	15. 58, 59	9
22	2. 23, 32	15. 46, 60	2. 24, 77	15. 51, 37	2. 27, 06	15. 58, 87	8
23	2. 23, 35	15. 46, 70	2. 24, 84	15. 51, 59	2. 27, 14	15. 59, 14	7
24	2. 23, 38	15. 46, 81	2. 24, 90	15. 51, 81	2. 27, 23	15. 59, 42	6
25	2. 23, 41	15. 46, 92	2. 24, 97	15. 52, 03	2. 27, 31	15. 59, 70	5
26	2. 23, 45	15. 47, 03	2. 25, 03	15. 52, 25	2. 27, 40	15. 59, 98	4
27	2. 23, 48	15. 47, 15	2. 25, 10	15. 52, 48	2. 27, 48	16. 0, 26	3
28	2. 23, 52	15. 47, 27	2. 25, 17	15. 52, 70	2. 27, 57	16. 0, 54	2
29	2. 23, 56	15. 47, 40	2. 25, 24	15. 52, 94	2. 27, 66	16. 0, 82	1
30	2. 23, 60	15. 47, 53	2. 25, 32	15. 53, 17	2. 27, 74	16. 1, 10	0
	Sig. XI.		Sig. X.		Sig. IX.		

TABLE XXXIV. *Continued.*

	Sig. III.		Sig. IV.		Sig. V.		
Deg.	Sun's hor. mot.	Sun's semid.	Sun's hor. mot.	Sun's semid.	Sun's hor. mot.	Sun's semid.	Deg.
0	2′. 27″, 74	16′. 1″, 10	2′. 30″, 28	16′. 9″, 30	2′. 32″, 20	16′. 15″, 49	30
1	2. 27, 83	16. 1, 38	2. 30, 35	16. 9, 55	2. 32, 24	16. 15, 63	29
2	2. 27, 92	16. 1, 67	2. 30, 43	16. 9, 80	2. 32, 29	16. 15, 78	28
3	2. 28, 00	16. 1, 95	2. 30, 51	16. 10, 05	2. 32, 33	16. 15, 92	27
4	2. 28, 09	16. 2, 23	2. 30, 58	16. 10, 29	2. 32, 37	16. 16, 05	26
5	2. 28, 18	16. 2, 51	2. 30, 66	16. 10, 54	2. 32, 41	16. 16, 18	25
6	2. 28, 26	16. 2, 79	2. 30, 73	16. 10, 77	2. 32, 45	16. 16, 30	24
7	2. 28, 35	16. 3, 07	2. 30, 80	16. 11, 01	2. 32, 49	16. 16, 42	23
8	2. 28, 44	16. 3, 36	2. 30, 88	16. 11, 24	2. 32, 53	16. 16, 54	22
9	2. 28, 52	16. 3, 64	2. 30, 95	16. 11, 45	2. 32, 56	16. 16, 65	21
10	2. 28, 61	16. 3, 92	2. 31, 02	16. 11, 68	2. 32, 59	16. 16, 75	20
11	2. 28, 70	16. 4, 20	2. 31, 09	16. 11, 92	2. 32, 62	16. 16, 85	19
12	2. 28, 78	16. 4, 48	2. 31, 16	16. 12, 14	2. 32, 65	16. 16, 95	18
13	2. 28, 87	16. 4, 76	2. 31, 22	16. 12, 36	2. 32, 68	16. 17, 04	17
14	2. 28, 96	16. 5, 04	2. 31, 29	16. 12, 57	2. 32, 71	16. 17, 12	16
15	2. 29, 04	16. 5, 31	2. 31, 36	16. 12, 78	2. 32, 73	16. 17, 20	15
16	2. 29, 13	16. 5, 59	2. 31, 42	16. 12, 99	2. 32, 76	16. 17, 28	14
17	2. 29, 21	16. 5, 86	2. 31, 48	16. 13, 20	2. 32, 78	16. 17, 35	13
18	2. 29, 30	16. 6, 14	2. 31, 55	16. 13, 39	2. 32, 80	16. 17, 41	12
19	2. 29, 38	16. 6, 41	2. 31, 61	16. 13, 59	2. 32, 82	16. 17, 47	11
20	2. 29, 46	16. 6, 68	2. 31, 67	16. 13, 78	2. 32, 84	16. 17, 53	10
21	2. 29, 54	16. 6, 91	2. 31, 73	16. 13, 97	2. 32, 86	16. 17, 58	9
22	2. 29, 63	16. 7, 22	2. 31, 78	16. 14, 16	2. 32, 87	16. 17, 62	8
23	2. 29, 71	16. 7, 49	2. 31, 84	16. 14, 34	2. 32, 88	16. 17, 66	7
24	2. 29, 80	16. 7, 75	2. 31, 90	16. 14, 51	2. 32, 89	16. 17, 70	6
25	2. 29, 88	16. 8, 02	2. 31, 95	16. 14, 69	2. 32, 90	16. 17, 73	5
26	2. 29, 96	16. 8, 28	2. 32, 00	16. 14, 85	2. 32, 91	16. 17, 75	4
27	2. 30, 04	16. 8, 54	2. 32, 05	16. 15, 02	2. 32, 91	16. 17, 77	3
28	2. 30, 12	16. 8, 79	2. 32, 10	16. 15, 18	2. 32, 92	16. 17, 78	2
29	2. 30, 20	16. 9, 05	2. 32, 15	16. 15, 34	2. 32, 92	16. 17, 79	1
30	2. 30, 28	16. 9, 30	2. 32, 20	16. 15, 49	2. 32, 92	16. 17, 79	0
	Sig. VIII.		Sig. VII.		Sig. VI.		

A:

	Sig. O.	Sig. VI.								
	−	Diff.	Var.	−						
0°. 0'	0°. 0'. 0",0	2'. 28",9	0",0	2°. 5'. 4[illegible]						
0. 30	0. 2. 28,9	2. 28, 9	0, 2	2. 7. 2,1						.
1. 0	0. 4. 57,8	2. 28, 8	0, 4	2. 8. 19,6						0
1. 30	0. 7. 26,6	2. 28, 7	0, 6	2. 9. 34,9	1					30
2. 0	0. 9. 55,3	2. 28, 5	0, 8	2. 10. 47,9	1. 1.					8. 0
2. 30	0. 12. 23,8	2. 26, 3	1, 0	2. 11. 58,7	1. 8,					27. 30
3. 0	0. 14. 52,1	2. 28, 1	1, 3	2. 13. 7,3	1. 6				1	27. 0
3. 30	0. 17. 20,2	2. 27, 8	1, 5	2. 14. 13,6	1. 4, 0	.			,9	26. 30
4. 0	0. 19. 48,0	2. 27, 5	1, 7	2. 15. 17,0	1. 1, 5	11			10, 8	26. 0
4. 30	0. 22. 15,5	2. 27, 1	1, 9	2. 16. 19,1	0. 59, 1	11, 8			10, 7	25. 30
5. 0	0. 24. 42,6	2. 26, 7	2, 1	2. 17. 18,2	0. 56, 9	11, 8	.	6	10, 5	25. 0
5. 30	0. 27. 9,3	2. 26, 3	2, 3	2. 18. 15,1	0. 54, 4	11, 9	1.	1, 9	10, 4	24. 30
6. 0	0. 29. 35,6	2. 25, 9	2, 5	2. 19. 9,5	0. 51, 9	12, 0	1. 53.	44, 2	10, 2	24. 0
6. 30	0. 32. 1,5	2. 25, 3	2, 7	2. 20. 1,4	0. 49, 6	12, 1	1. 51. 4	46, 3	10, 1	23. 30
7. 0	0. 34. 26,8	2. 24, 7	2, 9	2. 20. 51,0	0. 47, 1	12, 2	1. 49. 58,	1. 48, 5	9, 9	23. 0
7. 30	0. 36. 51,5	2. 24, 1	3, 1	2. 21. 38,1	0. 44, 5	12, 2	1. 48. 10,0	1. 50, 7	9, 8	22. 30
8. 0	0. 39. 15,6	2. 23, 5	3, 3	2. 22. 22,6	0. 42, 0	12, 3	1. 46. 19,3	1. 52, 8	9, 6	22. 0
8. 30	0. 41. 39,1	2. 22, 9	3, 5	2. 23. 4,6	0. 39, 5	12, 4	1. 44. 26,5	1. 55, 0	9, 4	21. 30
9. 0	0. 44. 2,0	2. 22. 2	3, 7	2. 23. 44,1	0. 36, 9	12, 5	1. 42. 31,5	1. 57, 0	9, 3	21. 0
9. 30	0. 46. 24,2	2. 21, 3	3, 9	2. 24. 21,0	0. 34, 5	12, 5	1. 40. 34,5	1. 59, 0	9, 1	20. 30
10. 0	0. 48. 45,5	2. 20, 6	4, 1	2. 24. 55,5	0. 31, 8	12, 6	1. 38. 35,5	2. 1. 0	8, 9	20. 0
10. 30	0. 51. 6,1	2. 19, 8	4, 3	2. 25. 27,3	0. 29, 3	12, 6	1. 36. 34,5	2. 3, 0	8, 8	19. 30
11. 0	0. 53. 25,9	2. 18, 9	4, 5	2. 25. 56,6	0. 26, 6	12, 7	1. 34. 31,5	2. 4, 8	8, 6	19. 0
11. 30	0. 55. 44,8	2. 18, 0	4, 7	2. 26. 23,2	0. 24, 0	12, 7	1. 32. 26,7	2. 6, 8	8, 4	18. 30
12. 0	0. 58. 2,8	2. 17, 1	4, 9	2. 26. 47,2	0. 21, 5	12, 8	1. 30. 19,9	2. 8, 7	8, 2	18. 0
12. 30	1. 0. 19,9	2. 16, 0	5, 1	2. 27. 8,7	0. 18, 6	12, 8	1. 28. 11,2	2. 10, 2	8, 0	17. 30
13. 0	1. 2. 35,9	2. 15, 0	5, 3	2. 27. 27,3	0. 16, 0	12, 9	1. 26. 1,0	2. 12, 1	7, 8	17. 0
13. 30	1. 4. 50,9	2. 14, 0	5, 5	2. 27. 43,3	0. 13, 5	12, 9	1. 23. 48,9	2. 13, 8	7, 6	16. 30
14. 0	1. 7. 4,9	2. 12, 9	5, 7	2. 27. 56,8	0. 10, 8	12, 9	1. 21. 35,1	2. 15, 4	7, 4	16. 0
14. 30	1. 9. 17,8	2. 11, 8	5, 8	2. 28. 7,6	0. 8, 0	13, 0	1. 19. 19,7	2. 17, 1	7, 2	15. 30
15. 0	1. 11. 29,6		6, 0	2. 28. 15,6		13, 0	1. 17. 2,6		7, 0	15. 0
	+			+			+			
	Sig. V.	Sig. XI.		Sig. IV.	Sig. X.		Sig. III.	Sig. IX.		

The Declination of the Points of the Ecliptic, according to the Obliquity of the Ecliptic 23°. 28'. 15"; with the Variation of Declination for the Variation of the Obliquity of One Minute.

Argument. Longitude of the Point of the Ecliptic.

	Sig. O. +	Sig. VI. —		Sig. I. +	Sig. VII. —		Sig. II. +	Sig. VIII. —		
	Declination.	Difference.	Variat.	Declination.	Difference.	Variat.	Declination.	Difference.	Variat.	
0°. 0'	0°. 0'. 0"	11'. 57",0	0",0	11°. 29'. 12",3	10'. 32",1	28,1	20°. 10'. 37",4	6'. 19",2	50',8	30°. 0'
0. 30	0. 11. 57, 0	11. 56, 8	0, 5	11. 39 44,4	10. 29, 3	28, 5	20. 16 56, 6	6. 13, 5	51, 1	29. 30
1. 0	0. 23. 53, 8	11 56, 7	1, 0	11. 50. 13 7	10. 26, 4	29, 0	20. 23 10, 1	6. 8, 0	51, 4	29. 0
1. 30	0. 35. 50, 5	11. 56, 6	1, 4	12. 0. 40,1	10. 23, 5	29, 4	20. 29. 18, 1	6. 2, 4	51, 7	28. 30
2. 0	0. 47. 47, 1	11. 56, 4	1, 9	12. 11. 3,6	10. 20, 5	29, 9	20. 35. 20, 5	5. 56, 7	51, 9	28. 0
2. 30	0. 59. 43, 5	11. 56, 3	2, 4	12. 21. 24,1	10. 17, 4	30, 3	20. 41. 17, 2	5. 51, 0	52, 2	27. 30
3. 0	1 11. 39, 8	11. 55, 9	2, 9	12. 31. 41,5	10. 14, 3	30, 7	20. 47. 8, 2	5. 45, 2	52, 4	27. 0
3. 30	1. 23 35, 7	11. 55, 6	3, 4	12. 41 55,8	10. 11, 3	31, 2	20. 52. 53, 4	5. 39, 5	52, 7	26. 30
4. 0	1. 35 31, 3	11. 55, 3	3, 8	12 52 7,1	10. 8, 1	31, 6	20. 58. 32, 9	5. 33, 7	53, 0	26 0
4. 30	1. 47. 26, 6	11. 54, 8	4, 3	13. 2. 15,2	10. 4, 8	32, 0	21. 4. 6, 6	5. 27, 8	53, 2	25 30
5. 0	1. 59. 21, 4	11. 54, 5	4, 8	13. 12. 20,0	10. 1, 5	32, 4	21. 9. 34, 4	5. 21, 9	53, 5	25. 0
5. 30	2 11. 15, 9	11. 53, 8	5, 3	13. 22 21,5	9. 58, 3	32, 8	21. 14. 56, 3	5. 16, 0	53, 7	24. 30
6. 0	2. 23. 9, 7	11. 53, 3	5, 8	13 32. 19,8	9. 54, 9	33, 3	21. 20. 12, 3	5. 10, 1	54, 0	24. 0
6. 30	2. 35. 3, 0	11. 52 7	6, 2	13. 42. 14,7	9. 51, 5	33, 7	21. 25. 22, 4	5. 4, 1	54, 2	23. 30
7. 0	2. 46. 55, 7	11. 52, 2	6, 7	13. 52. 6,2	9. 48, 0	34, 1	21. 30 26, 5	4. 58, 2	54, 5	23. 0
7. 30	2 58. 47, 9	11. 51, 3	7, 2	14. 1. 54,2	9. 44, 4	34, 5	21. 35. 24, 7	4. 51, 9	54, 7	22. 30
8. 0	3. 10. 39, 2	11. 50, 7	7, 7	14. 11. 38,6	9. 40, 8	35, 0	21. 40. 16, 6	4. 45, 9	54, 9	22. 0
8. 30	3. 22. 29, 9	11. 49, 8	8, 2	14. 21 19,4	9. 37, 5	35, 4	21. 45. 2, 5	4. 39, 8	55, 2	21. 30
9. 0	3. 34. 19, 7	11. 48, 9	8, 6	14. 30. 56,9	9. 33, 7	35, 8	21. 49. 42, 3	4. 33, 6	55, 4	21. 0
9. 30	3. 46. 8, 6	11. 48, 3	9, 1	14. 40. 30,6	9. 30, 0	36, 2	21. 54. 15, 9	4. 27, 6	55, 6	20. 30
10. 0	3. 57. 56, 9	11. 47, 3	9, 6	14. 50. 0,6	9. 26, 1	36, 6	21. 58. 43, 5	4. 21, 3	55, 8	20. 0
10. 30	4. 9. 44, 2	11. 46, 2	10, 1	14. 59. 26,7	9. 22, 5	37, 0	22. 3. 4, 8	4. 15, 1	56, 0	19 30
11. 0	4. 21. 30, 4	11. 45, 3	10, 6	15. 8. 49,2	9. 18, 6	37, 4	22. 7. 19, 9	4. 9, 0	56, 2	19. 0
11. 30	4. 33. 15, 7	11. 44, 2	11, 0	15. 18. 7,8	9. 14, 8	37, 8	22. 11. 28, 9	4. 2, 4	56, 4	18. 30
12. 0	4. 44. 59, 9	11. 43, 1	11, 5	15 27. 22,6	9. 10, 7	38, 2	22. 15. 31, 3	3. 56, 1	56, 6	18. 0
12. 30	4. 56. 43, 0	11. 41, 9	12, 0	15. 36. 33,3	9. 6, 8	38, 6	22. 19. 27, 4	3. 49, 9	56, 8	17. 30
13. 0	5. 8. 24, 9	11. 40, 8	12, 4	15. 45. 40,1	9. 2, 8	39, 0	22. 23. 17, 3	3. 43, 5	57, 0	17. 0
13. 30	5. 20. 5, 7	11. 39, 5	12, 9	15. 54. 42,9	8. 58, 7	39, 4	22. 27. 0, 8	3. 37, 2	57, 1	16 30
14. 0	5. 31. 45, 2	11. 38, 2	13, 4	16. 3. 41,6	8. 54, 6	39, 8	22. 30. 38, 0	3. 30, 6	57, 3	16. 0
14. 30	5. 43. 23, 4	11. 36, 8	13, 9	16. 12. 36,2	8. 50, 4	40, 2	22. 34. 8, 6	3. 24, 3	57, 5	15. 30
15. 0	5. 55. 0, 2		14, 3	16. 21. 26,6		40, 6	22. 37. 32, 9		57, 6	15. 0
	Sig. V. +	Sig. XI. —		Sig. IV. +	Sig. X. —		Sig III. +	Sig. IX. —		

TABLE XXXVI. *Continued.*

ARGUMENT. Longitude of the Point of the Ecliptic.

		Sig. VI. —		Sig. I. +	Sig. VII. —		Sig. II. +	Sig. VIII. —		
		Difference.	Variat.	Declination.	Difference.	Variat.	Declination.	Difference.	Variat.	
	…8	11'. 35",6	14",3	16°. 21'. 26",6	8'. 46",2	40",6	22°. 37'. 32",9	3'. 17",7	57",6	15°. 0'
	…3	11. 34, 0	14, 8	16. 30. 12, 8	8. 42, 0	41, 0	22. 40. 50, 6	3. 11, 4	57, 8	14. 30
	…, 8	11. 32, 6	15, 3	16. 38. 54, 8	8. 37, 6	41, 4	22. 44. 2, 0	3. 4, 8	57, 9	14. 0
	…2, 4	11. 31, 0	15, 7	16. 47. 32, 4	8. 33, 3	41, 7	22. 47. 6, 8	2. 58, 2	58, 0	13. 30
…0	…41. 13, 4	11. 29, 5	16, 2	16. 56. 5, 7	8. 28, 8	42, 1	22. 50. 5, 0	2. 51, 8	58, 2	13. 0
17. 30	6. 52. 42, 9	11. 27, 9	16, 7	17. 4. 34, 5	8. 24, 5	42, 5	22. 52. 56, 8	2. 45, 1	58, 3	12. 30
18. 0	7. 4. 10, 8	11. 26, 1	17, 1	17. 12. 59, 0	8. 20, 4	42, 9	22. 55. 41, 9	2. 38, 5	58, 4	12. 0
18. 30	7. 15. 36, 9	11. 24, 6	17, 6	17. 21. 19, 4	8. 15, 0	43, 2	22. 58. 20, 4	2. 31, 9	58, 6	11. 30
19. 0	7. 27. 1, 5	11. 22, 6	18, 1	17. 29. 34, 4	8. 10, 8	43, 6	23. 0. 52, 3	2. 25, 3	58, 7	11. 0
19. 30	7. 38. 24, 1	11. 21, 0	18, 5	17. 37. 45, 2	8. 6, 3	43, 9	23. 3. 17, 6	2. 18, 7	58, 8	10. 30
20. 0	7. 49. 45, 1	11. 19, 2	19, 0	17. 45. 51, 5	8. 1, 5	44, 3	23. 5. 36, 3	2. 12, 0	58, 9	10. 0
20. 30	8. 1. 4, 3	11. 17, 1	19, 4	17. 53. 53, 0	7. 56, 8	44, 6	23. 7. 48, 3	2. 5, 3	59, 0	9. 30
21. 0	8. 12. 21, 4	11. 15, 3	19, 9	18. 1. 49, 8	7. 52, 1	45, 0	23. 9. 53, 6	1. 58, 6	59, 1	9. 0
21. 30	8. 23. 36, 7	11. 13, 2	20, 4	18. 9. 41, 9	7. 47, 3	45, 3	23. 11. 52, 2	1. 52, 0	59, 2	8. 30
22. 0	8. 34. 49, 9	11. 11, 1	20, 8	18. 17. 29, 2	7. 42, 4	45, 7	23. 13. 44, 2	1. 45, 2	59, 3	8. 0
22. 30	8. 46. 1, 0	11. 9, 2	21, 3	18. 25. 11, 6	7. 37, 6	46, 0	23. 15. 29, 4	1. 38, 5	59, 4	7. 30
23. 0	8. 57. 10, 2	11. 7, 1	21, 7	18. 32. 49, 2	7. 32, 6	46, 4	23. 17. 7, 9	1. 31, 7	59, 5	7. 0
23. 30	9. 8. 17, 3	11. 4, 7	22, 2	18. 40. 21, 8	7. 27, 6	46, 7	23. 18. 39, 6	1. 25, 0	59, 5	6. 30
24. 0	9. 19. 22, 0	11. 2, 6	22, 7	18. 47. 49, 4	7. 22, 7	47, 1	23. 20. 4, 6	1. 18, 2	59, 6	6. 0
24. 30	9. 30. 24, 6	11. 0, 2	23, 2	18. 55. 12, 1	7. 17, 5	47, 4	23. 21. 22, 8	1. 11, 5	59, 6	5. 30
25. 0	9. 41. 24, 8	10. 58, 0	23, 6	19. 2. 29, 6	7. 12, 4	47, 7	23. 22. 34, 3	1. 4, 7	59, 7	5. 0
25. 30	9. 52. 22, 8	10. 55, 6	24, 1	19. 9. 42, 0	7. 7, 3	48, 0	23. 23. 39, 0	0. 57, 9	59, 7	4. 30
26. 0	10. 3. 18, 4	10. 53, 2	24, 5	19. 16. 49, 3	7. 2, 1	48, 4	23. 24. 36, 9	0. 51, 1	59, 8	4. 0
26. 30	10. 14. 11, 6	10. 50, 7	25, 0	19. 23. 51, 4	6. 56, 9	48, 7	23. 25. 28, 0	0. 44, 3	59, 8	3. 30
27. 0	10. 25. 2, 3	10. 48, 3	25, 5	19. 30. 48, 3	6. 51, 6	49, 0	23. 26. 12, 3	0. 37, 5	59, 8	3. 0
27. 30	10. 35. 50, 6	10. 45, 6	25, 9	19. 37. 39, 9	6. 46, 3	49, 3	23. 26. 49, 8	0. 30, 6	59, 9	2. 30
28. 0	10. 46. 36, 2	10. 42, 9	26, 3	19. 44. 26, 2	6. 41, 1	49, 6	23. 27. 20, 4	0. 23, 9	59, 9	2. 0
28. 30	10. 57. 19, 1	10. 40, 5	26, 8	19. 51. 7, 3	6. 35, 5	49, 9	23. 27. 44, 3	0. 17, 1	59, 9	1. 30
29. 0	11. 7. 59, 6	10. 37, 8	27, 2	19. 57. 42, 8	6. 30, 0	50, 2	23. 28. 1, 4	0. 10, 2	60, 0	1. 0
29. 30	11. 18. 37, 4	10. 34, 9	27, 7	20. 4. 12, 8	6. 24, 6	50, 5	23. 28. 11, 6	0. 3, 4	60, 0	0. 30
30. 0	11. 29. 12, 3		28, 1	20. 10. 37, 4		50, 8	23. 28. 15, 0		60, 0	0. 0
	Sig. V. +	Sig. XI. —		Sig. IV. +	Sig. X. —		Sig. III. +	Sig. IX. —		

TABLE XXXVII.

The Equation of Second Difference, for correcting the Equations of the Planetary Motions.

Minute of Argument.		SECOND DIFFERENCE.																
		1"	2"	3"	4"	5"	6"	7"	8	9"	10"	11"	12"	13"	14"	15"	16"	17"
1	59	0",0	0",0	0",0	0",0	0",0	0",0	0",1	0",1	0",1	0",1	0",1	0",1	0",1	0",1	0",1	0",1	0",1
2	58	0,0	0,0	0,0	0,1	0,1	0,1	0,1	0,1	0,1	0,2	0,2	0,2	0,2	0,2	0,2	0,3	0,3
3	57	0,0	0,0	0,1	0,1	0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,3	0,4	0,4	0,4
4	56	0,0	0,1	0,1	0,1	0,2	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,4	0,5	0,5	0,5
5	55	0,0	0,1	0,1	0,2	0,2	0,2	0,3	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6	0,6	0,6
6	54	0,0	0,1	0,1	0,2	0,2	0,3	0,3	0,4	0,4	0,5	0,5	0,5	0,6	0,6	0,7	0,7	0,8
7	53	0,1	0,1	0,2	0,2	0,3	0,3	0,4	0,4	0,5	0,5	0,6	0,6	0,7	0,7	0,8	0,8	0,9
8	52	0,1	0,1	0,2	0,2	0,3	0,3	0,4	0,5	0,5	0,6	0,6	0,7	0,8	0,8	0,9	0,9	1,0
9	51	0,1	0,1	0,2	0,3	0,3	0,4	0,4	0,5	0,6	0,6	0,7	0,8	0,8	0,9	1,0	1,0	1,1
10	50	0,1	0,1	0,2	0,3	0,3	0,4	0,5	0,6	0,6	0,7	0,8	0,8	0,9	1,0	1,0	1,1	1,2
11	49	0,1	0,1	0,2	0,3	0,4	0,4	0,5	0,6	0,7	0,7	0,8	0,9	1,0	1,0	1,1	1,2	1,3
12	48	0,1	0,2	0,2	0,3	0,4	0,5	0,6	0,6	0,7	0,8	0,9	1,0	1,0	1,1	1,2	1,3	1,4
13	47	0,1	0,2	0,3	0,3	0,4	0,5	0,6	0,7	0,8	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,4
14	46	0,1	0,2	0,3	0,4	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,3	1,4	1,5
15	45	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6
16	44	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7
17	43	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7
18	42	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8
19	41	0,1	0,2	0,3	0,4	0,5	0,6	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,5	1,6	1,7	1,8
20	40	0,1	0,2	0,3	0,4	0,6	0,7	0,8	0,9	1,0	1,1	1,2	1,3	1,4	1,6	1,7	1,8	1,9
21	39	0,1	0,2	0,3	0,5	0,6	0,7	0,8	0,9	1,0	1,1	1,3	1,4	1,5	1,6	1,7	1,8	1,9
22	38	0,1	0,2	0,3	0,5	0,6	0,7	0,8	0,9	1,0	1,2	1,3	1,4	1,5	1,6	1,7	1,9	2,0
23	37	0,1	0,2	0,4	0,5	0,6	0,7	0,8	0,9	1,1	1,2	1,3	1,4	1,5	1,7	1,8	1,9	2,0
24	36	0,1	0,2	0,4	0,5	0,6	0,7	0,8	1,0	1,1	1,2	1,3	1,4	1,6	1,7	1,8	1,9	2,0
25	35	0,1	0,2	0,4	0.5	0,6	0,7	0,9	1,0	1,1	1,2	1,3	1,5	1,6	1,7	1,8	1,9	2,1
26	34	0,1	0,2	0,4	0,5	0,6	0.7	0.9	1,0	1,1	1,2	1,4	1,5	1,6	1,7	1,8	2,0	2,1
27	33	0,1	0,2	0,4	0,5	0,6	0,7	0,9	1,0	1,1	1,2	1,4	1,5	1,6	1,7	1,9	2,0	2,1
28	32	0,1	0,2	0,4	0,5	0,6	0,7	0,9	1,0	1,1	1,2	1,4	1,5	1,6	1,7	1,9	2,0	2,1
29	31	0,1	0,2	0,4	0,5	0,6	0,7	0,9	1,0	1,1	1,2	1,4	1,5	1,6	1,7	1,9	2,0	2,1
30	30	0,1	0,3	0,4	0,5	0,6	0,8	0,9	1,0	1,1	1,3	1,4	1,5	1,6	1,8	1,9	2,0	2,1

TABLE [illegible] Continued.

[illegible]	[illegible]	[illegible]	2,2	2,3	2,4	2,5	2,6	2,7	2,8	3,0	3,1	3,2	3,3	3,4	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2,2	2,3	2,4	2,6	2,7	2,8	2,9	3,0	3,1	3,3	3,4	3,5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2,2	2,4	2,5	2,6	2,7	2,8	3,0	3,1	3,2	3,3	3,4	3,5	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2,3	2,4	2,5	2,6	2,8	2,9	3,0	3,1	3,2	3,4	3,5	3,6	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	2,3	2,4	2,6	2,7	2,8	2,9	3,0	3,2	3,3	3,4	3,5	3,6	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	2,5	2,6	2,7	2,8	2,9	3,1	3,2	3,3	3,4	3,6	3,7	3,8	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,6	2,7	2,8	3,0	3,1	3,2	3,3	3,5	3,6	3,7	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,6	2,7	2,9	3,0	3,1	3,2	3,4	3,5	3,6	3,7	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,7	2,9	3,0	3,1	3,2	3,4	3,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]
[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	2,8	2,9	3,0	3,1	3,3	3,4	3,5	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]	[illegible]

TABLE XXXVII. *Continued.*

Minute of Argument.		SECOND DIFFERENCE.															
		35″	36″	37″	38″	39″	40″	41″	42″	43″	44″	45″	46″	47″	48″	49″	50″
1	59	0″,3	0″,3	0″,3	0″,3	0″,3	0″,3	0″,3	0″,3	0″,4	0″,4	0″,4	0″,4	0″,4	0″,4	0″,4	0″,4
2	58	0,6	0,6	0,6	0,6	0,6	0,6	0,7	0,7	0,7	0,7	0,7	0,7	0,8	0,8	0,8	0,8
3	57	0,8	0,9	0,9	0,9	0,9	1,0	1,0	1,0	1,0	1,0	1,1	1,1	1,1	1,1	1,2	1,2
4	56	1,1	1,1	1,2	1,2	1,2	1,2	1,3	1,3	1,3	1,4	1,4	1,4	1,5	1,5	1,5	1,6
5	55	1,3	1,4	1,4	1,5	1,5	1,5	1,6	1,6	1,6	1,7	1,7	1,8	1,8	1,8	1,9	1,9
6	54	1,6	1,6	1,7	1,7	1,8	1,8	1,8	1,9	1,9	2,0	2,0	2,1	2,1	2,2	2,2	2,3
7	53	1,8	1,9	1,9	2,0	2,0	2,1	2,1	2,2	2,2	2,3	2,3	2,4	2,4	2,5	2,5	2,6
8	52	2,0	2,1	2,1	2,2	2,3	2,3	2,4	2,4	2,5	2,5	2,6	2,7	2,7	2,8	2,8	2,9
9	51	2,2	2,3	2,4	2,4	2,5	2,6	2,6	2,7	2,7	2,8	2,9	2,9	3,0	3,1	3,1	3,2
10	50	2,4	2,5	2,6	2,6	2,7	2,8	2,8	2,9	3,0	3,1	3,1	3,2	3,3	3,3	3,4	3,5
11	49	2,6	2,7	2,8	2,8	2,9	3,0	3,1	3,1	3,2	3,3	3,4	3,4	3,5	3,6	3,7	3,7
12	48	2,8	2,9	3,0	3,0	3,1	3,2	3,3	3,4	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0
13	47	3,0	3,1	3,1	3,2	3,3	3,4	3,5	3,6	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,2
14	46	3,1	3,2	3,3	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5
15	45	3,3	3,4	3,5	3,6	3,7	3,8	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7
16	44	3,4	3,5	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9
17	43	3,6	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	5,0	5,1
18	42	3,7	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,6	4,7	4,8	4,9	5,0	5,1	5,3
19	41	3,8	3,9	4,0	4,1	4,2	4,3	4,4	4,5	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,4
20	40	3,9	4,0	4,1	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,4	5,6
21	39	4,0	4,1	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7
22	38	4,1	4,2	4,3	4,4	4,5	4,6	4,8	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7	5,8
23	37	4,1	4,3	4,4	4,5	4,6	4,7	4,8	5,0	5,1	5,2	5,3	5,4	5,6	5,7	5,8	5,9
24	36	4,2	4,3	4,4	4,6	4,7	4,8	4,9	5,0	5,2	5,3	5,4	5,5	5,6	5,8	5,9	6,0
25	35	4,3	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,3	5,5	5,6	5,7	5,8	6,0	6,1
26	34	4,3	4,4	4,5	4,7	4,8	4,9	5,0	5,2	5,3	5,4	5,5	5,6	5,8	5,9	6,0	6,1
27	33	4,3	4,5	4,6	4,7	4,8	5,0	5,1	5,2	5,3	5,4	5,6	5,7	5,8	5,9	6,1	6,2
28	32	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,4	5,5	5,6	5,7	5,8	6,0	6,1	6,2
29	31	4,4	4,5	4,6	4,7	4,9	5,0	5,1	5,2	5,4	5,5	5,6	5,7	5,9	6,0	6,1	6,2
30	30	4,4	4,5	4,6	4,8	4,9	5,0	5,1	5,3	5,4	5,5	5,6	5,8	5,9	6,0	6,1	6,3

TABLE XXXVII. *Continued.*

minute of Argument.		SECOND DIFFERENCE. 51″	52″	53″	54″	55″	56″	57″	58″	59″	60″	61″	62″	63″	64″	65″	66″	67″
1	59	0″,4	0″,4	0″,4	0″,4	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5	0″,5
2	58	0,8	0,8	0,9	0,9	0,9	0,9	0,9	0,9	1,0	1,0	1,0	1,0	1,0	1,0	1,0	1,1	1,1
3	57	1,2	1,2	1,3	1,3	1,3	1,3	1,4	1,4	1,4	1,5	1,5	1,5	1,5	1,6	1,6	1,6	1,6
4	56	1,6	1,6	1,6	1,7	1,7	1,7	1,8	1,8	1,8	1,9	1,9	1,9	2,0	2,0	2,0	2,1	2,1
5	55	1,9	2,0	2,0	2,1	2,1	2,1	2,2	2,2	2,3	2,3	2,3	2,4	2,4	2,4	2,5	2,5	2,6
6	54	2,3	2,3	2,4	2,4	2,5	2,5	2,6	2,6	2,7	2,7	2,7	2,8	2,8	2,9	2,9	3,0	3,0
7	53	2,6	2,7	2,7	2,8	2,8	2,9	2,9	3,0	3,0	3,1	3,1	3,2	3,2	3,3	3,3	3,4	3,5
8	52	2,9	3,0	3,1	3,1	3,2	3,2	3,3	3,4	3,4	3,5	3,5	3,6	3,6	3,7	3,8	3,8	3,9
9	51	3,3	3,3	3,4	3,4	3,5	3,6	3,6	3,7	3,8	3,8	3,9	4,0	4,0	4,1	4,1	4,2	4,3
10	50	3,5	3,6	3,7	3,8	3,8	3,9	4,0	4,0	4,1	4,2	4,2	4,3	4,4	4,4	4,5	4,6	4,7
11	49	3,8	3,9	4,0	4,0	4,1	4,2	4,3	4,3	4,4	4,4	4,5	4,6	4,7	4,7	4,8	4,9	5,0
12	48	4,1	4,2	4,2	4,3	4,4	4,5	4,6	4,6	4,7	4,8	4,9	5,0	5,0	5,1	5,2	5,3	5,4
13	47	4,3	4,4	4,5	4,6	4,7	4,8	4,8	4,9	5,0	5,1	5,2	5,3	5,3	5,4	5,5	5,6	5,7
14	46	4,6	4,7	4,7	4,8	4,9	5,0	5,1	5,2	5,3	5,4	5,5	5,5	5,6	5,7	5,8	5,9	6,0
15	45	4,8	4,9	5,0	5,1	5,2	5,3	5,3	5,4	5,5	5,6	5,7	5,8	5,9	6,0	6,1	6,2	6,3
16	44	5,0	5,1	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	6,0	6,1	6,2	6,3	6,4	6,5	6,6
17	43	5,2	5,3	5,4	5,5	5,6	5,7	5,8	5,9	6,0	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8
18	42	5,4	5,5	5,6	5,7	5,8	5,9	6,0	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	7,0
19	41	5,5	5,6	5,7	5,8	6,0	6,1	6,2	6,3	6,4	6,5	6,6	6,7	6,8	6,9	7,0	7,1	7,2
20	40	5,7	5,8	5,9	6,0	6,1	6,2	6,3	6,4	6,6	6,7	6,8	6,9	7,0	7,1	7,2	7,3	7,4
21	39	5,8	5,9	6,0	6,1	6,3	6,4	6,5	6,6	6,7	6,8	6,9	7,1	7,2	7,3	7,4	7,5	7,6
22	38	5,9	6,0	6,2	6,3	6,4	6,5	6,6	6,7	6,9	7,0	7,1	7,2	7,3	7,4	7,5	7,7	7,8
23	37	6,0	6,1	6,3	6,4	6,5	6,6	6,7	6,9	7,0	7,1	7,2	7,3	7,4	7,6	7,7	7,8	7,9
24	36	6,1	6,2	6,4	6,5	6,6	6,7	6,8	7,0	7,1	7,2	7,3	7,4	7,6	7,7	7,8	7,9	8,0
25	35	6,2	6,3	6,4	6,6	6,7	6,8	6,9	7,0	7,2	7,3	7,4	7,5	7,7	7,8	7,9	8,0	8,1
26	34	6,3	6,4	6,5	6,6	6,8	6,9	7,0	7,1	7,2	7,4	7,5	7,6	7,7	7,9	8,0	8,1	8,2
27	33	6,3	6,4	6,6	6,7	6,8	6,9	7,1	7,2	7,3	7,4	7,5	7,7	7,8	7,9	8,0	8,2	8,3
28	32	6,3	6,5	6,6	6,7	6,8	7,0	7,1	7,2	7,3	7,5	7,6	7,7	7,8	8,0	8,1	8,2	8,3
29	31	6,4	6,5	6,6	6,7	6,9	7,0	7,1	7,2	7,4	7,5	7,6	7,7	7,9	8,0	8,1	8,2	8,4
30	30	6,4	6,5	6,6	6,8	6,9	7,0	7,1	7,3	7,4	7,5	7,6	7,8	7,9	8,0	8,1	8,3	8,4

Minute of Argument.		SECOND DIFFERENCE.																
		68″	69″	70″	71″	72″	73″	74″	75″	76″	77″	78″	79″	80″	81″	82″	83″	84″
1	59	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,6	0″,7	0″,7	0″,7	0″,7	0″,7
2	58	1,1	1,1	1,1	1,1	1,2	1,2	1,2	1,2	1,2	1,2	1,3	1,3	1,3	1,3	1,3	1,3	1,4
3	57	1,7	1,7	1,7	1,7	1,7	1,7	1,8	1,8	1,8	1,8	1,9	1,9	1,9	1,9	1,9	2,0	2,0
4	56	2,1	2,1	2,2	2,2	2,2	2,3	2,3	2,3	2,4	2,4	2,4	2,5	2,5	2,5	2,6	2,6	2,6
5	55	2,6	2,6	2,7	2,7	2,7	2,8	2,8	2,9	2,9	2,9	3,0	3,0	3,1	3,1	3,1	3,2	3,2
6	54	3,1	3,1	3,2	3,2	3,2	3,3	3,3	3,4	3,4	3,5	3,5	3,6	3,6	3,6	3,7	3,7	3,8
7	53	3,5	3,6	3,6	3,7	3,7	3,8	3,8	3,9	3,9	4,0	4,0	4,1	4,1	4,2	4,2	4,3	4,3
8	52	3,9	4,0	4,0	4,1	4,2	4,2	4,3	4,3	4,4	4,4	4,5	4,6	4,6	4,7	4,7	4,8	4,9
9	51	4,3	4,4	4,5	4,5	4,6	4,7	4,7	4,8	4,8	4,9	5,0	5,0	5,1	5,2	5,2	5,3	5,4
10	50	4,7	4,8	4,9	4,9	5,0	5,1	5,1	5,2	5,3	5,3	5,4	5,5	5,6	5,6	5,7	5,8	5,8
11	49	5,0	5,1	5,2	5,3	5,4	5,5	5,5	5,6	5,7	5,8	5,8	5,9	6,0	6,1	6,1	6,2	6,3
12	48	5,4	5,5	5,6	5,7	5,8	5,8	5,9	6,0	6,1	6,2	6,2	6,3	6,4	6,5	6,6	6,6	6,7
13	47	5,8	5,9	5,9	6,0	6,1	6,2	6,3	6,4	6,4	6,5	6,6	6,7	6,8	6,9	7,0	7,0	7,1
14	46	6,1	6,2	6,3	6,4	6,4	6,5	6,6	6,7	6,8	6,9	7,0	7,1	7,2	7,2	7,3	7,4	7,5
15	45	6,4	6,5	6,6	6,7	6,8	6,8	6,9	7,0	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9
16	44	6,6	6,7	6,8	6,9	7,0	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9	8,0	8,1	8,2
17	43	6,9	7,0	7,1	7,2	7,3	7,4	7,5	7,6	7,7	7,8	7,9	8,0	8,1	8,2	8,3	8,4	8,5
18	42	7,1	7,2	7,4	7,5	7,6	7,7	7,8	7,9	8,0	8,1	8,2	8,3	8,4	8,5	8,6	8,7	8,8
19	41	7,4	7,5	7,6	7,7	7,8	7,9	8,0	8,1	8,2	8,3	8,4	8,5	8,7	8,8	8,9	9,0	9,1
20	40	7,6	7,7	7,8	7,9	8,0	8,1	8,2	8,3	8,4	8,6	8,7	8,8	8,9	9,0	9,1	9,2	9,3
21	39	7,7	7,8	8,0	8,1	8,2	8,3	8,4	8,5	8,6	8,8	8,9	9,0	9,1	9,2	9,3	9,4	9,6
22	38	7,9	8,0	8,1	8,2	8,4	8,5	8,6	8,7	8,8	8,9	9,1	9,2	9,3	9,4	9,5	9,6	9,8
23	37	8,0	8,2	8,3	8,4	8,5	8,6	8,7	8,9	9,0	9,1	9,2	9,3	9,5	9,6	9,7	9,8	9,9
24	36	8,2	8,3	8,4	8,5	8,6	8,8	8,9	9,0	9,1	9,2	9,4	9,5	9,6	9.7	9,8	10,0	10,1
25	35	8,3	8,4	8,5	8,6	8,8	8,9	9,0	9,1	9,2	9,4	9,5	9,6	9,7	9,8	10,0	10,1	10,2
26	34	8,3	8,5	8,6	8,7	8,8	9,0	9,1	9,2	9,3	9,5	9,6	9,7	9,8	9,9	10,1	10,2	10,3
27	33	8,4	8,5	8,7	8,8	8,9	9,0	9,2	9,3	9,4	9,5	9,7	9,8	9,9	10,0	10,1	10,3	10,4
28	32	8,5	8,6	8,7	8,8	9,0	9,1	9,2	9,3	9,5	9,6	9,7	9,8	10,0	10,1	10,2	10,3	10,5
29	31	8,5	8,6	8,7	8,9	9,0	9,1	9,2	9,4	9,5	9,6	9,7	9,9	10,0	10,1	10,2	10,4	10,5
30	30	8,5	8,6	8,8	8,9	9,0	9,1	9,3	9,4	9,5	9,6	9,8	9,9	10,0	10,1	10,3	10,4	10,5

TABLE XXXVII. *Continued.*

		SECOND DIFFERENCE.															
		85"	86"	87"	88"	89"	90"	91"	92"	93"	94"	95"	96"	97"	98"	99"	100"
		0",7	0",7	0,"7	0".7	0",7	0,"7	0",7	0",8	0",8	0",8	0",8	0',8	0",8	0',8	0",8	0",8
	[illegible]	1 4	1, 4	1, 4	1, 4	1, 4	1, 5	1, 5	1, 5	1, 5	1, 5	1, 5	1, 5	1, 6	1, 6	1, 6	1, 6
[illegible]	57	2, 0	2, 0	2. 1	2, 1	2, 1	2, 1	2, 2	2 2	2, 2	2, 2	2, 3	2, 3	2, 3	2, 3	2, 4	2, 4
4	56	2, 6	2, 7	2, 7	2, 7	2, 8	2, 8	2, 8	2 9	2, 9	2, 9	3, 0	3, 0	3, 0	3 0	3, 1	3, 1
5	55	3, 2	3, 3	3, 3	3, 4	3, 4	3, 4	3, 5	3 5	3, 6	3, 6	3. 6	3, 7	3, 7	3. 7	3, 8	3, 8
6	54	3, 8	3, 9	3, 9	4, 0	4, 0	4, 1	4, 1	4, 1	4, 2	4, 2	4, 3	4, 3	4, 4	4, 4	4, 5	4, 5
7	53	4, 4	4, 4	4, 5	4, 5	4, 6	4, 6	4, 7	4, 7	4, 8	4, 8	4, 9	4, 9	5, 0	5, 0	5, 1	5, 2
8	52	4, 9	5, 0	5, 0	5, 1	5, 1	5, 2	5, 3	5, 3	5, 4	5, 4	5. 5	5, 5	5, 6	5, 7	5, 7	5, 8
9	51	5, 4	5, 5	5, 5	5, 6	5, 7	5, 7	5, 8	5, 9	5, 9	6, 0	6, 1	6, 1	6, 2	6, 2	6, 3	6, 4
10	50	5, 9	6, 0	6, 0	6, 1	6, 2	6, 3	6, 3	6, 4	6, 5	6, 5	6, 6	6, 7	6, 7	6, 8	6, 9	6, 9
11	49	6, 4	6, 4	6, 5	6, 6	6, 7	6, 7	6, 8	6, 9	7, 0	7, 0	7, 1	7, 2	7, 3	7, 3	7, 4	7, 5
12	48	6, 8	6, 9	7, 0	7, 0	7, 1	7, 2	7, 3	7, 4	7, 4	7, 5	7, 6	7, 7	7, 8	7, 8	7, 9	8, 0
13	47	7, 2	7, 3	7, 4	7, 5	7, 6	7, 6	7, 7	7, 8	7, 9	8, 0	8, 1	8, 1	8, 2	8, 3	8, 4	8, 5
14	46	7, 6	7, 7	7, 8	7, 9	8, 0	8, 1	8, 1	8, 2	8, 3	8, 4	8, 5	8, 6	8, 7	8, 8	8, 9	8, 9
15	45	8, 0	8, 1	8, 2	8, 3	8, 3	8, 4	8, 5	8, 6	8, 7	8, 8	9, 0	9, 0	9, 1	9, 2	9, 3	9, 4
16	44	8, 3	8, 4	8, 5	8, 6	8, 7	8, 8	8, 9	9, 0	9, 1	9, 2	9, 3	9, 4	9, 5	9, 6	9. 7	9, 8
17	43	8, 6	8, 7	8, 8	8, 9	9, 0	9, 1	9, 2	9, 3	9, 4	9, 5	9, 6	9, 7	9, 8	9. 9	10, 1	10, 2
18	42	8, 9	9, 0	9, 1	9, 2	9, 3	9, 5	9, 6	9, 7	9, 8	9, 9	10, 0	10, 1	10, 2	10, 3	10, 4	10, 5
19	41	9, 2	9, 3	9. 4	9, 5	9, 6	9, 7	9, 8	10, 0	10, 1	10, 2	10, 3	10, 4	10, 5	10, 6	10, 7	10, 8
20	40	9, 4	9. 6	9. 7	9, 8	9, 9	10, 0	10, 1	10, 2	10, 3	10, 4	10, 6	10, 7	10, 8	10, 9	11, 0	11, 1
21	39	9, 7	9, 8	9, 9	10, 0	10, 1	10, 2	10, 4	10, 5	10, 6	10, 7	10, 8	10, 9	11, 0	11, 1	11, 3	11, 4
22	38	9, 9	10, 0	10, 1	10, 2	10, 3	10, 5	10, 6	10, 7	10, 8	10, 9	11, 0	11, 1	11, 3	11, 4	11, 5	11, 6
23	37	10, 0	10, 2	10, 3	10, 4	10, 5	10, 6	10, 8	10, 9	11, 0	11, 1	11, 2	11, 3	11, 5	11, 6	11, 7	11, 8
24	36	10, 2	10, 3	10, 4	10, 6	10, 7	10, 8	10, 9	11, 0	11, 2	11, 3	11, 4	11, 5	11, 6	11, 8	11, 9	12, 0
25	35	10, 3	10, 5	10, 6	10, 7	10, 8	10, 9	11, 1	11, 2	11, 3	11, 4	11, 5	11, 7	11, 8	11, 9	12, 0	12, 2
26	34	10, 4	10, 6	10, 7	10, 8	10, 9	11, 1	11, 2	11, 3	11, 4	11, 5	11, 7	11, 8	11, 9	12, 0	12, 2	12, 3
27	33	10, 5	10, 6	10, 8	10, 9	11, 0	11, 1	11, 3	11, 4	11, 5	11, 6	11, 8	11, 9	12, 0	12, 1	12, 3	12, 4
28	32	10, 6	10, 7	10, 8	11, 0	11, 1	11, 2	11, 3	11, 4	11, 6	11, 7	11, 8	11, 9	12, 1	12, 2	12, 3	12, 4
29	31	10, 6	10, 7	10, 9	11, 0	11, 1	11, 2	11, 4	11, 5	11, 6	11, 7	11, 9	12, 0	12, 1	12, 2	12, 4	12, 5
30	30	10, 6	10, 8	10, 9	11, 0	11, 1	11, 3	11, 4	11, 5	11, 6	11, 8	11, 9	12, 0	12, 1	12, 3	12, 4	12, 5

TABLE XXXVIII.

The Equation of Second Difference, useful in computing the Moon's Place from the Nautical Ephemeris.

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.										
		0 Minute.					1 Minute.					
		10″	20″	30″	40″	50″	0′	10′	20″	30″	40″	50″
H. M.	H. M.	″	″	″	″	″	″	″	″	′	″	″
0. 0	12. 0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0. 10	11. 50	0,1	0,1	0,2	0,3	0,3	0,4	0,5	0,5	0,6	0,7	0,8
0. 20	11. 40	0,1	0,3	0,4	0,5	0,7	0,8	0,9	1,1	1,2	1,4	1,5
0. 30	11. 30	0,2	0,4	0,6	0,8	1,0	1,2	1,4	1,6	1,8	2,0	2,2
0. 40	11. 20	0,3	0,5	0,8	1,0	1,3	1,6	1,8	2,1	2,4	2,6	2,9
0. 50	11. 10	0,3	0,6	1,0	1,3	1,6	1,9	2,3	2,6	2,9	3,2	3,6
1. 0	11. 0	0,4	0,8	1,1	1,5	1,9	2,3	2,7	3,1	3,4	3,8	4,2
1. 10	10. 50	0,4	0,9	1,3	1,8	2,2	2,6	3,1	3,5	3,9	4,4	4,8
1. 20	10. 40	0,5	1,0	1,5	2,0	2,5	3,0	3,5	4,0	4,4	4,9	5,4
1. 30	10. 30	0,5	1,1	1,6	2,2	2,7	3,3	3,8	4,4	4,9	5,5	6,0
1. 40	10. 20	0,6	1,2	1,8	2,4	3,0	3,6	4,2	4,8	5,4	6,0	6,6
1. 50	10. 10	0,6	1,3	1,9	2,6	3,2	3,9	4,5	5,2	5,8	6,5	7,1
2. 0	10. 0	0,7	1,4	2,1	2,8	3,5	4,2	4,9	5,6	6,3	6,9	7,6
2. 10	9. 50	0,7	1,5	2,2	3,0	3,7	4,4	5,2	5,9	6,7	7,4	8,1
2. 20	9. 40	0,8	1,6	2,3	3,1	3,9	4,7	5,5	6,3	7,0	7,8	8,6
2. 30	9. 30	0,8	1,6	2,5	3,3	4,1	4,9	5,8	6,6	7,4	8,2	9,1
2. 40	9. 20	0,9	1,7	2,6	3,5	4,3	5,2	6,0	6,9	7,8	8,6	9,5
2. 50	9. 10	0,9	1,8	2,7	3,6	4,5	5,4	6,3	7,2	8,1	9,0	9,9
3. 0	9. 0	0,9	1,9	2,8	3,8	4,7	5,6	6,6	7,5	8,4	9,4	10,3
3. 10	8. 50	1,0	1,9	2,9	3,9	4,9	5,8	6,8	7,8	8,7	9,7	10,7
3. 20	8. 40	1,0	2,0	3,0	4,0	5,0	6,0	7,0	8,0	9,0	10,0	11,0
3. 30	8. 30	1,0	2,1	3,1	4,1	5,2	6,2	7,2	8,3	9,3	10,3	11,4
3. 40	8. 20	1,1	2,1	3,2	4,2	5,3	6,4	7,4	8,5	9,5	10,6	11,7
3. 50	8. 10	1,1	2,2	3,3	4,3	5,4	6,5	7,6	8,7	9,8	10,9	12,0
4. 0	8. 0	1,1	2,2	3,3	4,4	5,6	6,7	7,8	8,9	10,0	11,1	12,2
4. 10	7. 50	1,1	2,3	3,4	4,5	5,7	6,8	7,9	9,1	10,2	11,3	12,5
4. 20	7. 40	1,2	2,3	3,5	4,6	5,8	6,9	8,1	9,2	10,4	11,5	12,7
4. 30	7. 30	1,2	2,3	3,5	4,7	5,9	7,0	8,2	9,4	10,5	11,7	12,9
4. 40	7. 20	1,2	2,4	3,6	4,8	5,9	7,1	8,3	9,5	10,7	11,9	13,1
4. 50	7. 10	1,2	2,4	3,6	4,8	6,0	7,2	8,4	9,6	10,8	12,0	13,2
5. 0	7. 0	1,2	2,4	3,6	4,9	6,1	7,3	8,5	9,7	10,9	12,2	13,4
5. 10	6. 50	1,2	2,5	3,7	4,9	6,1	7,4	8,6	9,8	11,0	12,3	13,5
5. 20	6. 40	1,2	2,5	3,7	4,9	6,2	7,4	8,6	9,9	11,1	12,3	13,6
5. 30	6. 30	1,2	2,5	3,7	5,0	6,2	7,4	8,7	9,9	11,2	12,4	13,7
5. 40	6. 20	1,2	2,5	3,7	5,0	6,2	7,5	8,7	10,0	11,2	12,5	13,7
5. 50	6. 10	1,2	2,5	3,7	5,0	6,2	7,5	8,7	10,0	11,2	12,5	13,7
6. 0	6. 0	1,3	2,5	3,8	5,0	6,3	7,5	8,8	10,0	11,3	12,5	13,8

TABLE XXXVIII. *Continued.*

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.											
		2 Minutes.						3 Minutes.					
		0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″
H. M.	H. M.	″	″	″	″	″	″	″	″	″	″	″	″
0. 0	12. 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
0. 10	11. 50	0, 8	0, 9	1, 0	1, 0	1, 1	1, 2	1, 2	1, 3	1, 4	1, 4	1, 5	1, 6
0. 20	11. 40	1, 6	1, 8	1, 9	2, 0	2, 2	2, 3	2, 4	2, 6	2, 7	2, 8	3, 0	3, 1
0. 30	11. 30	2, 4	2, 6	2, 8	3, 0	3, 2	3, 4	3, 6	3, 8	4, 0	4, 2	4, 4	4, 6
0. 40	11. 20	3, 1	3, 4	3, 7	3, 9	4, 2	4, 5	4, 7	5, 0	5, 2	5, 5	5, 8	6, 0
0. 50	11. 10	3, 9	4, 2	4, 5	4, 8	5, 2	5, 5	5, 8	6, 1	6, 5	6, 8	7, 1	7, 4
1. 0	11. 0	4, 9	5, 0	5, 3	5, 7	6, 1	6, 5	6, 9	7, 3	7, 6	8, 0	8, 4	8, 8
1. 10	10. 50	5, 3	5, 7	6, 1	6, 6	7, 0	7, 5	7, 9	8, 3	8, 8	9, 2	9, 7	10, 1
1. 20	10. 40	5, 9	6, 4	6, 9	7, 4	7, 9	8, 4	8, 9	9, 4	9, 9	10, 4	10, 9	11, 4
1. 30	10. 30	6, 6	7, 1	7, 7	8, 2	8, 8	9, 3	9, 8	10, 4	10, 9	11, 5	12, 0	12, 6
1. 40	10. 20	7, 2	7, 8	8, 4	9, 0	9, 6	10, 2	10, 8	11, 4	12, 0	12, 6	13, 2	13, 8
1. 50	10. 10	7, 8	8, 4	9, 1	9, 7	10, 4	11, 0	11, 6	12, 3	12, 9	13, 6	14, 2	14, 9
2. 0	10. 0	8, 3	9, 0	9, 7	10, 4	11, 1	11, 8	12, 5	13, 2	13, 9	14, 6	15, 3	16, 0
2. 10	9. 50	8, 9	9, 6	10, 4	11, 1	11, 8	12, 6	13, 3	14, 1	14, 8	15, 5	16, 3	17, 0
2. 20	9. 40	9, 4	10, 2	11, 0	11, 7	12, 5	13, 3	14, 1	14, 9	15, 7	16, 4	17, 2	18, 0
2. 30	9. 30	9, 9	10, 7	11, 5	12, 4	13, 2	14, 0	14, 8	15, 7	16, 5	17, 3	18, 1	19, 0
2. 40	9. 20	10, 4	11, 2	12, 1	13, 0	13, 8	14, 7	15, 6	16, 4	17, 3	18, 1	19, 0	19, 9
2. 50	9. 10	10, 8	11, 7	12, 6	13, 5	14, 4	15, 3	16, 2	17, 1	18, 0	18, 9	19, 8	20, 7
3. 0	9. 0	11, 3	12, 2	13, 1	14, 1	15, 0	15, 9	16, 9	17, 8	18, 8	19, 7	20, 6	21, 6
3. 10	8. 50	11, 7	12, 6	13, 6	14, 6	15, 5	16, 5	17, 5	18, 5	19, 4	20, 4	21, 4	22, 3
3. 20	8. 40	12, 0	13, 0	14, 0	15, 0	16, 0	17, 1	18, 1	19, 1	20, 1	21, 1	22, 1	23, 1
3. 30	8. 30	12, 4	13, 4	14, 5	15, 5	16, 5	17, 6	18, 6	19 6	20, 7	21, 7	22, 7	23, 8
3. 40	8. 20	12, 7	13, 8	14, 9	15, 9	17, 0	18, 0	19, 1	20, 2	21, 2	22, 3	23, 3	24, 4
3. 50	8. 10	13, 0	14, 1	15, 2	16, 3	17, 4	18, 5	19, 6	20, 7	21, 7	22, 8	23, 9	25, 0
4. 0	8. 0	13, 3	14, 4	15, 6	16, 7	17, 8	18, 9	20, 0	21, 1	22. 2	23, 3	24, 4	25, 6
4. 10	7. 50	13, 6	14, 7	15, 9	17, 0	18, 1	19, 3	20, 4	21, 5	22, 7	23, 8	24, 9	26, 1
4. 20	7. 40	13, 8	15, 0	16, 1	17, 3	18, 5	19, 6	20, 8	21, 9	23, 1	24, 2	25, 4	26, 5
4. 30	7. 30	14, 1	15, 2	16, 4	17, 6	18, 8	19, 9	21, 1	22, 3	23, 4	24, 6	25, 8	27, 0
4. 40	7. 20	14, 3	15, 4	16, 6	17, 8	19, 0	20, 2	21, 4	22, 6	23, 8	25, 0	26, 1	27, 3
4. 50	7. 10	14, 4	15, 6	16, 8	18, 0	19, 2	20, 4	21, 6	22, 9	24, 1	25, 3	26, 5	27, 7
5. 0	7. 0	14, 6	15, 8	17, 0	18, 2	19, 4	20, 7	21, 9	23, 1	24, 3	25, 5	26, 7	28, 0
5. 10	6. 50	14, 7	15, 9	17, 2	18, 4	19, 6	20, 8	22, 1	23, 3	24, 5	25, 7	27, 0	28, 2
5. 20	6. 40	14, 8	16, 0	17, 3	18, 5	19, 8	21, 0	22, 2	23, 5	24, 7	25, 9	27, 2	28, 4
5. 30	6. 30	14, 9	16, 1	17, 4	18, 6	19, 9	21, 1	22, 3	23, 6	24, 8	26, 1	27, 3	28, 6
5. 40	6. 20	15, 0	16, 2	17, 4	18, 7	19, 9	21, 2	22, 4	23, 7	24, 9	26, 2	27, 4	28, 7
[illegible]	[illegible] 10	15, 0	16, 2	17, 5	18, 7	20, 0	21, 2	22, 5	23, 7	25, 0	26, 2	27, 5	28, 7
[illegible]	[illegible]	15, 0	16, 3	17, 5	18, 8	20, 0	21, 3	22, 5	23, 8	25, 0	26, 3	27, 5	28, 8

TABLE XXXVIII. *Continued.*

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.											
		4 Minutes.						5 Minutes.					
		0″	10″	20″	30″	40″	50″	0′	10″	20″	30″	40″	50″
H. M.	H. M.	″	″	″	″	″	″	″	″	″	″	″	″
0. 0	12. 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
0. 10	11. 50	1, 6	1, 7	1, 8	1, 8	1, 9	2, 0	2, 1	2, 1	2, 2	2, 3	2, 3	2, 4
0. 20	11. 40	3, 2	3, 4	3, 5	3, 6	3, 8	3, 9	4, 1	4, 2	4, 5	4, 5	4, 6	4, 7
0. 30	11. 30	4, 8	5, 0	5, 2	5, 4	5, 6	5, 8	6, 0	6, 2	6, 4	6, 6	6, 8	7, 0
0. 40	11. 20	6, 3	6, 6	6, 8	7, 1	7, 3	7, 6	7, 9	8, 1	8, 4	8, 7	8, 9	9, 2
0. 50	11. 10	7, 8	8, 1	8, 4	8, 7	9, 0	9, 4	9, 7	10, 0	10, 3	10, 7	11, 0	11, 3
1. 0	11. 0	9, 2	9, 5	9, 9	10, 3	10, 7	11, 1	11, 5	11, 8	12, 2	12, 6	13, 0	13, 4
1. 10	10. 50	10, 5	11, 0	11, 4	11, 8	12, 3	12, 7	13, 2	13, 6	14, 0	14, 5	14, 9	15, 4
1. 20	10. 40	11, 9	12, 3	12, 8	13, 3	13, 8	14, 3	14, 8	15, 3	15, 8	16, 3	16, 8	17, 3
1. 30	10. 30	13, 1	13, 7	14, 2	14, 8	15, 3	15, 9	16, 4	17, 0	17, 5	18, 0	18, 6	19, 1
1. 40	10. 20	14, 4	14, 9	15, 5	16, 1	16, 7	17, 3	17, 9	18, 5	19, 1	19, 7	20, 3	20, 9
1. 50	10. 10	15, 5	16, 2	16, 8	17, 5	18, 1	18, 8	19, 4	20, 1	20, 7	21, 4	22, 0	22, 7
2. 0	10. 0	16, 7	17, 4	18, 1	18, 8	19, 4	20, 1	20, 8	21, 5	22, 2	22, 9	23, 6	24, 3
2. 10	9. 50	17, 8	18, 5	19, 2	20, 0	20, 7	21, 5	22, 2	22, 9	23, 6	24, 4	25, 2	25, 9
2. 20	9. 40	18, 8	19, 6	20, 4	21, 1	21, 9	22, 7	23, 5	24, 3	25, 1	25, 8	26, 6	27, 4
2. 30	9. 30	19, 8	20, 6	21, 4	22, 3	23, 1	23, 9	24, 7	25, 6	26, 4	27, 2	28, 0	28, 9
2. 40	9. 20	20, 7	21, 6	22, 5	23, 3	24, 2	25, 1	25, 9	26, 8	27, 7	28, 5	29, 4	30, 2
2. 50	9. 10	21, 6	22, 5	23, 4	24, 3	25, 3	26, 2	27, 1	28, 0	28, 9	29, 8	30, 7	31, 6
3. 0	9. 0	22, 5	23, 4	24, 4	25, 3	26, 3	27, 2	28, 1	29, 1	30, 0	30, 9	31, 9	32, 8
3. 10	8. 50	23, 3	24, 3	25, 3	26, 2	27, 2	28, 2	29, 1	30, 1	31, 1	32, 1	33, 0	34, 0
3. 20	8. 40	24, 1	25, 1	26, 1	27, 1	28, 1	29, 1	30, 1	31, 1	32, 1	33, 1	34, 1	35, 1
3. 30	8. 30	24, 8	25, 8	26, 9	27, 9	28, 9	30, 0	31, 0	32, 0	33, 1	34, 1	35, 1	36, 2
3. 40	8. 20	25, 5	26, 5	27, 6	28, 6	29, 7	30, 8	31, 8	32, 9	34, 0	35, 0	36, 1	37, 1
3. 50	8. 10	26, 1	27, 2	28, 3	29, 3	30, 4	31, 5	32, 6	33, 7	34, 8	35, 9	37, 0	38, 0
4. 0	8. 0	26, 7	27, 8	28, 9	30, 0	31, 1	32, 2	33, 3	34, 4	35, 6	36, 7	37, 8	38, 9
4. 10	7. 50	27, 2	28, 3	29, 5	30, 6	31, 7	32, 9	34, 0	35, 1	36, 3	37, 4	38, 5	39, 7
4. 20	7. 40	27, 7	28, 8	30, 0	31, 1	32, 3	33, 5	34, 6	35, 8	36, 9	38, 1	39, 2	40, 4
4. 30	7. 30	28, 1	29, 3	30, 5	31, 6	32, 8	34, 0	35, 2	36, 3	37, 5	38, 7	39, 8	41, 0
4. 40	7. 20	28, 5	29, 7	20, 9	32, 1	33, 3	34, 5	35, 6	36, 8	38, 0	39, 2	40, 4	41, 6
4. 50	7. 10	28, 9	30, 1	31, 3	32, 5	33, 7	34, 9	36, 1	37, 3	38, 5	39, 7	40, 9	42, 1
5. 0	7. 0	29, 2	30, 4	31, 6	32, 8	34, 0	35, 2	36, 5	37, 7	38, 9	40, 1	41, 3	42, 5
5. 10	6. 50	29, 4	30, 6	31, 9	33, 1	34, 3	35, 6	36, 8	38, 0	39, 2	40, 5	41, 7	42, 9
5. 20	6. 40	29, 6	30, 9	32, 1	33, 3	34, 6	35, 8	37, 0	38, 3	39, 5	40, 7	42, 0	43, 2
5. 30	6. 30	29, 8	31, 0	32, 3	33, 5	34, 8	36, 0	37, 2	38, 5	39, 7	41, 0	42, 2	43, 4
5. 40	6. 20	29, 9	31, 2	32, 4	33, 6	34, 9	36, 1	37, 4	38, 6	39, 9	41, 1	42, 4	43, 6
5. 50	6. 10	30, 0	31, 2	32, 5	33, 7	35, 0	36, 2	37, 5	38, 7	40, 0	41, 2	42, 5	43, 7
6. 0	6. 0	30, 0	31, 3	32, 5	33, 8	35, 0	36, 3	37, 5	38, 8	40, 0	41, 3	42, 5	43, 8

TABLE XXXVIII. *Continued.*

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.											
		6 Minutes.						7 Minutes.					
		0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″
H. M.	H. M.	″	″	″	″	″	″	″	″	″	″	″	″
0. 0	12. 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
0. 10	11. 50	2, 5	2, 5	2, 6	2, 7	2, 7	2, 8	2, 9	2, 9	3, 0	3, 1	3, 2	3, 2
0. 20	11. 40	4, 9	5, 0	5, 1	5, 3	5, 4	5, 5	5, 7	5, 8	5, 9	6. 1	6, 2	6, 3
0. 30	11. 30	7, 2	7, 4	7, 6	7, 8	8, 0	8, 2	8, 4	8, 6	8, 8	9. 0	9, 2	9, 4
0. 40	11. 20	9, 4	9, 7	10, 0	10, 2	10, 5	10, 8	11, 0	11, 3	11, 5	11, 8	12, 1	12, 3
0. 50	11. 10	11, 6	12, 0	12, 3	12, 6	12, 9	13, 2	13, 6	13, 9	14, 2	14, 5	14, 9	15, 2
1. 0	11. 0	13, 8	14, 1	14, 5	14, 9	15, 3	15, 7	16, 0	16, 4	16, 8	17, 2	17, 6	18, 0
1. 10	10. 50	15, 8	16, 2	16, 7	17, 1	17, 6	18, 0	18, 4	18, 9	19, 3	19, 7	20, 2	20, 6
1. 20	10. 40	17, 8	18, 3	18, 8	19, 3	19, 8	20 2	20, 7	21, 2	21, 7	22, 2	22, 7	23, 2
1. 30	10. 30	19, 7	20, 2	20, 8	21, 3	21, 9	22, 4	23, 0	23, 5	24, 1	24, 6	25, 2	25, 7
1. 40	10. 20	21, 5	22, 1	22, 7	23, 3	23, 9	24. 5	25, 1	25, 7	26, 3	26, 9	27, 5	28, 1
1. 50	10. 10	23, 3	23, 9	24, 6	25, 2	25, 9	26, 5	27, 2	27, 8	28, 5	29, 1	29, 8	30, 4
2. 0	10. 0	25, 0	25, 7	26, 4	27, 1	27, 8	28, 5	29, 2	29, 9	30, 6	31, 3	31, 9	32, 6
2. 10	9. 50	26, 6	27, 4	28, 1	28, 9	29, 6	30, 3	31, 1	31, 8	32, 6	33. 3	34, 0	34, 8
2. 20	9. 40	28, 2	29, 0	29, 8	30, 5	31, 3	32, 1	32, 9	33, 7	34, 5	35, 2	36, 0	36, 8
2. 30	9. 30	29, 7	30, 5	31, 3	32, 2	33 0	33, 8	34, 6	35, 5	36, 3	37, 1	37, 9	38, 8
2. 40	9. 20	31, 1	32, 0	32, 8	33, 7	34 6	35, 4	36, 3	37, 2	38, 0	38, 9	39, 8	40, 6
2. 50	9. 10	32. 5	33, 4	34, 3	35, 2	36, 1	37, 0	37, 9	38, 8	39, 7	40, 6	41, 5	42, 4
3. 0	9. 0	33, 8	34. 7	35, 6	36. 6	37, 5	38, 4	39, 4	40, 3	41, 3	42, 2	43, 1	44, 1
3. 10	8. 50	35, 0	35, 9	36, 9	37, 9	38, 9	39, 8	40, 8	41, 8	42, 7	43, 7	44, 7	45, 6
3. 20	8. 40	36, 1	37, 1	38, 1	39. 1	40, 1	41, 1	42, 1	43, 1	44, 1	45. 1	46, 1	47, 1
3. 30	8. 30	37, 2	38, 2	39, 3	40, 3	41, 3	42, 4	43, 4	44, 4	45, 5	46 5	47, 5	48, 6
3. 40	8. 20	38, 2	39, 3	40, 3	41. 4	42, 4	43, 5	44, 6	45, 6	46, 7	47, 7	48, 8	49, 9
3. 50	8. 10	39, 1	40, 2	41, 3	42, 4	43, 5	44, 6	45, 7	46, 7	47, 8	48, 9	50, 0	51, 1
4. 0	8. 0	40, 0	41, 1	42, 2	43, 3	44, 4	45 6	46, 7	47, 8	48, 9	50, 0	51, 1	52, 2
4. 10	7. 50	40, 8	41, 9	43, 1	44, 2	45, 3	46, 5	47, 6	48, 7	49, 9	51, 0	52, 1	53, 3
4. 20	7. 40	41, 5	42, 7	43, 8	45, 0	46, 1	47 3	48, 4	49, 6	50, 8	51, 9	53, 1	54, 2
4. 30	7. 30	42, 2	43, 4	44. 5	45, 7	46. 9	48, 0	49, 2	50, 4	51, 6	52, 7	53, 9	55, 1
4. 40	7. 20	42. 8	44, 0	45, 2	46, 3	47, 5	48, 7	49, 9	51, 1	52, 3	53, 5	54, 7	55, 8
50	7. 10	43, 3	44 5	45, 7	46, 9	48, 1	49, 3	50, 5	51, 7	52, 9	54, 1	55, 3	56, 5
	7. 0	43, 8	45, 0	46, 2	47, 4	48, 6	49, 8	51, 0	52, 3	53, 5	54. 7	55, 9	57, 1
	6. 50	44, 1	45, 4	46, 6	47, 8	49, 0	50, 3	51, 5	52, 7	53, 9	55, 2	56, 4	57, 6
	6. 40	44, 4	45. 7	46, 9	48. 1	49, 4	50, 6	51, 9	53, 1	54, 3	55, 6	56, 8	58, 0
	6. 30	44, 7	45, 9	47, 2	48, 4	49. 7	50, 9	52, 1	53, 4	54, 6	55, 9	57, 1	58, 3
	6. 20	44, 9	46. 1	47, 4	48. 6	49. 8	51, 1	52, 3	53, 6	54. 8	56, 1	57, 3	58, 6
	6. 10	45, 0	46, 2	47, 5	48, 7	50, 0	51, 2	52, 5	53, 7	55, 0	56, 2	57, 5	58, 7
	6. 0	45, 0	46, 3	47, 5	48, 8	50, 0	51, 3	52, 5	53, 8	55, 0	56, 3	57, 5	58, 8

TABLE XXXVIII. *Continued.*

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.											
		8 Minutes.						9 Minutes.					
		0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″
H. M.	H. M.	″	″	″	″	″	″	″	″	″	″	″	″
0. 0	12. 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0	0, 0
0. 10	11. 50	3, 3	3, 4	3, 4	3, 5	3, 6	3, 6	3, 7	3, 8	3, 8	3, 9	4, 0	4, 0
0. 20	11. 40	6, 5	6, 6	6, 8	6, 9	7, 0	7, 2	7, 3	7, 4	7, 6	7, 7	7, 8	8, 0
0. 30	11. 30	9, 6	9, 8	10, 0	10, 2	10, 4	10, 6	10, 8	11, 0	11, 2	11, 4	11, 6	11, 8
0. 40	11. 20	12, 6	12, 9	13, 1	13, 4	13, 6	13, 9	14, 2	14, 4	14, 7	15, 0	15, 2	15, 5
0. 50	11. 10	15, 5	15, 8	16, 2	16, 5	16, 8	17, 1	17, 4	17, 8	18, 1	18, 4	18, 7	19, 1
1. 0	11. 0	18, 3	18, 7	19, 1	19, 5	19, 9	20, 2	20, 6	21, 0	21, 4	21, 8	22, 2	22, 5
1. 10	10. 50	21, 1	21, 5	21, 9	22, 4	22, 8	23, 3	23, 7	24, 1	24, 6	25, 0	25, 5	25, 9
1. 20	10. 40	23, 7	24, 2	24, 7	25, 2	25, 7	26, 2	26, 7	27, 2	27, 7	28, 1	28, 6	29, 1
1. 30	10. 30	26, 3	26, 8	27, 3	27, 9	28, 4	29, 0	29, 5	30, 1	30, 6	31, 2	31, 7	32, 3
1. 40	10. 20	28, 7	29, 3	29, 9	30, 5	31, 1	31, 7	32, 3	32, 9	33, 5	34, 1	34, 7	35, 3
1. 50	10. 10	31, 1	31, 7	32, 4	33, 0	33, 7	34, 3	34, 9	35, 6	36, 2	36, 9	37, 5	38, 2
2. 0	10. 0	33, 3	34, 0	34, 7	35, 4	36, 1	36, 8	37, 5	38, 2	38, 9	39, 6	40, 3	41, 0
2. 10	9. 50	35, 5	36, 2	37, 0	37, 7	38, 5	39, 2	39, 9	40, 7	41, 4	42, 2	42, 9	43, 6
2. 20	9. 40	37, 6	38, 4	39, 2	39, 9	40, 7	41, 5	42, 3	43, 1	43, 9	44, 6	45, 4	46, 2
2. 30	9. 30	39, 6	40, 4	41, 2	42, 1	42, 9	43, 7	44, 5	45, 4	46, 2	47, 0	47, 8	48, 7
2. 40	9. 20	41, 5	42, 3	43, 2	44, 1	44, 9	45, 8	46, 7	47, 5	48, 4	49, 3	50, 1	51, 0
2. 50	9. 10	43, 3	44, 2	45, 1	46, 0	46, 9	47, 8	48, 7	49, 6	50, 5	51, 4	52, 3	53, 2
3. 0	9. 0	45, 0	45, 9	46, 9	47, 8	48, 8	49, 7	50, 6	51, 6	52, 5	53, 4	54, 4	55, 3
3. 10	8. 50	46, 6	47, 6	48, 6	49, 5	50, 5	51, 5	52, 4	53, 4	54, 4	55, 4	56, 3	57, 3
3. 20	8. 40	48, 1	49, 2	50, 2	51, 2	52, 2	53, 2	54, 2	55, 2	56, 2	57, 2	58, 2	59, 2
3. 30	8. 30	49, 6	50, 6	51, 6	52, 7	53, 7	54, 7	55, 8	56, 8	57, 8	58, 9	59, 9	60, 9
3. 40	8. 20	50, 9	52, 0	53, 0	54, 1	55, 2	56, 2	57, 3	58, 4	59, 4	60, 5	61, 5	62, 6
3. 50	8. 10	52, 2	53, 3	54, 3	55, 4	56, 5	57, 6	58, 7	59, 8	60, 9	62, 0	63, 0	64, 1
4. 0	8. 0	53, 3	54, 4	55, 6	56, 7	57, 8	58, 9	60, 0	61, 1	62, 2	63, 3	64, 4	65, 6
4. 10	7. 50	54, 4	55, 5	56, 7	57, 8	58, 9	60, 1	61, 2	62, 3	63, 5	64, 6	65, 7	66, 9
4. 20	7. 40	55, 4	56, 5	57, 7	58, 8	60, 0	61, 1	62, 3	63, 4	64, 6	65, 8	66, 9	68, 1
4. 30	7. 30	56, 3	57, 4	58, 6	59, 8	60, 9	62, 1	63, 3	64, 5	65, 6	66, 8	68, 0	69, 1
4. 40	7. 20	57, 0	58, 2	59, 4	60, 6	61, 8	63, 0	64, 2	65, 4	66, 5	67, 7	68, 9	70, 1
4. 50	7. 10	57, 7	58, 9	60, 1	61, 3	62, 5	63, 7	64, 9	66, 2	67, 4	68, 6	69, 8	71, 0
5. 0	7. 0	58, 3	59, 5	60, 8	62, 0	63, 2	64, 4	65, 6	66, 8	68, 1	69, 3	70, 5	71, 7
5. 10	6. 50	58, 8	60, 1	61, 3	62, 5	63, 7	65, 0	66, 2	67, 4	68, 6	69, 9	71, 1	72, 3
5. 20	6. 40	59, 3	60, 5	61, 7	63, 0	64, 2	65, 4	66, 7	67, 9	69, 1	70, 4	71, 6	72, 8
5. 30	6. 30	59, 6	60, 8	62, 1	63, 3	64, 5	65, 8	67, 0	68, 3	69, 5	70, 8	72, 0	73, 2
5. 40	6. 20	59, 8	61, 1	62, 3	63, 6	64, 8	66, 0	67, 3	68, 5	69, 8	71, 0	72, 3	73, 5
5. 50	6. 10	50, 0	61, 2	62, 5	63, 7	64, 9	66, 2	67, 4	68, 7	69, 9	71, 2	72, 4	73, 7
6. 0	6. 0	60, 0	61, 3	62, 5	63, 8	65, 0	66, 3	67, 5	68, 8	70, 0	71, 3	72, 5	73, 8

TABLE XXXVIII. *Continued.*

Apparent Time after Noon or Midnight.		Second Difference of the Moon's Place.												
		10 Minutes.						11 Minutes.						12 Min
		0″	10″	20″	30″	40″	50″	0″	10″	20″	30″	40″	50″	0″
H. M.	H. M.	″	″	″	″	″	″	″	″	″	″	″	″	″
0. 0	12. 0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0	0,0
0. 10	11. 50	4,1	4,2	4,2	4,3	4,4	4,5	4,5	4,6	4,7	4,7	4,8	4,9	4,9
0. 20	11. 40	8,1	8,2	8,4	8,5	8,6	8,8	8,9	9,0	9,2	9,3	9,5	9,6	9,7
0. 30	11. 30	12,0	12,2	12,4	12,6	12,8	13,0	13,2	13,4	13,6	13,8	14,0	14,2	14,4
0. 40	11. 20	15,7	16,0	16,3	16,5	16,8	17,1	17,3	17,6	17,8	18,1	18,4	18,6	18,9
0 50	11. 10	19,4	19,7	20,0	20,4	20,7	21,0	21,3	21,6	22,0	22,3	22,6	22,9	23,3
1. 0	11. 0	22,9	23,3	23,7	24,1	24,4	24,8	25,2	25,6	26,0	26,4	26,7	27,1	27,5
1. 10	10. 50	26,3	26,8	27,2	27,6	28,1	28,5	29,0	29,4	29,8	30,3	30,7	31,2	31,6
1. 20	10. 40	29,6	30,1	30,6	31,1	31,6	32,1	32,6	33,1	33,6	34,1	34,6	35,1	35,6
1. 30	10. 30	32,8	33,4	33,9	34,5	35,0	35,5	36,1	36,6	37,2	37,7	38,3	38,8	39,4
1. 40	10. 20	35,9	36,5	37,1	37,7	38,3	38,9	39,5	40,1	40,7	41,3	41,9	42,5	43,1
1. 50	10. 10	38,8	39,5	40,1	40,8	41,4	42,1	42,7	43,4	44,0	44,7	45,3	46,0	46,6
2. 0	0. 0	41,7	42,4	43,1	43,8	44,4	45,1	45,8	46,5	47,2	47,9	48,6	49,3	50,0
2. 10	9. 50	44,4	45,1	45,9	46,6	47,3	48,1	48,8	49,6	50,3	51,0	51,8	52,5	53,3
2. 20	9. 40	47,0	47,8	48,6	49,3	50,1	50,9	51,7	52,5	53,3	54,0	54,8	55,6	56,4
2. 30	9. 30	49,5	50,3	51,1	52,0	52,8	53,6	54,4	55,3	56,1	56,9	57,7	58,6	59,4
2. 40	9. 20	51,9	52,7	53,6	54,4	55,3	56,2	57,0	57,9	58,8	59,6	60,5	61,4	62,2
2. 50	9. 10	54,1	55,0	55,9	56,8	57,7	58,6	59,5	60,4	61,3	62,2	63,1	64,0	64,9
3. 0	9. 0	56,3	57,2	58,1	59,1	60,0	60,9	61,9	62,8	63,8	64,7	65,6	66,6	67,5
3. 10	8. 50	58,3	59,2	60,2	61,2	62,2	63,1	64,1	65,1	66,0	67,9	68,0	69,0	69,9
3. 20	8. 40	60,2	61,2	62,2	63,2	64,2	65,2	66,2	67,2	68,2	69,2	70,2	71,2	72,2
3. 30	8. 30	62,0	63,0	64,0	65,1	66,1	67,1	68,2	69,2	70,2	71,3	72,3	73,3	74,4
3. 40	8. 20	63,7	64,7	65,8	66,8	67,9	69,0	70,0	71,1	72,1	73,2	74,3	75,3	76,4
3. 50	8. 10	65,2	66,3	67,4	68,5	69,6	70,7	71,7	72,8	73,9	75,0	76,1	77,2	78,3
4. 0	8. 0	66,7	67,8	68,9	70,0	71,1	72,2	73,3	74,4	75,6	76,7	77,8	78,9	80,0
4. 10	7. 50	68,0	69,1	70,3	71,4	72,5	73,7	74,8	75,9	77,1	78,2	79,3	80,5	81,6
4. 20	7. 40	69,2	70,4	71,5	72,7	73,8	75,0	76,1	77,3	78,4	79,6	80,7	81,9	83,1
4. 30	7. 30	70,3	71,5	72,7	73,8	75,0	76,2	77,3	78,5	79,7	80,9	82,0	83,2	84,4
4. 40	7. 20	71,3	72,5	73,7	74,9	76,0	77,2	78,4	79,6	80,8	82,0	83,2	84,4	85,6
4. 50	7. 10	72,2	73,4	74,6	75,8	77,0	78,2	79,4	80,7	81,8	83,0	84,2	85,4	86,6
5. 0	7. 0	72,9	74,1	75,3	76,6	77,8	79,0	80,2	81,4	82,6	83,9	85,1	86,3	87,5
5. 10	6. 50	73,6	74,8	76,0	77,2	78,5	79,7	80,9	82,1	83,4	84,6	85,8	87,0	88,3
5. 20	6. 40	74,1	75,3	76,5	77,8	79,0	80,2	81,5	82,7	84,0	85,2	86,4	87,7	88,9
5. 30	6 30	74,5	75,7	77,0	78,2	79,4	80,7	81,9	83,2	84,4	85,7	86,9	88,1	89,4
5. 40	6. 20	74,8	76,1	77,3	78,5	79,8	81,0	82,2	83,5	84,7	86,0	87,2	88,5	89,7
5. 50	6. 10	74,9	76,2	77,4	78,7	79,9	81,2	82,4	83,7	84,9	86,2	87,4	88,7	89,9
6. 0	6. 0	75,0	76,3	77,5	78,8	80,0	81,3	82,5	83,8	85,0	86,3	87,5	88,8	90,0

TABLE XXXIX.

The Equation of Second Difference, useful for interpolating the Moon's Distances from the Sun and Stars, for every third Hour between those computed at Noon and Midnight, for the Use of the Nautical Ephemeris.

Second Difference.	Equation at 3h. and 9h.	Equation at 6h.	Second Difference.	Equation at 3h. and 9h.	Equation at 6h.	Second Difference.	Equation at 3h. and 9h.	Equation at 6h.
M. S.	′ ″	′ ″	M. S.	′ ″	′ ″	′ ″	′ ″	′ ″
0. 0	0. 0,0	0 0,0	6 0	0 33,8	0. 45,0	12. 0	1. 7,5	1. 30 0
0. 10	0. 0.9	0. 1.3	6. 10	0 34 7	0 46,3	12 10	1. 8,4	1. 31,3
0. 20	0. 1,9	0. 2.5	6 20	0. 35 6	0 47,5	12. 20	1. 9,4	1. 32,5
0 30	0 2,8	0. 3,8	6 30	0 36 6	0. 48.8	12 30	1 10,3	1. 33,8
0. 40	0. 3,8	0. 5,0	6. 40	0 37.5	0 50 0	12 40	1. 11,3	1. 35,0
0. 50	0. 4,7	0. 6,3	6. 50	0. 38,4	0 51,3	12. 50	1. 12,2	1. 36,3
1. 0	0. 5,6	0. 7,5	7. 0	0 39,4	0. 52,5	13. 0	1. 13,1	1. 37,5
1. 10	0. 6,6	0. 8,8	7. 10	0 40,3	0. 53.8	13. 10	1. 14,1	1. 38,8
1. 20	0. 7,5	0 10,0	7 20	0 41,3	0. 55,0	13. 20	1. 15.0	1. 40,0
1. 30	0. 8 4	0. 11,3	7. 30	0 42,2	0. 56 3	13. 30	1. 15 9	1. 41,3
1. 40	0. 9,4	0. 12 5	7. 40	0 43,1	0. 57,5	13. 40	1. 16,9	1. 42,5
1. 50	0. 10,3	0. 13,8	7. 50	0. 44,1	0. 58,8	13. 50	1. 17,8	1. 43,8
2. 0	0 11,3	0 15 0	8. 0	0. 45,0	1. 0,0	14. 0	1. 18,8	1. 45,0
2. 10	0. 12 2	0 16,3	8. 10	0. 45,9	1. 1,3	14. 10	1. 19,7	1. 46,3
2. 20	0. 13 1	0. 17 5	8. 20	0. 46 9	1. 2 5	14. 20	1. 20,6	1. 47,5
2. 30	0. 14 1	0. 18 8	8. 30	0. 47,8	1. 3,8	14. 30	1. 21,6	1. 48,8
2. 40	0. 15,0	0. 20,0	8. 40	0 48,8	1. 5,0	14. 40	1. 22,5	1. 50,0
2. 50	0. 15,9	0. 21,3	8. 50	0. 49,7	1. 6,3	14 50	1. 23.4	1. 51,3
3. 0	0. 16,9	0. 22,5	9 0	0. 50 6	1. 7,5	15. 0	1. 24 4	1. 52,5
3. 10	0. 17,8	0. 23.8	9 10	0. 51,6	1. 8,9	15. 10	1. 25,3	1. 53,8
3 20	0. 18.8	0. 25,0	9 20	0. 52,5	1. 10,0	15. 20	1. 26,3	1. 55,0
3. 30	0. 19,7	0. 26,3	9 30	0. 53,4	1 11,3	15. 30	1. 27,2	1. 56 3
3. 40	0. 20,6	0. 27,5	9. 40	0 54 4	1. 12.5	15 40	1. 28,1	1 57,5
3. 50	0. 21,6	0. 28,8	9 50	0. 55,3	1. 13,8	15. 50	1. 29,1	1 58,8
4. 0	0 22,5	0. 30 0	10. 0	0. 56,3	1. 15,0	16. 0	1. 30,0	2. 0,0
4. 10	0. 23,4	0. 31,3	10 10	0. 57,2	1 16,3	16. 10	1. 30.9	2. 1,3
4. 20	0 24,4	0. 32,5	10 20	0. 58 1	1 17,5	16. 20	1. 31,9	2. 2,5
4. 30	0. 25 3	0. 33,8	10 30	0. 59,1	1. 18 8	16. 30	1 32,8	2. 3,8
4 40	0 26 3	0. 35,0	10. 40	1. 0,0	1. 20,0	16. 40	1 33,8	2. 5,0
4. 50	0. 27,2	0. 36,3	10. 50	1. 0,9	1. 21,3	16 50	1. 34 7	2. 6,3
5. 0	0. 28,1	0. 37,5	11. 0	1. 1,9	1. 22,5	17 0	1. 35 6	2 7 5
5. 10	0. 29,1	0. 38,8	11. 10	1. 2,8	1. 23,8	17. 10	1. 36 6	2 8,8
5 20	0 30,0	0. 40 0	11. 20	1. 3,8	1. 25,0	17. 20	1 37,5	2. 10,0
5. 30	0 30 9	0. 41,3	11. 30	1. 4 7	1 26 3	17. 30	1. 38,4	2 11,3
5. 40	0 31 9	0. 42,5	11. 40	1. 5,6	1. 27 5	17 40	1 39,4	2. 12,5
5. 50	0. 32,8	0 43,8	11 50	1. 6,6	1 28,8	17 50	1. 40,3	2 13,8
6. 0	0. 33,8	0. 45,0	12. 0	1. 7,5	1. 30,0	18. 0	1. 41,3	2. 15,0

TABLE XXXIX. *Continued.*

Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.	Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.	Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.
M S.	′ ″	′ ″	M. S.	′ ″	′ ″	M. S.	′ ″	′ ″
18. 0	1. 41,3	2. 15,0	24. 0	2. 15,0	3. 0,0	30. 0	2. 48,8	3. 45,0
18. 10	1. 42,2	2. 16,3	24. 10	2. 15,9	3. 1,3	30. 10	2. 49,7	3. 46,3
18. 20	1. 43,1	2. 17,5	24. 20	2. 16,9	3. 2,5	30. 20	2. 50,6	3. 47,5
18. 30	1. 44,1	2. 18,8	24. 30	2. 17,8	3. 3,8	30. 30	2. 51,6	3. 48,8
18. 40	1. 45,0	2. 20,0	24. 40	2. 18,8	3. 5,0	30. 40	2. 52,5	3. 50,0
18. 50	1. 45,9	2. 21,3	24. 50	2. 19,7	3. 6,3	30. 50	2. 53,4	3. 51,3
19. 0	1. 46,9	2. 22,5	25. 0	2. 20,6	3. 7,5	31. 0	2. 54,4	3. 52,5
19. 10	1. 47,8	2. 23,8	25. 10	2. 21,6	3. 8,8	31. 10	2. 55,3	3. 53,8
19. 20	1. 48,8	2. 25,0	25. 20	2. 22,5	3. 10,0	31. 20	2. 56,3	3. 55,0
19. 30	1. 49,7	2. 26,3	25. 30	2. 23,4	3. 11,3	51. 30	2. 57,2	3. 56,3
19. 40	1. 50,6	2. 27,5	25. 40	2. 24,4	3. 12,5	31. 40	2. 58,1	3. 57,5
19. 50	1. 51,6	2. 28,8	25. 50	2. 25,3	3. 13,8	31. 50	2. 59,1	3. 58,8
20. 0	1. 52,5	2. 30,0	26. 0	2. 26,3	3. 15,0	32. 0	3. 0,0	4. 0,0
20. 10	1. 53,4	2. 31,3	26. 10	2. 27,2	3. 16,3	32. 10	3. 0,9	4. 1,3
20. 20	1. 54,4	2. 32,5	26. 20	2. 28,1	3. 17,5	32. 20	3. 1,9	4. 2,5
20. 30	1. 55,3	2. 33,8	26. 30	2. 29,1	3. 18,8	32. 30	3. 2,8	4. 3,8
20. 40	1. 56,3	2. 35,0	26. 40	2. 30,0	3. 20,0	32. 40	3. 3,8	4. 5,0
20. 50	1. 57,2	2. 36,3	26. 50	2. 80,9	3. 21,3	32. 50	3. 4,7	4. 6,3
21. 0	1. 58,1	2. 37,5	27. 0	2. 31,9	3. 22,5	33. 0	3. 5,6	4. 7,5
21. 10	1. 59,1	2. 38,8	27. 10	2. 32,8	3. 23,8	33. 10	3. 6,6	4. 8,8
21. 20	2. 0,0	2. 40,0	27. 20	2. 33,8	3. 25,0	33. 20	3. 7,5	4. 10,0
21. 30	2. 0,9	2. 41,3	27. 30	2. 34,7	3. 26,3	33. 30	3. 8,4	4. 11,3
21. 40	2. 1,9	2. 42,5	27. 40	2. 35,6	3. 27,5	33. 40	3. 9,4	4. 12,5
21. 50	2. 2,8	2. 43,8	27. 50	2. 36,6	3. 28,8	33. 50	3. 10,3	4. 13,8
22. 0	2. 3,8	2. 45,0	28. 0	2. 37,5	3. 30,0	34. 0	3. 11,3	4. 15,0
22. 10	2. 4,7	2. 46,3	28. 10	2. 38,4	3. 31,3	34. 10	3. 12,2	4. 16,3
22. 20	2. 5,6	2. 47,5	28. 20	2. 39,4	3. 32,5	34. 20	3. 13,1	4. 17,5
22. 30	2. 6,6	2. 48,8	28. 30	2. 40,3	3. 33,8	34. 30	3. 14,1	4. 18,8
22. 40	2. 7,5	2. 50,0	28. 40	2. 41,3	3. 35,0	34. 40	3. 15,0	4. 20,0
22. 50	2. 8,4	2. 51,3	28. 50	2. 42,2	3. 36,3	34. 50	3. 15,9	4. 21,3
23. 0	2. 9,4	2. 52,5	29. 0	2. 43,1	3. 37,5	35. 0	3. 16,9	4. 22,5
23. 10	2. 10,3	2. 53,8	29. 10	2. 44,1	3. 38,8	35. 10	3. 17,8	4. 23,8
23. 20	2. 11,3	2. 55,0	29. 20	2. 45,0	3. 40,0	35. 20	3. 18,8	4. 25,0
23. 30	2. 12,2	2. 56,3	29. 50	2. 45,9	3. 41,3	35. 30	3. 19,7	4. 26,3
23. 40	2. 13,1	2. 57,5	29. 40	2. 46,9	3. 42,5	35. 40	3. 20,6	4. 27,5
23. 50	2. 14,1	2. 58,8	29. 50	2. 47,8	3. 43,8	35. 50	3. 21,6	4. 28,8
24. 0	2. 15,0	3. 0,0	30. 0	2. 48,8	3. 45,0	36. 0	3. 22,5	4. 30,0

TABLE XXXIX. *Continued.*

Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.	Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.
M. S.	′ ″	′ ″	M. S.	′ ″	′ ″
36. 0	3. 22, 5	4. 30, 0	42. 0	3. 56, 3	5. 15, 0
36. 10	3. 23, 4	4. 31, 3	42. 10	3. 57, 2	5. 16, 3
36. 20	3. 24, 4	4. 32, 5	42. 20	3. 58, 1	5. 17, 5
36. 30	3. 25, 3	4. 33, 8	42. 30	3. 59, 1	5. 18, 8
36. 40	3. 26, 3	4. 35, 0	42. 40	4. 0, 0	5. 20, 0
36. 50	3. 27, 2	4. 36, 3	42. 50	4. 0, 9	5. 21, 3
37. 0	3. 28, 1	4. 37, 5	43. 0	4. 1, 9	5. 22, 5
37. 10	3. 29, 1	4. 38, 8	43. 10	4. 2, 8	5. 23, 8
37. 20	3. 30, 0	4. 40, 0	43. 20	4. 3, 8	5. 25, 0
37. 30	3. 30, 9	4. 41, 3	43. 30	4. 4, 7	5. 26, 3
37. 40	3. 31, 9	4. 42, 5	43. 40	4. 5, 6	5. 27, 5
37. 50	3. 32, 8	4. 43, 8	43. 50	4. 6, 6	5. 28, 8
38. 0	3. 33, 8	4. 45, 0	44. 0	4. 7, 5	5. 30, 0
38. 10	3. 34, 7	4. 46, 3	44. 10	4. 8, 4	5. 31, 3
38. 20	3. 35, 6	4. 47, 5	44. 20	4. 9, 4	5. 32, 5
38. 30	3. 36, 6	4. 48, 8	44. 30	4. 10, 3	5. 33, 8
38. 40	3. 37, 5	4. 50, 0	44. 40	4. 11, 3	5. 35, 0
38. 50	3. 38, 4	4. 51, 3	44. 50	4. 12, 2	5. 36, 3
39. 0	3. 39, 4	4. 52, 5	45. 0	4. 13, 1	5. 37, 5
39. 10	3. 40, 3	4. 53, 8	45. 10	4. 14, 1	5. 38, 8
39. 20	3. 41, 3	4. 55, 0	45. 20	4. 15, 0	5. 40, 0
39. 30	3. 42, 2	4. 56, 3	45. 30	4. 15, 9	5. 41, 3
39. 40	3. 43, 1	4. 57, 5	45. 40	4. 16, 9	5. 42, 5
39. 50	3. 44, 1	4. 58, 8	45. 50	4. 17, 8	5. 43, 8
40. 0	3. 45, 0	5. 0, 0	46. 0	4. 18, 8	5. 45, 0
40. 10	3. 45, 9	5. 1, 3	46. 10	4. 19, 7	5. 46, 3
40. 20	3. 46, 9	5. 2, 5	46. 20	4. 20, 6	5. 47, 5
40. 30	3. 47, 8	5. 3, 8	46. 30	4. 21, 6	5. 48, 8
40. 40	3. 48, 8	5. 5, 0	46. 40	4. 22, 5	5. 50, 0
40. 50	3. 49, 7	5. 6, 3	46. 50	4. 23, 4	5. 51, 3
41. 0	3. 50, 6	5. 7, 5	47. 0	4. 24, 4	5. 52, 5
41. 10	3. 51, 6	5. 8, 8	47. 10	4. 25, 3	5. 53, 8
41. 20	3. 52, 5	5. 10, 0	47. 20	4. 26, 3	5. 55, 0
41. 30	3. 53, 4	5. 11, 3	47. 30	4. 27, 2	5. 56, 3
41. 40	3. 54, 4	5. 12, 5	47. 40	4. 28, 1	5. 57, 5
41. 50	3. 55, 3	5. 13, 8	47. 50	4. 29, 1	5. 58, 8
42. 0	3. 56, 3	5. 15, 0	48. 0	4. 30, 0	6. 0, 0

TABLE XXXIX. *Continued.*

Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.	Second Difference.	Equation at 3*h*. and 9*h*.	Equation at 6*h*.
M. S.	′ ″	′ ″	M. S.	′ ′	′ ″
48. 0	4. 30, 0	6. 0, 0	54. 0	5. 3, 8	6. 45, 0
48. 10	4. 30, 9	6. 1, 3	54. 10	5. 4, 7	6. 46, 3
48. 20	4. 31, 9	6. 2, 5	54. 20	5. 5, 6	6. 47, 5
48. 30	4. 32, 8	6. 3, 8	54. 30	5. 6, 6	6. 48, 8
48. 40	4. 33, 8	6. 5, 0	54. 40	5. 7, 5	6. 50, 0
48. 50	4. 34, 7	6. 6, 3	54. 50	5. 8, 4	6. 51, 3
49. 0	4. 35, 6	6. 7, 5	55. 0	5. 9, 4	6. 52, 5
49. 10	4. 36, 6	6. 8, 8	55. 10	5. 10, 3	6. 53, 8
49. 20	4. 37, 5	6. 10, 0	55. 20	5. 11, 3	6. 55, 0
49. 30	4. 38, 4	6. 11, 3	55. 30	5. 12, 2	6. 56, 3
49. 40	4. 39, 4	6. 12, 5	55. 40	5. 13, 1	6. 57, 5
49. 50	4. 40, 3	6. 13, 8	55. 50	5. 14, 1	6. 58, 8
50. 0	4. 41, 3	6. 15, 0	56. 0	5. 15, 0	7. 0, 0
50. 10	4. 42, 2	6. 16, 3	56. 10	5. 15, 9	7. 1, 3
50. 20	4. 43, 1	6. 17, 5	56. 20	5. 16, 9	7. 2, 5
50. 30	4. 44, 1	6. 18, 8	56. 30	5. 17, 8	7. 3, 8
50. 40	4. 45, 0	6. 20, 0	56. 40	5. 18, 8	7. 5, 0
50. 50	4. 45, 9	6. 21, 3	56. 50	5. 19, 7	7. 6, 3
51. 0	4. 46, 9	6. 22, 5	57. 0	5. 20, 6	7. 7, 5
51. 10	4. 47, 8	6. 23, 8	57. 10	5. 21, 6	7. 8, 8
51. 20	4. 48, 8	6. 25, 0	57. 20	5. 22, 5	7. 10, 0
51. 30	4. 49, 7	6. 26, 3	57. 30	5. 23, 4	7. 11, 3
51. 40	4. 50, 6	6. 27, 5	57. 40	5. 24, 4	7. 12, 5
51. 50	4. 51, 6	6. 28, 8	57. 50	5. 25, 3	7. 13, 8
52. 0	4. 52, 5	6. 30, 0	58. 0	5. 26, 3	7. 15, 0
52. 10	4. 53, 4	6. 31, 3	58. 10	5. 27, 2	7. 16, 3
52. 20	4. 54, 4	6. 32, 5	58. 20	5. 28, 1	7. 17, 5
52. 30	4. 55, 3	6. 33, 8	58. 30	5. 29, 1	7. 18, 8
52. 40	4. 56, 3	6. 35, 0	58. 40	5. 30, 0	7. 20, 0
52. 50	4. 57, 2	6. 36, 3	58. 50	5. 30, 9	7. 21, 3
53. 0	4. 58, 1	6. 37, 5	59. 0	5. 31, 9	7. 22, 5
53. 10	4. 59, 1	6. 38, 8	59. 10	5. 32, 8	7. 23, 8
53. 20	5. 0, 0	6. 40, 0	59. 20	5. 33, 8	7. 25, 0
53. 30	5. 0, 9	6. 41, 3	59. 30	5. 34, 7	7. 26, 3
53. 40	5. 1, 9	6. 42, 5	59. 40	5. 35, 6	7. 27, 5
53. 50	5. 2, 8	6. 43, 8	59. 50	5. 36, 6	7. 28, 8
54. 0	4. 3, 8	6. 45, 0	60. 0	5. 37, 5	7. 30, 0

TABLE XL.

Decimal Parts of a Degree.

Min.	Dec.	Min.	Dec.	Sec.	Dec.	Sec.	Dec.
1	,01667	31	,51667	1	,00028	31	,00861
2	,03333	32	,53333	2	,00056	32	,00889
3	,05000	33	,55000	3	,00083	33	,00917
4	,06667	34	,56667	4	,00111	34	,00944
5	,08333	35	,58333	5	,00138	35	,00972
6	,10000	36	,60000	6	,00167	36	,01000
7	,11667	37	,61667	7	,00194	37	,01028
8	,13333	38	,63333	8	,00222	38	,01056
9	,15000	39	,65000	9	,00250	39	,01083
10	,16667	40	,66667	10	,00278	40	,01111
11	,18333	41	,68333	11	,00306	41	,01139
12	,20000	42	,70000	12	,00333	42	,01167
13	,21667	43	,71667	13	,00361	43	,01194
14	,23333	44	,73333	14	,00389	44	,01222
15	,25000	45	,75000	15	,00417	45	,01250
16	,26667	46	,76667	16	,00444	46	,01278
17	,28333	47	,78333	17	,00472	47	,01306
18	,30000	48	,80000	18	,00500	48	,01333
19	,31667	49	,81667	19	,00528	49	,01361
20	,33333	50	,83333	20	,00556	50	,01389
21	,35000	51	,85000	21	,00583	51	,01417
22	,36667	52	,86667	22	,00611	52	,01444
23	,38333	53	,88333	23	,00639	53	,01472
24	,40000	54	,90000	24	,00667	54	,01500
25	,41667	55	,91667	25	,00694	55	,01528
26	,43333	56	,93333	26	,00722	56	,01556
27	,45000	57	,95000	27	,00750	57	,01583
28	,46667	58	,96667	28	,00778	58	,01611
29	,48333	59	,98333	29	,00806	59	,01639
30	,50000	60	1,00000	30	,00833	60	,01667

TABLE XLI.

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Observations.						
		H.—M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		1. 30	1. 40	1. 50	2. 0	2. 10	2. 20	2. 30
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	— 0	15, 47	15, 57	15, 68	15, 79	15, 92	16, 06	16, 21
	5	15, 38	15, 48	15, 58	15, 70	15, 83	15, 97	16, 12
	10	15, 19	15, 28	15, 38	15, 50	15, 62	15, 76	15, 91
	15	14, 90	14, 99	15, 09	15, 20	15, 32	15, 46	15, 61
	20	14, 51	14, 60	14, 69	14, 80	14, 92	15, 05	15, 20
	25	14, 02	14, 10	14, 20	14, 31	14, 42	14, 55	14, 69
I.	0	13, 44	13, 53	13, 62	13, 72	13, 83	13, 95	14, 08
	5	12, 76	12, 84	12, 93	13, 03	13, 14	13, 25	13, 37
	10	11, 99	12, 06	12, 14	12, 23	12, 33	12, 44	12, 56
	15	11, 12	11, 19	11, 27	11, 35	11, 44	11, 54	11, 65
	20	10, 16	10, 22	10, 29	10, 37	10, 46	10, 55	10, 65
	25	9, 12	9, 17	9, 23	9, 30	9, 38	9, 46	9, 55
II.	— 0	7, 99	8, 04	8, 09	8, 15	8, 22	8, 29	8, 37
	5	6, 78	6, 82	6, 87	6, 92	6, 98	7, 04	7, 11
	10	5, 51	5, 55	5, 59	5, 63	5, 67	5, 72	5, 78
	15	4, 18	4, 21	4, 24	4, 27	4, 30	4, 34	4, 38
	20	2, 81	2, 83	2, 85	2, 87	2, 90	2, 92	2, 95
	25	1, 41	1, 42	1, 43	1, 44	1, 46	1, 47	1, 48
III.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 41	1, 42	1, 43	1, 44	1, 45	1, 47	1, 48
	10	2, 81	2, 83	2, 85	2, 87	2, 89	2, 92	2, 95
	15	4, 17	4, 20	4, 23	4, 26	4, 29	4, 33	4, 37
	20	5, 49	5, 52	5, 56	5, 60	5, 65	5, 70	5, 75
	25	6, 75	6, 79	6, 84	6, 89	6, 95	7, 01	7, 07
IV.	+ 0	7, 94	7, 99	8, 05	8, 11	8, 17	8, 24	8, 32
	5	9, 06	9, 11	9, 17	9, 24	9, 32	9, 40	9, 49
	10	10, 09	10, 15	10, 32	10, 30	10, 38	10, 47	10, 57
	15	11, 03	11, 10	11, 18	11, 26	11, 35	11, 45	11, 56
	20	11, 89	11, 96	12, 04	12, 13	12, 23	12, 33	12, 45
	25	12, 65	12, 72	12, 81	12, 91	13, 01	13, 12	13, 25
V.	+ 0	13, 31	13, 40	13, 49	13, 59	13, 70	13, 82	13, 95
	5	13, 88	13, 97	14, 07	14, 17	14, 28	14, 41	14, 54
	10	14, 35	14, 44	14, 54	14, 65	14, 77	14, 90	15, 04
	15	14, 74	14, 83	14, 93	15, 04	15, 16	15, 30	15, 44
	20	15, 03	15, 12	15, 22	15, 33	15, 46	15, 59	15, 74
	25	15, 22	15, 31	15, 41	15, 53	15, 65	15, 79	15, 94
VI.	+ 0	15, 31	15, 40	15, 51	15, 62	15, 75	15, 89	16, 04

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		2. 40	2. 50	3. 0	3. 10	3. 20	3. 30	3. 40
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	— 0	16, 38	16, 56	16, 75	16, 96	17, 18	17, 42	17, 67
	5	16, 28	16, 46	16, 65	16, 86	17, 08	17, 32	17, 57
	10	16, 08	16, 25	16, 44	16, 64	16, 86	17, 10	17, 35
	15	15, 77	15, 94	16, 12	16, 32	16, 54	16, 77	17, 01
	20	15, 35	15, 52	15, 70	15, 89	16, 10	16, 32	16, 56
	25	14, 84	15, 00	15, 17	15, 36	15, 56	15, 78	16, 01
I.	— 0	14, 22	14, 37	14, 54	14, 72	14, 92	15, 13	15, 35
	5	13, 51	13, 65	13, 81	13, 98	14, 17	14, 36	14, 57
	10	12, 69	12, 83	12, 98	13, 14	13, 31	13, 49	13, 69
	15	11, 77	11, 90	12, 04	12, 19	12, 35	12, 52	12, 70
	20	10, 75	10, 87	11, 00	11, 14	11, 28	11, 44	11, 61
	25	9, 65	9, 76	9, 87	9, 99	10, 12	10, 26	10, 41
II.	— 0	8, 46	8, 55	8, 65	8, 76	8, 87	8, 99	9, 12
	5	7, 18	7, 26	7, 35	7, 44	7, 54	7, 64	7, 75
	10	5, 84	5, 90	5, 97	6, 04	6, 12	6, 21	6, 30
	15	4, 43	4, 48	4, 53	4, 59	4, 65	4, 71	4, 78
	20	2, 98	3, 01	3, 05	3, 09	3, 13	3, 17	3, 22
	25	1, 50	1, 51	1, 53	1, 55	1, 57	1, 59	1, 62
III.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 49	1, 51	1, 53	1, 55	1, 57	1, 59	1, 61
	10	2, 98	3, 01	3, 04	3, 08	3, 12	3, 16	3, 21
	15	4, 42	4, 47	4, 52	4, 57	4, 63	4, 70	4, 77
	20	5, 81	5, 87	5, 94	6, 02	6, 10	6, 18	6, 27
	25	7, 14	7, 22	7, 31	7, 40	7, 50	7, 60	7, 71
IV.	+ 0	8, 41	8, 50	8, 60	8, 71	8, 82	8, 94	9, 07
	5	9, 59	9, 69	9, 80	9, 93	10, 06	10, 20	10, 34
	10	10, 68	10, 80	10, 93	11, 06	11, 20	11, 36	11, 53
	15	11, 68	11, 81	11, 94	12, 09	12, 25	12, 42	12, 60
	20	12, 58	12, 72	12, 86	13, 02	13, 19	13, 38	13, 57
	25	13, 38	13, 53	13, 69	13, 86	14, 04	14, 23	14, 44
V.	+ 0	14, 09	14, 24	14, 41	14, 59	14, 78	14, 98	15, 20
	5	14, 69	14, 85	15, 03	15, 21	15, 41	15, 62	15, 85
	10	15, 20	15, 36	15, 54	15, 73	15, 94	16, 16	16, 39
	15	15, 60	15, 77	15, 95	16, 15	16, 36	16, 59	16, 83
	20	15, 90	16, 07	16, 26	16, 46	16, 68	16, 91	17, 16
	25	16, 10	16, 28	16, 47	16, 67	16, 89	17, 12	17, 37
VI.	+ 0	16, 20	16, 38	16, 57	16, 78	17, 00	17, 23	17, 48

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		3. 50	4. 0	4. 10	4. 20	4. 30	4. 40	4. 50
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	— 0	17, 94	18, 23	18, 54	18, 87	19, 22	19, 60	20, 01
	5	17, 84	18, 13	18, 44	18, 77	19, 12	19, 49	19, 89
	10	17, 62	17, 90	18, 20	18, 53	18, 88	19, 25	19, 64
	15	17, 27	17, 55	17, 85	18, 17	18, 51	18, 87	19, 26
	20	16, 82	17, 09	17, 38	17, 69	18, 02	18, 37	18, 75
	25	16, 26	16, 52	16, 80	17, 10	17, 42	17, 76	18, 12
I.	— 0	15, 59	15, 84	16, 11	16, 40	16, 70	17, 03	17, 38
	5	14, 80	15, 04	15, 29	15, 56	15, 86	16, 17	16, 50
	10	13, 90	14, 13	14, 37	14, 63	14, 90	15, 19	15, 50
	15	12, 90	13, 11	13, 33	13, 57	13, 82	14, 09	14, 38
	20	11, 79	11, 98	12, 18	12, 40	12, 63	12, 88	13, 14
	25	10, 57	10, 74	10, 93	11, 12	11, 33	11, 55	11, 79
II.	— 0	9, 26	9, 41	9, 57	9, 74	9, 93	10, 12	10, 33
	5	7, 87	8, 00	8, 14	8, 28	8, 43	8, 60	8, 77
	10	6, 40	6, 50	6, 61	6, 72	6, 85	6, 99	7, 13
	15	4, 85	4, 93	5, 01	5, 10	5, 20	5, 30	5, 41
	20	3, 27	3, 32	3, 37	3, 43	3, 50	3, 57	3, 64
	25	1, 64	1, 67	1, 70	1, 73	1, 76	1, 79	1, 83
III.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 64	1, 66	1, 69	1, 72	1, 76	1, 79	1, 83
	10	3, 26	3, 31	3, 37	3, 43	3, 50	3, 56	3, 63
	15	4, 84	4, 92	5, 00	5, 09	5, 19	5, 29	5, 40
	20	6, 37	6, 47	6, 58	6, 70	6, 82	6, 96	7, 10
	25	7, 83	7, 96	8, 10	8, 24	8, 39	8, 56	8, 73
IV.	+ 0	9, 21	9, 36	9, 52	9, 69	9, 87	10, 06	10, 27
	5	10, 50	10, 68	10, 86	11, 05	11, 26	11, 48	11, 71
	10	11, 70	11, 89	12, 10	12, 31	12, 54	12, 79	13, 05
	15	12, 79	13, 00	13, 22	13, 46	13, 71	13, 98	14, 27
	20	13, 78	14, 01	14, 25	14, 50	14, 77	15, 06	15, 37
	25	14, 66	14, 90	15, 15	15, 42	15, 71	16, 02	16, 35
V.	+ 0	15, 44	15, 69	15, 96	16, 24	16, 54	16, 86	17, 21
	5	16, 10	16, 36	16, 64	16, 94	17, 26	17, 59	17, 95
	10	16, 64	16, 91	17, 20	17, 51	17, 84	18, 19	18, 56
	15	17, 09	17, 37	17, 67	17, 98	18, 31	18, 67	19, 06
	20	17, 42	17, 70	18, 00	18, 32	18, 67	19, 04	19, 43
	25	17, 64	17, 93	18, 23	18, 56	18, 91	19, 28	19, 67
VI.	+ 0	17, 75	18, 04	18, 35	18, 68	19, 03	19, 40	19, 79

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		5. 0	5. 10	5. 20	5. 30	5. 40	5. 50	6. 0
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	— 0	20, 44	20, 90	21, 38	21, 90	22, 46	23, 05	23, 69
	5	20, 32	20, 77	21, 25	21, 77	22, 32	22, 91	23, 55
	10	20, 06	20, 51	20, 99	21, 50	22, 04	22, 62	23, 25
	15	19, 67	20, 11	20, 58	21, 08	21, 62	22, 19	22, 80
	20	19, 16	19, 59	20, 05	20, 54	21, 06	21, 61	22, 21
	25	18, 51	18, 93	19, 37	19, 84	20, 34	20, 88	21, 46
I.	— 0	17, 75	18, 15	18, 57	19, 02	19. 50	20, 02	20, 57
	5	16, 85	17, 23	17, 63	18, 06	18, 52	19, 01	19, 53
	10	15, 83	16, 19	16, 57	16, 97	17, 40	17, 86	18, 35
	15	14, 69	15, 02	15, 37	15, 74	16, 14	16, 57	17, 03
	20	13, 42	13, 72	14, 04	14, 38	14, 75	15, 14	15, 56
	25	12, 04	12, 31	12, 59	12, 90	13, 23	13, 58	13, 96
II.	— 0	10, 55	10, 78	11, 03	11, 30	11, 59	11, 90	12, 23
	5	8, 96	9, 16	9, 37	9, 60	9, 85	10, 11	10, 39
	10	7, 28	7, 44	7, 62	7, 80	8, 00	8, 21	8, 44
	15	5, 53	5, 65	5, 78	5, 92	6, 07	6, 23	6, 41
	20	3, 72	3, 80	3, 89	3, 98	4, 08	4, 19	4, 31
	25	1, 87	1, 91	1, 95	2, 00	2, 05	2, 10	2, 16
III.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 87	1, 91	1, 95	2, 00	2, 05	2, 10	2, 16
	10	3, 71	3, 79	3, 88	3, 97	4, 07	4, 18	4, 30
	15	5, 51	5, 63	5, 77	5, 91	6, 06	6, 22	6, 39
	20	7, 25	7, 41	7, 59	7, 77	7, 97	8, 18	8, 41
	25	8, 92	9, 12	9, 33	9, 56	9, 80	10, 06	10, 34
IV.	+ 0	10, 49	10, 73	10, 98	11, 25	11, 54	11, 84	12, 16
	5	11, 96	12, 23	12, 52	12, 82	13, 14	13, 49	13, 86
	10	13, 33	13, 63	13, 94	14, 28	14, 65	15, 04	15, 45
	15	14, 57	14, 90	15, 25	15, 62	16, 01	16, 43	16, 89
	20	15, 70	16, 05	16, 42	16, 82	17, 25	17, 71	18, 19
	25	16, 70	17, 07	17, 47	17, 90	18, 35	18, 84	19, 36
V.	+ 0	17, 58	17, 98	18, 40	18, 85	19, 32	19, 83	20, 38
	5	18, 34	18, 75	19, 18	19, 64	20, 14	20, 67	21, 25
	10	18, 96	19, 39	19, 84	20, 32	20, 84	21, 39	21, 97
	15	19, 47	19, 90	20, 36	20, 86	21, 39	21, 96	22, 56
	20	19, 84	20, 28	20, 76	21, 26	21, 80	22, 38	23, 00
	25	20, 09	20, 54	21, 02	21, 53	22, 08	22, 66	23, 29
VI.	+ 0	20, 22	20, 67	21, 15	21, 67	22, 22	22, 81	24, 44

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		1. 30	1. 40	1. 50	2. 0	2. 10	2. 20	2. 30
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	15, 31	15, 41	15, 51	15, 62	15, 75	15, 89	16, 04
	5	15, 30	15, 40	15, 50	15, 61	15, 74	15, 88	16, 03
	10	15, 20	15, 29	15, 39	15, 51	15, 63	15, 77	15, 92
	15	14, 99	15, 08	15, 19	15, 30	15, 42	15, 56	15, 71
	20	14, 69	14, 78	14, 88	14, 99	15, 11	15, 24	15, 39
	25	14, 28	14, 37	14, 46	14, 57	14, 69	14, 82	14, 96
VII.	+ 0	13, 76	13, 85	13, 95	14, 05	14, 16	14, 29	14, 42
	5	13, 14	13, 22	13, 31	13, 41	13, 52	13, 64	13, 77
	10	12, 41	12, 49	12, 57	12, 66	12, 76	12, 87	13, 00
	15	11, 57	11, 64	11, 71	11, 80	11, 90	12, 01	12, 12
	20	10, 62	10, 68	10, 75	10, 83	10, 92	11, 02	11, 13
	25	9, 57	9, 63	9, 69	9, 76	9, 84	9, 93	10, 03
VIII.	+ 0	8, 42	8, 47	8, 53	8, 59	8, 66	8, 74	8, 82
	5	7, 18	7, 22	7, 27	7, 32	7, 38	7, 45	7, 52
	10	5, 85	5, 88	5, 92	5, 97	6, 02	6, 07	6, 13
	15	4, 45	4, 48	4, 51	4, 54	4, 58	4, 62	4, 66
	20	3, 00	3, 02	3, 04	3, 06	3, 09	3, 11	3, 14
	25	1, 51	1, 52	1, 53	1, 54	1, 55	1, 56	1, 58
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 51	1, 52	1, 53	1, 54	1, 55	1, 57	1, 58
	10	3, 00	3, 02	3, 04	3, 07	3, 09	3, 12	3, 15
	15	4, 46	4, 49	4, 52	4, 55	4, 59	4, 63	4, 67
	20	5, 87	5, 90	5, 94	5, 99	6, 04	6, 09	6, 15
	25	7, 20	7, 24	7, 29	7, 35	7, 41	7, 48	7, 55
X.	— 0	8, 46	8, 51	8, 57	8, 63	8, 70	8, 78	8, 86
	5	9, 63	9, 69	9, 75	9, 82	9, 90	9, 99	10, 09
	10	10, 70	10, 76	10, 83	10, 91	11, 00	11, 10	11, 21
	15	11, 66	11, 73	11, 81	11, 90	12, 00	12, 10	12, 21
	20	12, 51	12, 59	12, 68	12, 77	12, 87	12, 99	13, 11
	25	13, 26	13, 34	13, 43	13, 53	13, 64	13, 76	13, 89
XI.	— 0	13, 89	13, 98	14, 08	14, 18	14, 29	14, 42	14, 55
	5	14, 42	14, 51	14, 61	14, 72	14, 84	14, 97	15, 11
	10	14, 84	14, 93	15, 03	15, 14	15, 27	15, 40	15, 55
	15	15, 15	15, 25	15, 35	15, 46	15, 59	15, 73	15, 88
	20	15, 36	15, 46	15, 57	15, 68	15, 80	15, 94	16, 10
	25	15, 47	15, 56	15, 67	15, 78	15, 91	16, 05	16, 21
XII.	— 0	15, 48	15, 57	15, 68	15, 79	15, 92	16, 06	16, 22

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		2. 40	2. 50	3. 0	3. 10	3. 20	3. 30	3. 40
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	16, 21	16, 38	16, 57	16, 78	17, 00	17, 28	17, 48
	5	16, 20	16, 37	16, 56	16, 77	16, 99	17, 22	17, 47
	10	16, 08	16, 26	16, 45	16, 65	16, 87	17, 10	17, 35
	15	15, 87	16, 04	16, 23	16, 43	16, 64	16, 87	17, 12
	20	15, 55	15, 72	15, 90	16, 09	16, 30	16, 53	16, 77
	25	15, 11	15, 28	15, 46	15, 65	15, 86	16, 08	16, 31
VII.	+ 0	14, 57	14, 73	14, 90	15, 08	15, 28	15, 49	15, 72
	5	13, 91	14, 06	14, 22	14, 40	14, 59	14, 79	15, 01
	10	13, 13	13, 27	13, 42	13, 59	13, 77	13, 96	14, 17
	15	12, 24	12, 37	12, 51	12, 67	12, 84	13, 02	13, 21
	20	11, 24	11, 36	11, 49	11, 63	11, 79	11, 96	12, 13
	25	10, 13	10, 24	10, 36	10, 48	10, 62	10, 77	10, 93
VIII.	+ 0	8, 91	9, 01	9, 11	9, 22	9, 34	9, 47	9, 61
	5	7, 59	7, 67	7, 76	7, 86	7, 96	8, 07	8, 19
	10	6, 19	6, 26	6, 33	6, 41	6, 49	6, 58	6, 68
	15	4, 71	4, 76	4, 82	4, 88	4, 94	5, 01	5, 08
	20	3, 18	3, 21	3, 25	3, 29	3, 33	3, 38	3, 43
	25	1, 60	1, 62	1, 64	1, 66	1, 68	1, 70	1, 73
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 60	1, 62	1, 64	1, 66	1, 68	1, 70	1, 73
	10	3, 18	3, 21	3, 25	3, 23	3, 33	3, 38	3, 43
	15	4, 72	4, 77	4, 83	4, 89	4, 95	5, 02	5, 09
	20	6, 21	6, 28	6, 35	6, 43	6, 51	6, 60	6, 70
	25	7, 62	7, 70	7, 79	7, 89	8, 00	8, 11	8, 23
X.	— 0	8, 95	9, 05	9, 15	9, 27	9, 39	9, 52	9, 66
	5	10, 19	10, 30	10, 42	10, 55	10, 69	10, 83	10, 99
	10	11, 32	11, 44	11, 58	11, 72	11, 87	12, 04	12, 21
	15	12, 34	12, 48	12, 62	12, 77	12, 94	13, 12	13, 31
	20	13, 24	13, 39	13, 54	13, 71	13, 89	14, 08	14, 29
	25	14, 03	14, 18	14, 35	14, 53	14, 72	14, 92	15, 14
XI.	— 0	14, 70	14, 86	15, 04	15, 23	15, 42	15, 63	15, 86
	5	15, 26	15, 43	15, 61	15, 80	16, 01	16, 23	16, 47
	10	15, 71	15, 88	16, 06	16, 26	16, 48	16, 71	16, 95
	15	16, 04	16, 21	16, 40	16, 60	16, 82	17, 06	17, 31
	20	16, 26	16, 44	16, 63	16, 83	17, 05	17, 29	17, 54
	25	16, 37	16, 55	16, 74	16, 95	17, 17	17, 41	17, 66
XII.	— 0	16, 38	16, 56	16. 75	16, 96	17, 18	17, 42	17, 67

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		3. 50	4. 0	4. 10	4. 20	4. 30	4. 40	4. 50
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	17, 75	18, 04	18, 35	18, 67	19, 02	19, 40	19, 80
	5	17, 74	18, 03	18, 34	18, 66	19, 01	19, 39	19, 79
	10	17, 62	17, 91	18, 22	18, 54	18, 88	19, 25	19, 65
	15	17, 39	17, 67	17, 97	18, 29	18, 63	18, 99	19, 38
	20	17, 03	17, 31	17, 60	17, 92	18, 25	18, 61	18, 99
	25	16, 56	16, 82	17, 11	17, 42	17, 75	18, 10	18, 47
VII.	+ 0	15, 96	16, 22	16, 49	16, 79	17, 11	17, 44	17, 80
	5	15, 24	15, 49	15, 75	16, 03	16, 33	16, 65	16, 99
	10	14, 39	14, 62	14, 87	15, 14	15, 42	15, 72	16, 04
	15	13, 41	13, 63	13, 86	14, 11	14, 37	14, 65	14, 95
	20	12, 31	12, 51	12, 72	12, 95	13, 19	13, 45	13, 73
	25	11, 09	11, 27	11, 46	11, 67	11, 89	12, 12	12, 37
VIII.	+ 0	9, 76	9, 92	10, 09	10, 27	10, 46	10, 66	10, 88
	5	8, 32	8, 45	8, 59	8, 75	8, 92	9, 09	9, 27
	10	6, 78	6, 89	7, 01	7, 14	7, 27	7, 41	7, 56
	15	5, 16	5, 24	5, 33	5, 43	5, 53	5, 64	5, 76
	20	3, 48	3, 54	3, 60	3, 66	3, 73	3, 80	3, 88
	25	1, 75	1, 78	1, 81	1, 84	1, 88	1, 91	1, 95
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 75	1, 78	1, 81	1, 84	1, 88	1, 92	1, 95
	10	3, 48	3, 54	3, 60	3, 66	3, 73	3, 81	3, 89
	15	6, 17	5, 26	5, 35	5, 44	5, 54	5, 65	5, 77
	20	6, 80	6, 91	7, 03	7, 16	7, 29	7, 43	7, 59
	25	8, 36	8, 49	8, 63	8, 79	8, 95	9, 12	9, 31
X.	— 0	9, 81	9, 97	10, 14	10, 32	10, 51	10, 72	10, 94
	5	11, 16	11, 34	11, 54	11, 74	11, 96	12, 20	12, 45
	10	12, 40	12, 60	12, 82	13, 05	13, 29	13, 55	13, 83
	15	13, 52	13, 74	13, 97	14, 22	14, 49	14, 77	15, 07
	20	14, 51	14, 74	14, 99	15, 26	15, 55	15, 86	16, 18
	25	15, 37	15, 62	15, 89	16, 17	16, 47	16, 79	17, 14
XI.	— 0	16, 11	16, 37	16, 65	16, 95	17, 26	17, 60	17, 96
	5	16, 72	16, 99	17, 28	17, 59	17, 92	18, 27	18, 65
	10	17, 21	17, 49	17, 79	18, 10	18, 44	18, 80	19, 19
	15	17, 57	17, 86	18, 16	18, 48	18, 83	19, 20	19, 59
	20	17, 81	18, 10	18, 41	18, 74	19, 09	19, 46	19, 86
	25	17, 94	18, 23	18, 54	18, 87	19, 22	19, 60	20, 00
XII.	— 0	17, 95	18, 24	18, 55	18, 88	19, 23	19, 61	20, 01

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table I.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		5. 0	5. 10	5. 20	5. 30	5. 40	5. 50	6. 0
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	20, 22	20, 67	21, 15	21, 67	22, 22	22, 81	23, 43
	5	20, 21	20, 66	21, 14	21, 66	22, 21	22, 80	23, 42
	10	20, 07	20, 52	21, 00	21, 51	22, 05	22, 64	23, 26
	15	19, 80	20, 24	20, 71	21, 22	21, 76	22, 33	22, 95
	20	19, 40	19, 83	20, 29	20, 79	21, 32	21, 88	22, 48
	25	18, 86	19, 28	19, 73	20, 21	20, 72	21, 27	21, 85
VII.	+ 0	18, 18	18, 58	19, 02	19, 48	19, 97	20, 50	21, 06
	5	17, 35	17, 74	18, 15	18, 60	19, 07	19, 57	20, 11
	10	16, 38	16, 75	17, 14	17, 56	18, 00	18, 48	18, 99
	15	15, 27	15, 61	15, 98	16, 37	16, 78	17, 23	17, 70
	20	14, 03	14, 34	14, 67	15, 03	15, 41	15, 82	16, 25
	25	12, 64	12, 92	13, 22	13, 54	13, 88	14, 25	14, 65
VIII.	+ 0	11, 11	11, 36	11, 63	11, 91	12, 21	12, 54	12, 89
	5	9, 47	9, 68	9, 91	10, 15	10, 41	10, 68	10, 98
	10	7, 72	7, 89	8, 08	8, 28	8, 49	8, 71	8, 95
	15	5, 88	6, 01	6, 15	6, 30	6, 46	6, 63	6, 81
	20	3, 96	4, 05	4, 14	4, 24	4, 35	4, 47	4, 59
	25	1, 99	2, 04	2, 09	2, 14	2, 19	2, 25	2, 31
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	1, 99	2, 04	2, 09	2, 14	2, 19	2, 25	2, 31
	10	3, 97	4, 06	4, 15	4, 25	4, 36	4, 48	4, 60
	15	5, 89	6, 02	6, 17	6, 32	6, 48	6, 65	6, 83
	20	7, 75	7, 92	8, 11	8, 31	8, 52	8, 74	8, 98
	25	9, 51	9, 72	9, 95	10, 19	10, 45	10, 73	11, 02
X.	— 0	11, 17	11, 42	11, 69	11, 97	12, 28	12, 60	12, 94
	5	12, 72	13, 00	13, 30	13, 62	13, 97	14, 34	14, 73
	10	14, 12	14, 44	14, 78	15, 14	15, 52	15, 93	16, 37
	15	15, 39	15, 74	16, 11	16, 50	16, 92	17, 37	17, 84
	20	16, 52	16, 89	17, 29	17, 71	18, 16	18, 64	19, 15
	25	17, 51	17, 90	18, 32	18, 76	19, 24	19, 75	20, 29
XI.	— 0	18, 35	18, 76	19, 19	19, 66	20, 16	20, 69	21, 26
	5	19, 05	19, 47	19, 92	20, 41	20, 93	21, 48	22, 07
	10	19, 60	20, 04	20, 50	21, 00	21, 54	22, 11	22, 72
	15	20, 01	20, 45	20, 94	21, 45	21, 99	22, 57	23, 20
	20	20, 28	20, 73	21, 22	21, 74	22, 29	22, 88	23, 52
	25	20, 43	20, 89	21, 37	21, 89	22, 45	33, 04	23, 68
XII.	— 0	20, 44	20, 90	21, 38	21, 90	22, 46	23, 05	23, 69

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE II.

Sun's Longitude.	Half Interval between the Observations.						
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
	1. 30	1. 40	1. 50	2. 0	2. 10	2. 20	2. 30
S. D.	S.	S.	S.	S.	S.	S.	S.
O. + 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
5	0, 49	0, 49	0, 48	0, 47	0, 46	0, 45	0, 44
10	0, 97	0, 96	0, 95	0, 93	0, 91	0, 90	0, 88
15	1, 43	1, 41	1, 39	1, 36	1, 34	1, 31	1, 28
20	1, 84	1, 82	1, 79	1, 76	1, 73	1, 70	1, 66
25	2, 21	2, 18	2, 15	2, 12	2, 08	2, 04	1, 99
I. + 0	2, 52	2, 49	2, 45	2, 41	2, 37	2, 32	2, 27
5	2, 77	2, 73	2, 69	2, 65	2, 60	2, 55	2, 49
10	2, 93	2, 89	2, 85	2, 81	2, 76	2, 70	2, 64
15	3, 01	2, 97	2, 93	2, 88	2, 83	2, 77	2, 71
20	3, 01	2, 97	2, 93	2, 88	2, 83	2, 77	2, 71
25	2, 91	2, 87	2, 83	2, 78	2, 73	2, 67	2, 61
II. + 0	2, 71	2, 68	2, 64	2, 60	2, 55	2, 50	2, 44
5	2, 42	2, 39	2, 36	2, 32	2, 28	2, 23	2, 18
10	2, 06	2, 03	2, 00	1, 97	1, 93	1, 89	1, 85
15	1, 61	1, 59	1, 57	1, 54	1, 51	1, 48	1, 45
20	1, 11	1, 10	1, 08	1, 06	1, 04	1, 02	1, 00
25	0, 57	0, 56	0, 55	0, 54	0, 53	0, 52	0, 51
III. — 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
5	0, 56	0, 56	0, 55	0, 54	0, 53	0, 52	0, 51
10	1, 11	1, 10	1, 08	1, 06	1, 04	1, 02	1, 00
15	1, 61	1, 59	1, 57	1, 54	1, 51	1, 48	1, 45
20	2, 05	2, 02	1, 99	1, 96	1, 92	1, 88	1, 84
25	2, 41	2, 38	2, 35	2, 31	2, 27	2, 22	2, 17
IV. — 0	2, 70	2, 66	2, 62	2, 58	2, 53	2, 48	2, 43
5	2, 89	2, 85	2, 81	2, 76	2, 71	2, 66	2, 60
10	2, 99	2, 95	2, 90	2, 85	2, 80	2, 75	2, 69
15	2, 99	2, 95	2, 91	2, 86	2, 81	2, 75	2, 69
20	2, 91	2, 87	2, 83	2, 78	2, 73	2, 68	2, 62
25	2, 74	2, 70	2, 66	2, 62	2, 57	2, 52	2, 47
V. — 0	2, 50	2, 47	2, 43	2, 39	2, 35	2, 30	2, 25
5	2, 19	2, 16	2, 13	2, 09	2, 05	2, 01	1, 97
10	1, 82	1, 80	1, 77	1, 74	1, 71	1, 68	1, 64
15	1, 41	1, 39	1, 37	1, 35	1, 33	1, 30	1, 27
20	0, 96	0, 95	0, 94	0, 92	0, 90	0. 88	0, 86
25	0, 49	0, 48	0, 47	0, 47	0, 46	0. 45	0, 44
VI. — 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		2. 40	2 . 50	3. 0	3. 10	3. 20	3. 30	3. 40
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 43	0, 42	0, 41	0, 40	0, 38	0, 37	0, 35
	10	0, 85	0, 83	0, 81	0, 78	0, 75	0, 72	0, 69
	15	1, 25	1, 22	1, 18	1, 14	1, 10	1, 06	1, 01
	20	1, 62	1, 58	1, 53	1, 48	1, 43	1, 37	1, 31
	25	1, 94	1, 89	1, 83	1, 77	1, 71	1, 64	1, 57
I.	+ 0	2, 21	2, 15	2, 09	2, 02	1, 95	1, 87	1, 79
	5	2, 43	2, 36	2, 29	2, 22	2, 14	2, 05	1, 96
	10	2, 57	2, 50	2, 43	2, 35	2, 26	2, 17	2, 08
	15	2, 64	2, 57	2, 50	2, 42	2, 33	2, 24	2, 14
	20	2, 64	2, 57	2, 49	2, 41	2, 32	2, 23	2, 13
	25	2, 55	2, 48	2, 41	2, 33	2, 24	2, 15	2, 06
II.	+ 0	2, 38	2, 31	2, 24	2, 17	2, 09	2, 01	1, 92
	5	2, 13	2, 07	2, 01	1, 94	1, 87	1, 80	1, 72
	10	1, 80	1, 75	1, 70	1, 65	1, 59	1, 53	1, 46
	15	1, 41	1, 37	1, 33	1, 29	1, 24	1, 19	1, 14
	20	0, 98	0, 95	0, 92	0, 89	0, 86	0, 83	0, 79
	25	0, 50	0, 48	0, 47	0, 45	0, 44	0, 42	0, 40
III.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 49	0, 48	0, 47	0, 45	0, 44	0, 42	0, 40
	10	0, 97	0, 94	0, 91	0, 88	0, 85	0, 82	0, 78
	15	1, 41	1, 37	1, 33	1, 29	1, 24	1, 19	1, 14
	20	1, 80	1, 75	1, 70	1, 64	1, 58	1, 52	1, 45
	25	2, 12	2, 06	2, 00	1, 93	1, 86	1, 79	1, 71
IV.	— 0	2, 37	2, 30	2, 23	2, 16	2, 08	2, 00	1, 91
	5	2, 54	2, 47	2, 39	2, 31	2, 23	2, 14	2, 05
	10	2, 62	2, 55	2, 47	2, 39	2, 31	2, 22	2, 12
	15	2, 62	2, 55	2, 48	2, 40	2, 31	2, 22	2, 12
	20	2, 55	2, 48	2, 41	2, 33	2, 25	2, 16	2, 06
	25	2, 41	2, 34	2, 27	2, 20	2, 12	2, 03	1, 94
V.	— 0	2, 19	2, 13	2, 07	2, 00	1, 93	1, 85	1, 77
	5	1, 92	1, 87	1, 81	1, 75	1, 69	1, 62	1, 55
	10	1, 60	1, 56	1, 51	1, 46	1, 41	1, 35	1, 29
	15	1, 24	1, 21	1, 17	1, 13	1, 09	1, 05	1, 00
	20	0, 84	0, 82	0, 80	0, 77	0, 74	0, 71	0, 68
	25	0, 43	0, 42	0, 40	0, 39	0, 38	0, 36	0, 35
VI.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		3. 50	4. 0	4. 10	4. 20	4. 30	4. 40	4. 50
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 33	0, 31	0, 30	0, 28	0, 25	0, 23	0, 21
	10	0, 66	0, 62	0, 58	0, 54	0, 50	0, 46	0, 41
	15	0, 96	0, 91	0, 85	0, 79	0, 73	0, 67	0, 60
	20	1, 24	1, 17	1, 10	1, 03	0, 95	0, 87	0, 78
	25	1, 49	1, 41	1, 32	1, 23	1, 14	1, 04	0, 93
I.	+ 0	1, 70	1, 61	1, 51	1, 41	1, 30	1, 18	1, 06
	5	1, 86	1, 76	1, 65	1, 54	1, 42	1, 29	1, 16
	10	1, 98	1, 87	1, 76	1, 64	1, 51	1, 37	1, 23
	15	2, 03	1, 92	1, 80	1, 68	1, 55	1, 41	1, 27
	20	2, 03	1, 92	1, 80	1, 68	1, 55	1, 41	1, 26
	25	1, 96	1, 85	1, 74	1, 62	1, 49	1, 36	1, 22
II.	+ 0	1, 83	1, 73	1, 62	1, 51	1, 39	1, 27	1, 14
	5	1, 64	1, 55	1, 45	1, 35	1, 25	1, 14	1, 02
	10	1, 39	1, 31	1, 23	1, 15	1, 06	0, 96	0, 86
	15	1, 09	1, 03	0, 97	0, 90	0, 83	0, 76	0, 68
	20	0, 75	0, 71	0, 67	0, 62	0, 57	0, 52	0, 47
	25	0, 38	0, 36	0, 34	0, 32	0, 29	0, 26	0, 24
III.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 38	0, 36	0, 34	0, 31	0, 29	0, 26	0, 24
	10	0, 75	0, 71	0, 67	0, 62	0, 57	0, 52	0, 47
	15	1, 08	1, 02	0, 96	0, 90	0, 83	0, 76	0, 68
	20	1, 38	1, 31	1, 23	1, 14	1, 05	0, 96	0, 86
	25	1, 63	1, 54	1, 45	1, 35	1, 24	1, 13	1, 02
IV.	— 0	1, 82	1, 72	1, 61	1, 50	1, 38	1, 26	1, 13
	5	1, 95	1, 84	1, 73	1, 61	1, 48	1, 35	1, 21
	10	2, 01	1, 90	1, 79	1, 67	1, 54	1, 40	1, 26
	15	2, 02	1, 91	1, 79	1, 67	1, 54	1, 40	1, 26
	20	1, 96	1, 85	1, 74	1, 62	1, 49	1, 36	1, 22
	25	1, 85	1, 75	1, 64	1, 53	1, 41	1, 28	1, 15
V.	— 0	1, 68	1, 59	1, 50	1, 40	1, 29	1, 17	1, 05
	5	1, 48	1, 40	1, 31	1, 22	1, 13	1, 03	0, 92
	10	1, 23	1, 16	1, 09	1, 02	0, 94	0, 86	0, 77
	15	0, 95	0, 90	0, 85	0, 79	0, 73	0, 66	0, 59
	20	0, 65	0, 62	0, 58	0, 54	0, 50	0, 45	0, 40
	25	0, 33	0, 31	0, 29	0, 27	0, 25	0, 23	0, 21
VI.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		5. 0	5. 10	5. 20	5. 30	5. 40	5. 50	6. 0
S.	D.	S.	S.	S.	S.	S.	S.	S.
O.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 18	0, 16	0, 13	0, 10	0, 07	0, 04	0, 00
	10	0, 36	0, 31	0, 25	0, 19	0, 13	0, 07	0, 00
	15	0, 53	0, 45	0, 37	0, 29	0, 20	0, 10	0, 00
	20	0, 68	0, 58	0, 48	0, 37	0, 25	0, 13	0, 00
	25	0, 82	0, 70	0, 57	0, 44	0, 30	0, 16	0, 00
I.	+ 0	0, 93	0, 80	0, 66	0, 51	0, 35	0, 18	0, 00
	5	1, 02	0, 87	0, 71	0, 55	0, 38	0, 19	0, 00
	10	1, 08	0, 92	0, 76	0, 59	0, 41	0, 21	0, 00
	15	1, 12	0, 95	0, 78	0, 60	0, 41	0, 21	0, 00
	20	1, 11	0, 95	0, 78	0, 60	0, 41	0, 21	0, 00
	25	1, 07	0, 91	0, 75	0, 58	0, 40	0, 20	0, 00
II.	+ 0	1, 00	0, 85	0, 70	0, 54	0, 37	0, 19	0, 00
	5	0, 90	0, 77	0, 63	0, 48	0, 33	0, 17	0, 00
	10	0, 76	0, 65	0, 53	0, 41	0, 28	0, 14	0, 00
	15	0, 60	0, 51	0, 42	0, 32	0, 22	0, 11	0, 00
	20	0, 41	0, 35	0, 29	0, 22	0, 15	0, 08	0, 00
	25	0, 21	0, 18	0, 15	0, 11	0, 08	0, 04	0, 00
III.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 21	0, 18	0, 15	0, 11	0, 08	0, 04	0, 00
	10	0, 41	0, 35	0, 29	0, 22	0, 15	0, 08	0, 00
	15	0, 60	0, 51	0, 42	0, 32	0, 22	0, 11	0, 00
	20	0, 76	0, 65	0, 53	0, 41	0, 28	0, 14	0, 00
	25	0, 90	0, 77	0, 63	0, 48	0, 33	0, 17	0, 00
IV.	— 0	0, 99	0, 85	0, 70	0, 54	0, 37	0, 19	0, 00
	5	1, 06	0, 91	0, 75	0, 58	0, 40	0, 20	0, 00
	10	1, 11	0, 95	0, 78	0, 60	0, 41	0, 21	0, 00
	15	1, 11	0, 95	0, 78	0, 60	0, 41	0, 21	0, 00
	[illegible]	1, 10	0, 92	0, 76	0, 58	0, 39	0, 20	0, 00
	25	1, 01	0, 86	0, 71	0, 55	0, 38	0, 19	0, 00
V.	— 0	0, 92	0, 79	0, 65	0, 50	0, 34	0, 17	0, 00
	5	0, 81	0, 69	0, 57	0, 44	0, 30	0, 15	0, 00
	10	0, 68	0, 58	0, 47	0, 36	0, 25	0, 13	0, 00
	15	0, 52	0, 45	0, 37	0, 28	0, 19	0, 10	0, 00
	[illegible]	0, 35	0, 35	0, 25	0, 19	0, 13	0, 07	0, 00
	25	0, 18	0, 15	0, 13	0, 10	0, 07	0, 04	0, 00
VI.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table II.

Sun's Longitude.	Half Interval between the Observations.						
	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
	1. 30	1. 40	1. 50	2. 0	2. 10	2. 20	2. 30
S. D.	S.	S.	S.	S.	S.	S.	S.
VI. + 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
5	0, 49	0, 48	0, 48	0, 47	0, 46	0, 45	0, 44
10	0, 97	0, 96	0, 95	0, 93	0, 91	0, 89	0, 87
15	1, 43	1, 42	1, 40	1, 37	1, 35	1, 32	1, 29
20	1, 84	1, 84	1, 81	1, 78	1, 75	1, 72	1, 68
25	2, 25	2, 22	2, 19	2, 15	2, 11	2, 07	2, 03
VII. + 0	2, 58	2, 55	2, 51	2, 47	2, 43	2, 38	2, 33
5	2, 85	2, 81	2, 77	2, 72	2, 67	2, 62	2, 56
10	3, 04	3, 00	2, 95	2, 90	2, 85	2, 79	2, 73
15	3, 14	3, 10	3, 05	3, 00	2, 94	2, 88	2, 82
20	3, 14	3, 10	3, 06	3, 01	2, 95	2, 89	2, 83
25	3, 05	3, 01	2, 97	2, 92	2, 87	2, 81	2, 75
VIII. + 0	2, 86	2, 82	2, 78	2, 73	2, 68	2, 63	2, 57
5	2, 57	2, 53	2, 49	2, 45	2, 41	2, 36	2, 31
10	2, 18	2, 15	2, 12	2, 08	2, 05	2, 01	1, 96
15	1, 71	1, 69	1, 67	1, 64	1, 61	1, 58	1, 54
20	1, 18	1, 17	1, 15	1, 13	1, 11	1, 09	1, 06
25	0, 60	0, 60	0, 59	0, 58	0, 57	0, 55	0, 54
IX. — 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
5	0, 60	0, 60	0, 59	0, 58	0, 57	0, 55	0, 54
10	1, 18	1, 17	1, 15	1, 13	1, 11	1, 09	1, 06
15	1, 72	1, 70	1, 67	1, 64	1, 61	1, 58	1, 54
20	2, 19	2, 16	2, 13	2, 09	2, 05	2, 01	1, 97
25	2, 58	2, 54	2, 50	2, 46	2, 42	2, 37	2, 32
X. — 0	2, 87	2, 83	2, 79	2, 79	2, 70	2, 64	2, 58
5	3, 06	3, 03	2, 99	2, 94	2, 88	2, 82	2, 76
10	3, 16	3, 12	3, 08	3, 03	2, 97	2, 91	2, 85
15	3, 16	3, 12	3, 07	3, 02	2, 97	2, 91	2, 84
20	3, 06	3, 02	2, 98	2, 93	2, 88	2, 82	2, 76
25	2, 87	2, 84	2, 80	2, 75	2, 70	2, 64	2, 58
XI. — 0	2, 60	2, 57	2, 54	2, 50	2, 45	2, 40	2, 35
5	2, 27	2, 24	2, 21	2, 18	2, 14	2, 10	2, 05
10	1, 88	1, 86	1, 83	1, 80	1, 77	1, 73	1, 69
15	1, 45	1, 43	1, 41	1, 39	1, 36	1, 33	1, 30
20	0, 99	0, 97	0, 96	0, 94	0, 92	0, 90	0, 88
25	0, 50	0, 49	0, 48	0, 47	0, 47	0, 46	0, 45
XII. — 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		2. 40	2. 50	3. 0	3. 10	3. 20	3. 30	3. 40
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 43	0, 42	0, 41	0, 39	0, 38	0, 36	0, 35
	10	0, 85	0, 83	0, 81	0, 78	0, 75	0, 72	0, 69
	15	1, 26	1, 22	1, 19	1, 15	1, 11	1, 06	1, 02
	20	1, 64	1, 59	1, 54	1, 49	1, 44	1, 38	1, 32
	25	1, 98	1, 92	1, 86	1, 80	1, 74	1, 67	1, 60
VII.	+ 0	2, 27	2, 21	2, 14	2, 07	2, 00	1, 92	1, 83
	5	2, 50	2, 43	2, 36	2, 28	2, 20	2, 11	2, 02
	10	2, 66	2, 59	2, 55	2, 43	2, 34	2, 25	2, 15
	15	2, 75	2, 68	2, 60	2, 51	2, 42	2, 32	2, 22
	20	2, 76	2, 68	2, 60	2, 52	2, 43	2, 33	2, 23
	25	2, 68	2, 60	2, 52	2, 44	2, 35	2, 26	2, 16
VIII.	+ 0	2, 51	2, 44	2, 37	2, 29	2, 21	2, 12	2, 03
	5	2, 25	2, 19	2, 12	2, 05	1, 98	1, 90	1, 82
	10	1, 91	1, 86	1, 81	1, 75	1, 69	1, 62	1, 55
	15	1, 50	1, 46	1, 42	1, 37	1, 32	1, 27	1, 22
	20	1, 04	1, 01	0, 98	0, 95	0, 91	0, 88	0, 84
	25	0, 53	0, 51	0, 50	0, 48	0, 47	0, 45	0, 43
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 53	0, 51	0, 50	0, 48	0, 47	0, 46	0, 43
	10	1, 04	1, 01	0, 98	0, 95	0, 91	0, 88	0, 84
	15	1, 51	1, 47	1, 42	1, 38	1, 33	1, 27	1, 22
	20	1, 92	1, 87	1, 81	1, 75	1, 69	1, 62	1, 55
	25	2, 26	2, 20	2, 13	2, 06	1, 99	1, 91	1, 83
X.	— 0	2, 52	2, 45	2, 38	2, 30	2, 22	2, 13	2, 04
	5	2, 69	2, 62	2, 54	2, 46	2, 37	2, 28	2, 18
	10	2, 78	2, 70	2, 62	2, 53	2, 44	2, 34	2, 24
	15	2, 77	2, 70	2, 62	2, 53	2, 44	2, 34	2, 24
	20	2, 69	2, 61	2, 53	2, 45	2, 36	2, 27	2, 17
	25	2, 52	2, 45	2, 38	2, 30	2, 22	2, 13	2, 04
XI.	— 0	2, 29	2, 23	2, 16	2, 09	2, 01	1, 93	1, 85
	5	2, 00	1, 94	1, 88	1, 82	1, 76	1, 69	1, 61
	10	1, 65	1, 61	1, 56	1, 51	1, 46	1, 40	1, 34
	15	1, 27	1, 24	1, 20	1, 16	1, 12	1, 08	1, 03
	20	0, 86	0, 84	0, 81	0, 79	0, 76	0, 73	0, 70
	25	0, 43	0, 42	0, 41	0, 40	0, 38	0, 37	0, 35
XII.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. TABLE II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		3. 50	4. 0	4. 10	4. 20	4. 30	4. 40	4. 50
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI.	+ 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 33	0, 31	0, 29	0, 27	0, 25	0, 23	0, 21
	10	0, 66	0, 62	0, 58	0, 54	0, 50	0, 46	0, 41
	15	0, 97	0, 91	0, 86	0, 80	0, 74	0, 67	0, 60
	20	1, 26	1, 19	1, 12	1, 04	0, 96	0, 87	0, 78
	25	1, 52	1, 44	1, 35	1, 26	1, 16	1, 06	0, 95
VII.	+ 0	1, 74	1, 65	1, 55	1, 44	1, 33	1, 21	1, 09
	5	1, 92	1, 82	1, 71	1, 59	1, 47	1, 34	1, 20
	10	2, 05	1, 94	1, 82	1, 69	1, 56	1, 42	1, 28
	15	2, 11	2, 00	1, 88	1, 75	1, 61	1, 47	1, 32
	20	2, 12	2, 00	1, 88	1, 75	1, 62	1, 47	1, 32
	25	2, 06	1, 95	1, 83	1, 70	1, 57	1, 43	1, 28
VIII.	+ 0	1, 93	1, 82	1, 71	1, 59	1, 47	1, 34	1, 20
	5	1, 73	1, 63	1, 53	1, 43	1, 32	1, 20	1, 08
	10	1, 47	1, 39	1, 31	1, 22	1, 12	1, 02	0, 92
	15	1, 16	1, 09	1, 02	0, 96	0, 88	0, 80	0, 72
	20	0, 80	0, 75	0, 71	0, 66	0, 61	0, 55	0, 50
	25	0, 41	0, 38	0, 36	0, 34	0, 31	0, 28	0, 25
IX.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00
	5	0, 41	0, 38	0, 36	0, 34	0, 31	0, 28	0, 25
	10	0, 80	0, 75	0, 71	0, 66	0, 61	0, 55	0, 50
	15	1, 16	1, 10	1, 03	0, 96	0, 88	0, 80	0, 72
	20	1, 47	1, 39	1, 31	1, 22	1, 13	1, 03	0, 92
	25	1, 74	1, 64	1, 54	1, 44	1, 33	1, 21	1, 08
X.	— 0	1, 94	1, 83	1, 72	1, 60	1, 48	1, 35	1, 21
	5	2, 07	1, 96	1, 84	1, 71	1, 58	1, 44	1, 29
	10	2, 13	2, 02	1, 90	1, 77	1, 63	1, 48	1, 33
	15	2, 13	2, 01	1, 89	1, 76	1, 62	1, 48	1, 33
	20	2, 06	1, 95	1, 83	1, 71	1, 58	1, 44	1, 29
	25	1, 94	1, 83	1, 72	1, 60	1, 48	1, 35	1, 21
XI.	— 0	1, 76	1, 66	1, 56	1, 45	1, 34	1, 22	1, 09
	5	1, 53	1, 45	1, 36	1, 27	1, 17	1, 07	0, 96
	10	1, 27	1, 20	1, 13	1, 05	0, 97	0, 88	0, 79
	15	0, 98	0, 93	0, 87	0, 81	0, 75	0, 68	0, 61
	20	0, 66	0, 63	0, 59	0, 55	0, 51	0, 46	0, 41
	25	0, 33	0, 32	0, 30	0, 28	0, 26	0, 23	0, 21
XII.	— 0	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00	0, 00

TABLE XLI. *Continued.*

Equations to Equal Altitudes. Table II.

Sun's Longitude.		Half Interval between the Observations.						
		H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
		5. 0	5. 10	5. 20	5. 30	5. 40	5. 50	6. 0
S.	D.	S.	S.	S.	S.	S.	S.	S.
VI. +	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	5	0,18	0,15	0,13	0,10	0,07	0,04	0,00
	10	0,36	0,31	0,25	0,19	0,13	0,07	0,00
	15	0,53	0,45	0,37	0,29	0,20	0,10	0,00
	20	0,69	0,59	0,48	0,37	0,26	0,13	0,00
	25	0,83	0,71	0,58	0,45	0,31	0,16	0,00
VII. +	0	0,96	0,82	0,67	0,52	0,35	0,18	0,00
	5	1,05	0,90	0,74	0,57	0,39	0,20	0,00
	10	1,12	0,96	0,79	0,61	0,42	0,21	0,00
	15	1,16	0,99	0,81	0,63	0,43	0,22	0,00
	20	1,16	0,99	0,82	0,63	0,43	0,22	0,00
	25	1,13	0,96	0,79	0,61	0,42	0,21	0,00
VIII. +	0	1,05	0,90	0,74	0,57	0,39	0,20	0,00
	5	0,95	0,81	0,66	0,51	0,35	0,18	0,00
	10	0,81	0,69	0,57	0,44	0,30	0,15	0,00
	15	0,63	0,54	0,44	0,34	0,23	0,12	0,00
	20	0,44	0,37	0,31	0,24	0,16	0,08	0,00
	25	0,22	0,19	0,16	0,12	0,08	0,04	0,00
IX. —	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00
	5	0,22	0,19	0,16	0,12	0,08	0,04	0,00
	10	0,44	0,37	0,31	0,24	0,16	0,08	0,00
	15	0,64	0,55	0,45	0,35	0,24	0,12	0,00
	20	0,81	0,69	0,57	0,44	0,30	0,15	0,00
	25	0,95	0,81	0,67	0,51	0,35	0,18	0,00
X. —	0	1,06	0,91	0,75	0,57	0,39	0,20	0,00
	5	1,13	0,97	0,80	0,61	0,42	0,22	0,00
	10	1,17	1,00	0,82	0,63	0,43	0,22	0,00
	15	1,17	1,00	0,82	0,63	0,43	0,22	0,00
	20	1,13	0,96	0,79	0,61	0,42	0,21	0,00
	25	1,06	0,91	0,75	0,57	0,39	0,20	0,00
XI. —	0	0,96	0,82	0,67	0,52	0,36	0,18	0,00
	5	0,84	0,72	0,59	0,45	0,31	0,16	0,00
	10	0,70	0,60	0,49	0,38	0,26	0,13	0,00
	15	0,54	0,46	0,38	0,29	0,20	0,10	0,00
	20	0,36	0,31	0,25	0,19	0,13	0,07	0,00
	25	0,18	0,16	0,13	0,10	0,07	0,04	0,00
XII. —	0	0,00	0,00	0,00	0,00	0,00	0,00	0,00

TABLE XLII.

Equations to Equal Altitudes, where extreme Accuracy is not required.

N. or S. Declination of Sun.	Latitude N. or S. 20°						Latitude N. or S. 30°						Latitude N. or S. 40°.						
	Hours between Observations.						Hours between Observations.						Hours between Observations.						
	9	8	7	6	5	4	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	
23	1	0	0	0	0	0	2	1	1	0	0	0	3	3	2	2	2	1	0
22	1	1	0	0	0	0	3	2	2	1	1	0	5	4	4	4	3	2	2
21	2	1	1	1	0	0	4	3	3	2	2	1	7	6	5	5	4	3	3
20	2	2	1	1	1	0	4	4	3	3	2	2	8	7	6	6	5	5	4
19	3	2	2	1	1	1	5	4	4	3	3	2	9	8	7	7	6	5	5
18	3	2	2	1	1	1	5	5	4	4	3	3	10	9	8	8	6	6	5
17	4	3	2	2	2	1	6	5	5	4	4	3	11	10	9	8	7	7	6
16	4	3	3	2	2	1	6	5	5	5	4	4	11	10	9	9	8	7	7
15	4	3	3	3	2	2	7	6	5	5	5	4	12	11	10	9	8	8	7
14	4	4	3	3	2	2	7	6	6	5	5	4	12	11	10	10	9	8	8
13	5	4	3	3	3	2	7	7	6	6	5	5	13	12	11	10	9	9	8
12	5	4	4	3	3	2	7	7	6	6	6	5	13	12	11	11	10	9	9
11	5	4	4	4	3	3	8	7	7	6	6	5	14	13	12	11	10	10	9
10	5	5	4	4	3	3	8	8	7	7	6	5	14	13	12	11	11	10	10
9	5	5	4	4	4	3	8	8	7	7	6	6	15	14	12	12	11	10	10
8	5	5	4	4	4	3	9	8	8	7	7	6	15	14	13	12	11	11	10
7	6	5	5	5	4	4	9	8	8	7	7	6	16	14	13	12	12	11	11
6	6	5	5	5	4	4	9	9	8	8	7	6	16	15	13	13	12	11	11
5	6	6	5	5	4	4	9	9	8	8	7	7	16	15	14	13	12	12	11
4	6	6	5	5	5	4	10	9	9	8	8	7	17	15	14	13	13	12	12
3	6	6	6	5	5	5	10	9	9	8	8	7	17	15	14	14	13	12	12
2	6	6	6	5	5	5	10	9	9	9	8	8	17	16	14	14	13	13	12
1	7	6	6	5	5	5	10	10	9	9	9	8	17	16	15	14	14	13	12
0	7	6	6	6	5	5	10	10	10	9	9	8	17	16	15	14	14	13	13

TABLE XLII. *Continued.*

N. or S. Declination of Sun.	Latitude N. or S. 50°							Latitude N. or S. 60°.						
	Hours between Observations.							Hours between Observations.						
	10	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
23	4	4	3	3	3	2	2	7	6	6	5	5	5	4
22	8	7	7	6	5	5	5	12	11	10	9	8	8	8
21	10	9	8	8	7	6	6	15	14	13	13	12	11	10
20	11	11	10	9	8	8	7	18	16	15	14	14	13	12
19	13	12	11	10	9	9	8	20	18	17	16	15	14	13
18	14	13	12	11	10	10	9	21	20	18	17	16	15	15
17	15	14	13	12	11	11	10	23	21	20	19	17	17	16
16	16	15	14	13	12	12	11	24	23	21	20	18	18	17
15	17	16	15	13	13	12	12	26	24	22	21	19	19	18
14	18	17	16	14	13	13	12	—	26	23	22	20	20	19
13	19	18	17	15	14	14	13	—	27	24	23	21	21	20
12	20	19	17	16	15	14	14	—	28	25	23	22	21	21
11	20	19	18	16	15	15	14	—	30	26	24	23	22	22
10	21	20	18	17	16	15	15	—	31	27	25	24	23	22
9	21	20	19	17	16	15	15	—	—	28	26	25	23	23
8	21	21	19	18	17	16	16	—	—	28	27	26	24	23
7	22	21	20	18	17	16	16	—	—	29	27	26	25	24
6	22	22	20	19	17	17	16	—	—	29	28	27	25	24
5	23	22	20	19	18	17	17	—	—	29	28	27	26	25
4	23	22	20	19	18	17	17	—	—	30	29	27	26	25
3	23	22	21	20	18	18	17	—	—	30	29	28	27	26
2	23	22	21	20	19	18	18	—	—	30	29	28	27	26
1	23	22	21	20	19	19	18	—	—	31	29	28	27	27
0	23	22	21	21	20	19	18	—	—	31	29	28	28	27

TABLE XLII. *Continued.*

S. or N. Declination of Sun.	Latitude N. or S. 20°.						Latitude N. or S. 30°.						Latitude N. or S. 40°.						
	Hours between Observations.						Hours between Observations.						Hours between Observations.						
	9	8	7	6	5	4	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.
0	7	6	6	6	5	5	10	10	10	9	9	8	—	16	15	14	14	13	13
1	7	6	6	6	6	6	10	10	10	9	9	9	—	16	15	14	14	13	13
2	7	7	7	6	6	6	11	10	10	9	9	9	—	16	15	15	14	14	13
3	7	7	7	6	6	6	11	10	10	10	9	9	—	16	15	15	14	14	13
4	7	7	7	6	6	6	11	10	10	10	10	9	—	16	15	15	14	14	13
5	7	7	7	6	7	6	11	10	10	10	10	10	—	16	16	15	15	14	14
6	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	15	15	14	14
7		7	7	7	7	7	—	10	10	10	10	10	—	—	16	16	15	14	14
8	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	16	15	14	14
9	—	7	7	7	7	7	—	10	10	10	10	10	—	—	16	15	15	14	14
10	—	8	8	7	7	7	—	11	10	10	10	10	—	—	16	15	15	14	14
11	-	7	7	8	7	7	—	11	11	10	10	10	—	—	15	15	14	14	14
12		7	7	7	7	7	—	10	11	10	10	10	—	—	15	15	14	14	14
13		7	7	7	7	7	—	10	10	10	10	9	—	—	15	14	14	14	14
14	—	7	7	7	7	7	—	10	10	10	9	9	—	—	15	14	14	14	14
15	—	7	7	7	7	7	—	10	10	9	9	9	—	—	14	14	14	14	13
16	—	—	6	6	6	6	—	—	9	9	9	9	—	—	—	13	13	13	13
17	—	—	6	6	6	6	—	—	9	9	9	8	—	—	—	13	12	12	12
18	—	—	6	6	6	6	—	—	9	9	8	8	—	—	—	12	12	12	12
19	—	—	6	6	6	6	—	—	8	8	8	8	—	—	—	11	11	11	11
20	—	—	5	5	5	5	—	—	8	8	7	7	—	—	—	10	10	10	10
21	—	—	5	5	5	5	—	—	7	7	7	7	—	—	—	—	9	9	9
22	—	—	4	4	4	4	—	—	6	6	6	5	—	—	—	—	7	7	7
23	—	—	2	2	2	2	—	—	3	3	3	3	—	—	—	—	4	4	4

TABLE XLII. *Continued.*

S. or N. Declination of Sun.	Latitude N. or S. 50°. Hours between Observations.							Latitude N. or S. 60°. Hours between Observations.						
	10	9	8	7	6	5	4	10	9	8	7	6	5	4
Deg.	—	Sec.	Sec.	Sec.	Sec.	Sec.	Sec.	—	—	—	Sec.	Sec.	Sec.	Sec.
0	—	22	21	21	20	19	18	—	—	—	29	28	28	27
1	—	22	21	21	20	19	18	—	—	—	29	29	28	27
2	—	22	22	21	20	19	18	—	—	—	30	29	28	27
3	—	—	22	21	20	19	18	—	—	—	30	29	28	27
4	—	—	22	21	20	19	19	—	—	—	30	29	29	28
5	—	—	22	21	20	19	19	—	—	—	30	29	29	28
6	—	—	—	21	20	20	19	—	—	—	—	29	29	28
7	—	—	—	21	20	20	19	—	—	—	—	29	28	27
8	—	—	—	21	20	20	19	—	—	—	—	29	28	27
9	—	—	—	21	20	20	19	—	—	—	—	28	28	27
10	—	—	—	—	20	19	19	—	—	—	—	28	28	27
11	—	—	—	—	19	19	18	—	—	—	—	—	27	27
12	—	—	—	—	19	19	18	—	—	—	—	—	27	26
13	—	—	—	—	19	19	18	—	—	—	—	—	26	26
14	—	—	—	—	18	18	17	—	—	—	—	—	25	25
15	—	—	—	—	18	18	17	—	—	—	—	—	—	24
16	—	—	—	—	17	17	17	—	—	—	—	—	—	23
17	—	—	—	—	16	16	16	—	—	—	—	—	—	22
18	—	—	—	—	15	15	15	—	—	—	—	—	—	21
19	—	—	—	—	14	14	14	—	—	—	—	—	—	19
20	—	—	—	—	13	13	13	—	—	—	—	—	—	17
21	—	—	—	—	—	11	11	—	—	—	—	—	—	14
22	—	—	—	—	—	9	9	—	—	—	—	—	—	11
23	—	—	—	—	—	5	5	—	—	—	—	—	—	7

TABLE XLIII.

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.								
	1°	2°	3°	4°	5'	6°	7°	8°	9°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 2	6. 2	6. 2	6. 2	6. 2	6. 3	6. 3	6. 3	6. 3
2	6. 2	6. 2	6. 3	6. 3	6. 3	6. 3	6. 3	6. 3	6. 3
3	6. 2	6. 3	6. 3	6. 3	6. 3	6. 3	6. 4	6. 4	6. 4
4	6. 2	6. 3	6. 3	6. 3	6. 4	6. 4	6. 4	6. 4	6. 5
5	6. 2	6. 3	6. 3	6. 4	6. 4	6. 4	6. 5	6. 5	6. 5
6	6. 3	6. 3	6. 3	6. 4	6. 4	6. 4	6. 5	6. 6	6. 6
7	6. 3	6. 3	6. 4	6. 4	6. 5	6. 5	6. 6	6. 6	6. 7
8	6. 3	6. 3	6. 4	6. 4	6. 5	6. 5	6. 6	6. 7	6. 7
9	6. 3	6. 3	6. 4	6. 5	6. 5	6. 6	6. 7	6. 7	6. 8
10	6. 3	6. 4	6. 4	6. 5	6. 6	6. 6	6. 7	6. 8	6. 9
11	6. 3	6. 4	6. 4	6. 5	6. 6	6. 7	6. 8	6. 8	6. 9
12	6. 3	6. 4	6. 5	6. 6	6. 6	6. 7	6. 8	6. 9	6. 10
13	6. 3	6. 4	6. 5	6. 6	6. 7	6. 8	6. 9	6. 10	6. 11
14	6. 3	6. 4	6. 5	6. 6	6. 7	6. 8	6. 9	6. 10	6. 11
15	6. 3	6. 4	6. 5	6. 6	6. 8	6. 9	6. 10	6. 11	6. 12
16	6. 3	6. 4	6. 6	6. 7	6. 8	6. 9	6. 10	6. 11	6. 13
17	6. 3	6. 5	6. 6	6. 7	6. 8	6. 10	6. 11	6. 12	6. 13
18	6. 4	6. 5	6. 6	6. 7	6. 9	6. 10	6. 11	6. 13	6. 14
19	6. 4	6. 5	6. 6	6. 8	6. 9	6. 11	6. 12	6. 13	6. 15
20	6. 4	6. 5	6. 7	6. 8	6. 9	6. 11	6. 12	6. 14	6. 15
21	6. 4	6. 5	6. 7	6. 8	6. 10	6. 12	6. 13	6. 15	6. 16
22	6. 4	6. 6	6. 7	6. 8	6. 10	6. 12	6. 14	6. 15	6. 17
23	6. 4	6. 6	6. 7	6. 9	6. 11	6. 13	6. 14	6. 16	6. 18
24	6. 4	6. 6	6. 8	6. 9	6. 11	6. 13	6. 15	6. 17	6. 19
25	6. 4	6. 6	6. 8	6. 10	6. 12	6. 14	6. 15	6. 17	6. 19
26	6. 4	6. 6	6. 8	6. 10	6. 12	6. 14	6. 16	6. 18	6. 20
27	6. 4	6. 6	6. 8	6. 11	6. 13	6. 15	6. 17	6. 19	6. 21
28	6. 5	6. 7	6. 9	6. 11	6. 13	6. 15	6. 17	6. 20	6. 22
29	6. 5	6. 7	6. 9	6. 11	6. 14	6. 16	6. 18	6. 20	6. 23
30	6. 5	6. 7	6. 9	6. 12	6. 14	6. 16	6. 19	6. 21	6. 23
31	6. 5	6. 7	6. 10	6. 12	6. 15	6. 17	6. 19	6. 23	6. 24
32	6. 5	6. 7	6. 10	6. 12	6. 15	6. 18	6. 20	6. 23	6. 25

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.								
	10°	11°	12°	13°	14°	15°	16°	17°	18°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 3	6. 3	6. 3	6. 3	6. 3	6. 3	6. 3	6. 3	6. 4
2	6. 4	6. 4	6. 4	6. 4	6. 4	6. 4	6. 4	6. 5	6. 5
3	6. 4	6. 4	6. 5	6. 5	6. 5	6. 5	6. 6	6. 6	6. 6
4	6. 5	6. 5	6. 6	6. 6	6. 6	6. 6	6. 7	6. 7	6. 7
5	6. 6	6. 6	6. 6	6. 7	6. 7	6. 8	6. 8	6. 8	6. 9
6	6. 6	6. 7	6. 7	6. 8	6. 8	6. 9	6. 9	6. 10	6. 10
7	6. 7	6. 8	6. 8	6. 9	6. 9	6. 10	6. 10	6. 11	6. 11
8	6. 8	6. 8	6. 9	6. 10	6. 10	6. 11	6. 11	6. 12	6. 13
9	6. 9	6. 9	6. 10	6. 11	6. 11	6. 12	6. 13	6. 13	6. 14
10	6. 9	6. 10	6. 11	6. 12	6. 12	6. 13	6. 14	6. 15	6. 15
11	6. 10	6. 11	6. 12	6. 13	6. 13	6. 14	6. 15	6. 16	6. 17
12	6. 11	6. 12	6. 13	6. 14	6. 14	6. 15	6. 16	6. 17	6. 18
13	6. 12	6. 12	6. 13	6. 15	6. 15	6. 16	6. 17	6. 18	6. 19
14	6. 12	6. 13	6. 14	6. 15	6. 16	6. 18	6. 19	6. 20	6. 21
15	6. 13	6. 14	6. 15	6. 16	6. 18	6. 19	6. 20	6. 21	6. 22
16	6. 14	6. 15	6. 16	6. 17	6. 19	6. 20	6. 21	6. 22	6. 24
17	6. 15	6. 16	6. 17	6. 18	6. 20	6. 21	6. 22	6. 24	6. 25
18	6. 15	6. 17	6. 18	6. 19	6. 21	6. 22	6. 24	6. 25	6. 27
19	6. 16	6. 18	6. 19	6. 21	6. 22	6. 23	6. 25	6. 27	6. 28
20	6. 17	6. 19	6. 20	6. 22	6. 23	6. 25	6. 26	6. 28	6. 30
21	6. 18	6. 19	6. 21	6. 23	6. 24	6. 26	6. 28	6. 29	6. 31
22	6. 19	6. 20	6. 22	6. 24	6. 25	6. 27	6. 29	6. 31	6. 33
23	6. 19	6. 21	6. 23	6. 25	6. 27	6. 28	6. 30	6. 32	6. 34
24	6. 20	6. 22	6. 24	6. 26	6. 28	6. 30	6. 32	6. 34	6. 36
25	6. 21	6. 23	6. 25	6. 27	6. 29	6. 31	6. 33	6. 35	6. 37
26	6. 22	6. 24	6. 26	6. 28	6. 30	6. 32	6. 35	6. 37	6. 39
27	6. 23	6. 25	6. 27	6. 29	6. 32	6. 34	6. 36	6. 38	6. 41
28	6. 24	6. 26	6. 28	6. 31	6. 33	6. 35	6. 38	6. 40	6. 42
29	6. 25	6. 27	6. 30	6. 32	6. 34	6. 37	6. 39	6. 42	6. 44
30	6. 26	6. 28	6. 31	6. 33	6. 36	6. 38	6. 41	6. 45	6. 46
31	6. 27	6. 29	6. 32	6. 34	6. 37	6. 40	6. 42	6. 45	6. 48
32	6. 28	6. 30	6. 33	6. 36	6. 38	6. 41	6. 44	6. 47	6. 50

... the same kind.

... UDE.

		23°	24°	25°	26°
		H. M.	H. M.	H. M.	H. M.
	[illegible]	6. 4	6. 4	6. 4	6. 4
	[illegible]	6. 6	6. 6	6. 6	6. 6
	[illegible]	6. 7	6. 8	6. 8	6. 8
	[illegible]	6. 9	6. 9	6. 10	6. 10
	[illegible] 10	6. 11	6. 11	6. 12	6. 12
	[illegible] 12	6. 13	6. 13	6. 14	6. 14
	[illegible] 14	6. 14	6. 15	6. 15	6. 16
	[illegible] 15	6. 16	6. 17	6. 17	6. 18
	6. 17	6. 18	6. 19	6. 19	6. 20
	6. 19	6. 19	6. 20	6. 21	6. 22
	6. 20	6. 21	6. 22	6. 23	6. 24
	6. 22	6. 23	6. 24	6. 25	6. 26
	6. 24	6. 25	6. 26	6. 27	6. 28
	6. 25	6. 27	6. 28	6. 29	6. 30
	6. 27	6. 28	6. 30	6. 31	6. 32
	6. 29	6. 30	6. 32	6. 33	6. 35
[illegible]	6. 31	6. 32	6. 34	6. 35	6. 37
[illegible]	6. 33	6. 34	6. 36	6. 37	6. 39
[illegible]	6. 34	6. 36	6. 38	6. 39	6. 41
[illegible]	6. 36	6. 38	6. 40	6. 42	6. 43
[illegible] 36	6. 38	6. 40	6. 42	6. 44	6. 46
[illegible] 38	6. 40	6. 42	6. 44	6. 46	6. 48
[illegible] 40	6. 42	6. 44	6. 46	6. 48	6. 50
[illegible] 42	6. 44	6. 46	6. 48	6. 51	6. 53
6. 44	6. 46	6. 48	6. 51	6. 53	6. 55
6. 46	6. 48	6. 50	6. 53	6. 55	6. 58
6. 48	6. 50	6. 53	6. 55	6. 58	7. 0
6. 50	6. 52	6. 55	6. 57	7. 0	7. 3
6. 52	6. 54	6. 57	7. 0	7. 3	7. 6
6. 54	6. 57	6. 59	7. 2	7. 5	7. 8
6. 56	6. 59	7. 2	7. 5	7. 8	7. 11
[illegible]	[illegible]	7. 4	7. 7	7. 11	7. 14

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLINATION.	LONGITUDE.							
	27°	28°	29°	30°	31°	32°	33°	34°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 4	6. 5	6. 5	6. 5	6. 5	6. 5	6. 5	6. 5
2	6. 6	6. 7	6. 7	6. 7	6. 7	6. 7	6. 8	6. 8
3	6. 8	6. 9	6. 9	6. 9	6. 10	6. 10	6. 10	6. 11
4	6. 11	6. 11	6. 11	6. 11	6. 12	6. 12	6. 13	6. 13
5	6. 13	6. 13	6. 14	6. 14	6. 15	6. 15	6. 16	6. 16
6	6. 15	6. 15	6. 16	6. 16	6. 17	6. 18	6. 18	6. 19
7	6. 17	6. 17	6. 18	6. 19	6. 19	6. 20	6. 21	6. 22
8	6. 19	6. 20	6. 20	6. 21	6. 22	6. 23	6. 23	6. 24
9	6. 21	6. 22	6. 23	6. 23	6. 24	6. 25	6. 26	6. 27
10	6. 23	6. 24	6. 25	6. 26	6. 27	6. 28	6. 29	6. 30
11	6. 25	6. 26	6. 27	6. 28	6. 29	6. 30	6. 32	6. 33
12	6. 27	6. 28	6. 30	6. 31	6. 32	6. 33	6. 34	6. 36
13	6. 30	6. 31	6. 32	6. 33	6. 34	6. 36	6. 37	6. 38
14	6. 32	6. 33	6. 34	6. 36	6. 37	6. 38	6. 40	6. 41
15	6. 34	6. 35	6. 37	6. 38	6. 40	6. 41	6. 43	6. 44
16	6. 36	6. 38	6. 39	6. 41	6. 42	6. 44	6. 46	6. 47
17	6. 38	6. 40	6. 42	6. 43	6. 45	6. 47	6. 48	6. 50
18	6. 41	6. 42	6. 44	6. 46	6. 48	6. 50	6. 51	6. 53
19	6. 43	6. 45	6. 47	6. 48	6. 50	6. 52	6. 54	6. 56
20	6. 45	6. 47	6. 49	6. 51	6. 53	6. 55	6. 57	7. 0
21	6. 48	6. 50	6. 52	6. 54	6. 56	6. 58	7. 0	7. 3
22	6. 50	6. 52	6. 54	6. 57	6. 59	7. 1	7. 4	7. 6
23	6. 53	6. 55	6. 57	6. 59	7. 2	7. 4	7. 7	7. 9
24	6. 55	6. 57	7. 0	7. 2	7. 5	7. 7	7. 10	7. 13
25	6. 58	7. 0	7. 3	7. 5	7. 8	7. 11	7. 13	7. 16
26	7. 0	7. 3	7. 6	7. 8	7. 11	7. 14	7. 17	7. 20
27	7. 3	7. 6	7. 8	7. 11	7. 14	7. 17	7. 20	7. 23
28	7. 6	7. 8	7. 11	7. 14	7. 17	7. 21	7. 24	7. 27
29	7. 9	7. 11	7. 14	7. 18	7. 21	7. 24	7. 28	7. 31
[illegible]	7. 11	7. 14	7. 18	7. 21	7. 24	7. 28	7. 31	7. 35
[illegible]	7. 14	7. 17	7. 21	7. 24	7. 28	7. 31	7. 35	7. 39
[illegible]	[illegible]	7. 21	7. 24	7. 28	7. 31	7. 35	7. 39	7. 43

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.							
	35°	36°	37°	38°	39°	40°	41°	42°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 5	6. 6	6. 6	6. 6	6. 6	6. 6	6. 6	6. 6
2	6. 8	6. 8	6. 9	6. 9	6. 9	6. 9	6. 10	6. 10
3	6. 11	6. 11	6. 12	6. 12	6. 12	6. 13	6. 13	6. 14
4	6. 14	6. 14	6. 15	6. 15	6. 16	6. 16	6. 17	6. 17
5	6. 17	6. 17	6. 18	6. 18	6. 19	6. 20	6. 20	6. 21
6	6. 19	6. 20	6. 21	6. 22	6. 22	6. 23	6. 24	6. 25
7	6. 22	6. 24	6. 24	6. 25	6. 26	6. 26	6. 27	6. 28
8	6. 25	6. 26	6. 27	6. 28	6. 29	6. 30	6. 31	6. 32
9	6. 28	6. 29	6. 30	6. 31	6. 32	6. 33	6. 35	6. 36
10	6. 31	6. 32	6. 33	6. 34	6. 36	6. 37	6. 38	6. 39
11	6. 34	6. 35	6. 36	6. 38	6. 39	6. 40	6. 42	6. 43
12	6. 37	6. 38	6. 40	6. 41	6. 42	6. 44	6. 45	6. 47
13	6. 40	6. 41	6. 43	6. 44	6. 46	6. 48	6. 49	6. 51
14	6. 43	6. 44	6. 46	6. 48	6. 49	6. 51	6. 53	6. 55
15	6. 46	6. 48	6. 49	6. 51	6. 53	6. 55	6. 57	6. 59
16	6. 49	6. 51	6. 53	6. 55	6. 57	6. 59	7. 1	7. 3
17	6. 52	6. 54	6. 56	6. 58	7. 0	7. 2	7. 5	7. 7
18	6. 55	6. 57	7. 0	7. 2	7. 4	7. 6	7. 9	7. 11
19	6. 59	7. 1	7. 3	7. 5	7. 8	7. 10	7. 13	7. 15
20	7. 2	7. 4	7. 7	7. 9	7. 12	7. 14	7. 17	7. 20
21	7. 5	7. 8	7. 10	7. 13	7. 15	7. 18	7. 21	7. 24
22	7. 9	7. 11	7. 13	7. 17	7. 19	7. 22	7. 25	7. 29
23	7. 12	7. 15	7. 18	7. 21	7. 24	7. 27	7. 30	7. 33
24	7. 16	7. 19	7. 21	7. 25	7. 28	7. 31	7. 34	7. 38
25	7. 19	7. 22	7. 25	7. 29	7. 32	7. 35	7. 39	7. 43
26	7. 23	7. 26	7. 29	7. 33	7. 36	7. 40	7. 44	7. 48
27	7. 27	7. 30	7. 34	7. 37	7. 41	7. 45	7. 49	7. 53
28	7. 31	7. 34	7. 38	7. 42	7. 45	7. 49	7. 54	7. 58
29	7. 35	7. 38	7. 42	7. 46	7. 50	7. 54	7. 59	8. 4
30	7. 39	7. 43	7. 47	7. 51	7. 55	8. 0	8. 5	8. 9
31	7. 43	7. 47	7. 51	7. 56	8. 0	8. 5	8. 10	8. 15
32	7. 47	7. 51	7. 56	8. 1	8. 5	8. 10	8. 16	8. 21

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.							
	43°	44°	45°	46°	47°	48°	49°	50°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 7	6. 7	6. 7	6. 7	6. 7	6. 8	6. 8	6. 8
2	6. 10	6. 11	6. 11	6. 11	6. 12	6. 12	6. 12	6. 13
3	6. 14	6. 15	6. 15	6. 15	6. 16	6. 17	6. 17	6. 18
4	6. 18	6. 18	6. 19	6. 20	6. 20	6. 21	6. 22	6. 22
5	6. 22	6. 22	6. 23	6. 24	6. 25	6. 25	6. 26	6. 27
6	6. 25	6. 26	6. 27	6. 28	6. 29	6. 30	6. 31	6. 32
7	6. 29	6. 30	6. 31	6. 32	6. 33	6. 34	6. 36	6. 37
8	6. 33	6. 34	6. 35	6. 37	6. 38	6. 39	6. 41	6. 42
9	6. 37	6. 38	6. 40	6. 41	6. 42	6. 44	6. 45	6. 47
10	6. 41	6. 42	6. 44	6. 45	6. 47	6. 48	6. 50	6. 52
11	6. 45	6. 46	6. 48	6. 50	6. 51	6. 53	6. 55	6. 57
12	6. 49	6. 50	6. 52	6. 55	6. 56	6. 58	7. 0	7. 2
13	6. 53	6. 55	6. 57	6. 59	7. 1	7. 3	7. 5	7. 7
14	6. 57	6. 59	7. 1	7. 3	7. 5	7. 8	7. 10	7. 13
15	7. 1	7. 3	7. 5	7. 8	7. 10	7. 13	7. 15	7. 18
16	7. 5	7. 7	7. 10	7. 12	7. 15	7. 18	7. 21	7. 24
17	7. 9	7. 12	7. 14	7. 17	7. 20	7. 23	7. 26	7. 29
18	7. 14	7. 16	7. 19	7. 22	7. 25	7. 28	7. 31	7. 35
19	7. 18	7. 21	7. 24	7. 27	7. 30	7. 34	7. 37	7. 41
20	7. 23	7. 26	7. 29	7. 32	7. 35	7. 39	7. 43	7. 47
21	7. 27	7. 30	7. 34	7. 37	7. 41	7. 45	7. 49	7. 53
22	7. 32	7. 35	7. 39	7. 43	7. 46	7. 50	7. 55	7. 59
23	7. 37	7. 40	7. 44	7. 48	7. 52	7. 56	8. 1	8. 6
24	7. 42	7. 45	7. 49	7. 54	7. 58	8. 3	8. 7	8. 12
25	7. 47	7. 51	7. 55	7. 59	8. 4	8. 9	8. 14	8. 19
26	7. 52	7. 56	8. 1	8. 5	8. 10	8. 15	8. 21	8. 27
27	7. 57	8. 2	8. 6	8. 12	8. 17	8. 22	8. 28	8. 34
28	8. 3	8. 7	8. 12	8. 18	8. 23	8. 29	8. 35	8. 42
29	8. 9	8. 13	8. 19	8. 24	8. 30	8. 37	8. 43	8. 50
30	8. 14	8. 20	8. 25	8. 31	8. 38	8. 44	8. 52	8. 59
31	8. 20	8. 26	8. 32	8. 38	8. 45	8. 52	9. 0	9. 9
32	8. 27	8. 33	8. 39	8. 46	8. 53	9. 1	9. 9	9. 19

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.							
	51°	52°	53°	54°	55°	56°	57°	58°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 8	6. 9	6. 9	6. 9	6. 9	6. 10	6. 10	6. 10
2	6. 13	6. 14	6. 14	6. 15	6. 15	6. 16	6. 16	6. 17
3	6. 18	6. 19	6. 19	6. 20	6. 21	6. 22	6. 22	6. 23
4	6. 22	6. 24	6. 25	6. 26	6. 27	6. 28	6. 29	6. 30
5	6. 27	6. 29	6. 30	6. 31	6. 32	6. 34	6. 35	6. 36
6	6. 33	6. 34	6. 36	6. 37	6. 38	6. 40	6. 41	6. 43
7	6. 38	6. 40	6. 41	6. 43	6. 44	6. 46	6. 48	6. 49
8	6. 43	6. 45	6. 47	6. 48	6. 50	6. 52	6. 54	6. 56
9	6. 48	6. 50	6. 52	6. 54	6. 56	6. 58	7. 1	7. 3
10	6. 54	6. 56	6. 58	7. 0	7. 2	7. 5	7. 7	7. 10
11	6. 59	7. 1	7. 3	7. 6	7. 8	7. 11	7. 14	7. 17
12	7. 4	7. 7	7. 9	7. 12	7. 15	7. 18	7. 21	7. 24
13	7. 10	7. 12	7. 15	7. 18	7. 21	7. 24	7. 28	7. 31
14	7. 15	7. 18	7. 21	7. 24	7. 28	7. 31	7. 35	7. 39
15	7. 21	7. 24	7. 27	7. 31	7. 34	7. 39	7. 42	7. 46
16	7. 27	7. 30	7. 33	7. 37	7. 41	7. 45	7. 49	7. 54
17	7. 33	7. 36	7. 40	7. 44	7. 48	7. 52	7. 57	8. 2
18	7. 38	7. 42	7. 46	7. 51	7. 55	8. 0	8. 5	8. 10
19	7. 45	7. 49	7. 53	7. 58	8. 2	8. 7	8. 13	8. 19
20	7. 51	7. 55	8. 0	8. 5	8. 10	8. 15	8. 21	8. 28
21	7. 57	8. 2	8. 7	8. 12	8. 18	8. 24	8. 30	8. 37
22	8. 4	8. 9	8. 14	8. 20	8. 26	8. 32	8. 39	8. 47
23	8. 11	8. 16	8. 22	8. 28	8. 34	8. 41	8. 49	8. 57
24	8. 18	8. 24	8. 30	8. 36	8. 43	8. 51	8. 59	9. 8
25	8. 25	8. 31	8. 38	8. 45	8. 53	9. 1	9. 10	9. 20
26	8. 33	8. 39	8. 47	8. 54	9. 2	9. 11	9. 21	9. 33
27	8. 41	8. 48	8. 56	9. 4	9. 13	9. 23	9. 34	9. 46
28	8. 49	8. 57	9. 5	9. 14	9. 24	9. 35	9. 48	10. 2
29	8. 58	9. 6	9. 14	9. 25	9. 36	9. 49	10. 3	10. 20
30	9. 8	9. 17	9. 26	9. 38	9. 50	10. 4	10. 21	10. 43
31	9. 18	9. 28	9. 38	9. 51	10. 5	10. 22	10. 44	11. 17
32	9. 28	9. 39	9. 52	10. 6	10. 23	10. 44	11. 17	

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of the *same* kind.

DECLI-NATION.	LATITUDE.							
	59°	60°	61°	62°	63°	64°	65°	66°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 11	6. 11	6. 12	6. 13	6. 13	6. 13	6. 14	6. 14
2	6. 17	6. 18	6. 19	6. 20	6. 20	6. 21	6. 22	6. 23
3	6. 24	6. 25	6. 26	6. 27	6. 28	6. 30	6. 31	6. 32
4	6. 31	6. 32	6. 33	6. 35	6. 36	6. 38	6. 40	6. 41
5	6. 38	6. 39	6. 41	6. 42	6. 44	6. 46	6. 48	6. 51
6	6. 44	6. 46	6. 48	6. 50	6. 52	6. 55	6. 57	7. 0
7	6. 51	6. 53	6. 55	6. 58	7. 1	7. 3	7. 6	7. 10
8	6. 58	7. 1	7. 3	7. 6	7. 9	7. 12	7. 15	7. 19
9	7. 5	7. 8	7. 11	7. 14	7. 17	7. 21	7. 25	7. 29
10	7. 13	7. 16	7. 19	7. 22	7. 26	7. 30	7. 34	7. 39
11	7. 20	7. 23	7. 27	7. 31	7. 35	7. 39	7. 44	7. 49
12	7. 27	7. 31	7. 35	7. 39	7. 44	7. 49	7. 54	8. 0
13	7. 35	7. 39	7. 43	7. 48	7. 53	7. 59	8. 5	8. 11
14	7. 43	7. 47	7. 52	7. 57	8. 3	8. 9	8. 15	8. 23
15	7. 51	7. 56	8. 1	8. 6	8. 13	8. 19	8. 27	8. 35
16	7. 59	8. 4	8. 10	8. 16	8. 23	8. 30	8. 38	8. 48
17	8. 7	8. 13	8. 19	8. 26	8. 34	8. 42	8. 51	9. 1
18	8. 16	8. 22	8. 29	8. 37	8. 45	8. 54	9. 4	9. 16
19	8. 25	8. 32	8. 40	8. 48	8. 57	9. 7	9. 18	9. 32
20	8. 35	8. 42	8. 50	8. 59	9. 10	9. 21	9. 34	9. 49
21	8. 45	8. 53	9. 2	9. 12	9. 23	9. 37	9. 51	10. 10
22	8. 55	9. 4	9. 14	9. 25	9. 38	9. 53	10. 12	10. 35
23	9. 6	9. 16	9. 27	9. 40	9. 55	10. 13	10. 36	11. 12
24	9. 18	9. 29	9. 42	9. 57	10. 15	10. 38	11. 13	
25	9. 31	9. 44	9. 58	10. 16	10. 39	11. 14		
26	9. 45	10. 0	10. 17	10. 40	11. 14			
27	10. 1	10. 18	10. 41	11. 14				
28	10. [illegible]	10. 42	11. 15					
29	[illegible]	11. 16						
30	[illegible]							
31	[illegible]							
32	[illegible]							

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.								
	1°	2°	3°	4°	5°	6°	7°	8°	9°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 2	6. 2	6. 2	6. 2	6. 2	6. 2	6. 2	6. 1	6. 1
2	6. 2	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1
3	6. 2	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 0	6. 0
4	6. 2	6. 1	6. 1	6. 1	6. 1	6. 0	6. 0	6. 0	6. 0
5	6. 2	6. 1	6. 1	6. 1	6. 0	6. 0	6. 0	5. 59	5. 59
6	6. 2	6. 1	6. 1	6. 0	6. 0	6. 0	5. 59	5. 59	5. 58
7	6. 2	6. 1	6. 1	6. 0	6. 0	5. 59	5. 58	5. 58	5. 58
8	6. 1	6. 1	6. 0	6. 0	5. 59	5. 59	5. 58	5. 57	5. 57
9	6. 1	6. 1	6. 0	6. 0	5. 59	5. 58	5. 57	5. 56	5. 57
10	6. 1	6. 1	6. 0	5. 59	5. 59	5. 58	5. 57	5. 56	5. 56
11	6. 1	6. 1	6. 0	5. 59	5. 58	5. 57	5. 57	5. 56	5. 55
12	6. 1	6. 1	6. 0	5. 59	5. 58	5. 57	5. 56	5. 55	5. 54
13	6. 1	6. 0	5. 59	5. 58	5. 57	5. 57	5. 56	5. 55	5. 54
14	6. 1	6. 0	5. 59	5. 58	5. 57	5. 56	5. 55	5. 54	5. 53
15	6. 1	6. 0	5. 59	5. 58	5. 57	5. 56	5. 55	5. 54	5. 52
16	6. 1	6. 0	5. 59	5. 58	5. 56	5. 55	5. 54	5. 53	5. 52
17	6. 1	6. 0	5. 59	5. 57	5. 56	5. 55	5. 53	5. 52	5. 51
18	6. 1	6. 0	5. 58	5. 57	5. 56	5. 54	5. 53	5. 52	5. 50
19	6. 1	5. 59	5. 58	5. 57	5. 55	5. 54	5. 52	5. 51	5. 50
20	6. 1	5. 59	5. 58	5. 56	5. 55	5. 53	5. 52	5. 51	5. 49
21	6. 1	5. 59	5. 58	5. 56	5. 55	5. 53	5. 51	5. 50	5. 48
22	6. 1	5. 59	5. 57	5. 56	5. 54	5. 53	5. 51	5. 49	5. 48
23	6. 1	5. 59	5. 57	5. 55	5. 54	5. 52	5. 50	5. 49	5. 47
24	6. 1	5. 59	5. 57	5. 55	5. 53	5. 52	5. 50	5. 48	5. 46
25	6. 1	5. 59	5. 57	5. 55	5. 53	5. 51	5. 49	5. 47	5. 46
26	6. 0	5. 58	5. 56	5. 55	5. 53	5. 51	5. 49	5. 47	5. 45
27	6. 0	5. 58	5. 56	5. 54	5. 52	5. 50	5. 48	5. 46	5. 44
28	6. 0	5. 58	5. 56	5. 54	5. 52	5. 50	5. 47	5. 45	5. 43
29	6. 0	5. 58	5. 56	5. 54	5. 51	5. 49	5. 47	5. 45	5. 42
30	6. 0	5. 58	5. 55	5. 53	5. 51	5. 49	5. 46	5. 44	5. 41
31	6. 0	5. 58	5. 55	5. 53	5. 50	5. 48	5. 46	5. 43	5. 41
32	6. 0	5. 57	5. 55	5. 52	5. 50	5. 47	5. 45	5. 43	5. 40

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.								
	10°	11°	12°	13°	14°	15°	16°	17°	18°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1	6. 1
2	6. 1	6. 1	6. 0	6. 0	6. 0	6. 0	6. 0	6. 0	6. 0
3	6. 0	6. 0	6. 0	5. 59	5. 59	5. 59	5. 59	5. 59	5. 58
4	5. 59	5. 59	5. 59	5. 59	5. 58	5. 58	5. 58	5. 57	5. 57
5	5. 59	5. 58	5. 58	5. 57	5. 57	5. 57	5. 56	5. 56	5. 56
6	5. 58	5. 57	5. 57	5. 57	5. 56	5. 56	5. 55	5. 55	5. 54
7	5. 57	5. 57	5. 56	5. 56	5. 55	5. 55	5. 54	5. 54	5. 53
8	5. 56	5. 56	5. 55	5. 55	5. 54	5. 54	5. 53	5. 52	5. 52
9	5. 56	5. 55	5. 54	5. 54	5. 53	5. 52	5. 52	5. 51	5. 50
10	5. 55	5. 54	5. 54	5. 53	5. 52	5. 51	5. 51	5. 50	5. 49
11	5. 54	5. 54	5. 53	5. 52	5. 51	5. 50	5. 49	5. 49	5. 48
12	5. 54	5. 53	5. 52	5. 51	5. 50	5. 49	5. 48	5. 47	5. 46
13	5. 53	5. 52	5. 51	5. 50	5. 49	5. 48	5. 47	5. 46	5. 45
14	5. 52	5. 51	5. 50	5. 49	5. 48	5. 47	5. 46	5. 45	5. 44
15	5. 51	5. 50	5. 49	5. 48	5. 47	5. 46	5. 45	5. 43	5. 42
16	5. 51	5. 49	5. 48	5. 47	5. 46	5. 45	5. 43	5. 42	5. 41
17	5. 50	5. 49	5. 47	5. 46	5. 45	5. 43	5. 42	5. 41	5. 40
18	5. 49	5. 48	5. 46	5. 45	5. 44	5. 42	5. 41	5. 40	5. 38
19	5. 48	5. 47	5. 45	5. 44	5. 43	5. 41	5. 40	5. 38	5. 37
20	5. 48	5. 46	5. 45	5. 43	5. 41	5. 40	5. 38	5. 37	5. 35
21	5. 47	5. 45	5. 44	5. 42	5. 40	5. 39	5. 37	5. 35	5. 34
22	5. 46	5. 44	5. 43	5. 41	5. 39	5. 38	5. 36	5. 34	5. 32
23	5. 45	5. 43	5. 42	5. 40	5. 38	5. 36	5. 34	5. 33	5. 31
24	5. 44	5. 42	5. 41	5. 39	5. 37	5. 35	5. 33	5. 31	5. 29
25	5. 43	5. 42	5. 40	5. 38	5. 36	5. 34	5. 32	5. 30	5. 28
26	5. 43	5. 41	5. 39	5. 37	5. 34	5. 32	5. 30	5. 28	5. 26
27	5. 42	5. 40	5. 38	5. 36	5. 33	5. 31	5. 29	5. 26	5. 24
28	5. 41	5. 39	5. 37	5. 34	5. 32	5. 30	5. 27	5. 25	5. 22
29	5. 40	5. 38	5. 36	5. 33	5. 31	5. 28	5. 26	5. 24	5. 21
30	5. 39	5. 37	5. 35	5. 32	5. 29	5. 27	5. 24	5. 22	5. 19
31	5. 38	5. 36	5. 34	5. 31	5. 28	5. 26	5. 23	5. 20	5. 18
32	6. 37	5. 35	6. 32	5. 29	5. 27	5. 24	5. 21	5. 19	5. 16

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.							
	27°	28°	29°	30°	31°	32°	33°	34°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 0	6. 0	6. 0	6. 0	6. 0	6. 0	6. 0	6. 0
2	5. 58	5. 58	5. 58	5. 58	5. 58	5. 57	5. 57	5. 57
3	5. 56	5. 56	5. 56	5. 55	5. 55	5. 55	5. 55	5. 54
4	5. 54	5. 54	5. 54	5. 53	5. 53	5. 52	5. 52	5. 52
5	5. 52	5. 52	5. 51	5. 51	5. 50	5. 50	5. 49	5. 49
6	5. 50	5. 50	5. 49	5. 49	5. 48	5. 47	5. 47	5. 46
7	5. 48	5. 47	5. 47	5. 46	5. 46	5. 45	5. 44	5. 44
8	5. 46	5. 45	5. 45	5. 44	5. 43	5. 43	5. 42	5. 41
9	5. 44	5. 43	5. 42	5. 41	5. 41	5. 40	5. 39	5. 38
10	5. 42	5. 41	5. 40	5. 39	5. 38	5. 37	5. 36	5. 35
11	5. 40	5. 39	5. 38	5. 37	5. 36	5. 35	5. 34	5. 32
12	5. 38	5. 36	5. 36	5. 35	5. 34	5. 32	5. 31	5. 30
13	5. 36	5. 34	5. 33	5. 32	5. 31	5. 29	5. 28	5. 27
14	5. 33	5. 32	5. 31	5. 29	5. 28	5. 27	5. 25	5. 24
15	5. 31	5. 30	5. 28	5. 27	5. 26	5. 24	5. 23	5. 21
16	5. 29	5. 27	5. 26	5. 24	5. 23	5. 21	5. 20	5. 18
17	5. 26	5. 25	5. 24	5. 22	5. 20	5. 19	5. 17	5. 15
18	5. 24	5. 23	5. 21	5. 19	5. 18	5. 16	5. 14	5. 12
19	5. 22	5. 21	5. 19	5. 17	5. 15	5. 13	5. 11	5. 9
20	5. 20	5. 18	5. 16	5. 14	5. 12	5. 10	5. 8	5. 6
21	5. 17	5. 15	5. 13	5. 11	5. 9	5. 7	5. 5	5. 3
22	5. 15	5. 13	5. 11	5. 9	5. 7	5. 4	5. 2	5. 0
23	5. 13	5. 10	5. 8	5. 6	5. 4	5. 1	4. 59	4. 56
24	5. 10	5. 8	5. 6	5. 3	5. 1	4. 58	4. 56	4. 53
25	5. 8	5. 5	5. 3	5. 0	4. 58	4. 55	4. 52	4. 50
26	5. 5	5. 3	5. 0	4. 57	4. 55	4. 52	4. 49	4. 46
27	5. 3	5. 0	4. 57	4. 54	4. 52	4. 49	4. 46	4. 43
28	5. 0	4. 57	4. 54	4. 51	4. 48	4. 45	4. 42	4. 39
29	4. 57	4. 54	4. 51	4. 48	4. 45	4. 42	4. 39	4. 35
30	4. 54	4. 51	4. 49	4. 45	4. 42	4. 38	4. 35	4. 31
31	4. 52	4. 48	4. 45	4. 42	4. 38	4. 35	4. 31	4. 28
32	4. 49	4. 45	4. 42	4. 38	4. 35	4. 31	4. 27	4. 24

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLINATION.	LATITUDE.							
	35°	36°	37°	38°	39°	40°	41°	42°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	6. 0	6. 0	6. 0	6. 0	5. 59	5. 59	5. 59	5. 59
2	5. 57	5. 57	5. 57	5. 56	5. 56	5. 56	5. 56	5. 56
3	5. 54	5. 54	5. 54	5. 53	5. 53	5. 53	5. 52	5. 52
4	5. 51	5. 51	5. 51	5. 50	5. 50	5. 49	5. 49	5. 48
5	5. 49	5. 48	5. 48	5. 47	5. 46	5. 46	5. 45	5. 45
6	5. 46	5. 45	5. 44	5. 44	5. 43	5. 43	5. 42	5. 41
7	5. 43	5. 42	5. 41	5. 41	5. 40	5. 39	5. 38	5. 37
8	5. 40	5. 39	5. 38	5. 37	5. 37	5. 36	5. 35	5. 34
9	5. 37	5. 36	5. 35	5. 34	5. 33	5. 32	5. 31	5. 30
10	5. 34	5. 33	5. 32	5. 31	5. 30	5. 29	5. 28	5. 26
11	5. 31	5. 30	5. 29	5. 28	5. 27	5. 25	5. 24	5. 23
12	5. 28	5. 27	5. 26	5. 25	5. 23	5. 22	5. 20	5. 19
13	5. 25	5. 24	5. 23	5. 21	5. 20	5. 18	5. 17	5. 15
14	5. 22	5. 21	5. 19	5. 18	5. 16	5. 15	5. 13	5. 11
15	5. 19	5. 18	5. 16	5. 14	5. 13	5. 11	5. 9	5. 7
16	5. 16	5. 15	5. 13	5. 11	5. 9	5. 7	5. 5	5. 3
17	5. 13	5. 11	5. 10	5. 8	5. 6	5. 4	5. 1	4. 59
18	5. 10	5. 8	5. 6	5. 4	5. 2	4. 59	4. 57	4. 55
19	5. 7	5. 5	5. 3	5. 0	4. 58	4. 56	4. 53	4. 51
20	5. 4	5. 2	4. 59	4. 57	4. 54	4. 52	4. 49	4. 47
21	5. 1	4. 58	4. 56	4. 53	4. 51	4. 48	4. 45	4. 42
22	4. 57	4. 55	4. 52	4. 49	4. 47	4. 44	4. 41	4. 38
23	4. 54	4. 51	4. 49	4. 46	4. 43	4. 40	4. 37	4. 33
24	4. 50	4. 48	4. 45	4. 42	4. 39	4. 35	4. 32	4. 29
25	4. 47	4. 44	4. 41	4. 38	4. 34	4. 31	4. 28	4. 24
26	4. 43	4. 40	4. 36	4. 34	4. 30	4. 27	4. 23	4. 19
27	4. 39	4. 36	4. 33	4. 29	4. 26	4. 22	4. 18	4. 14
28	4. 36	4. 32	4. 29	4. 25	4. 21	4. 17	4. 13	4. 9
29	4. 32	4. 28	4. 25	4. 21	4. 17	4. 13	4. 8	4. 4
30	4. [illegible]		4. 20	4. 16	4. 12	4. 8	4. 3	3. 59
31	4[illegible]			4. 12	4. 7	4. 8	3. 58	3. 53
32				4. 7	4. 2	3. 57	3. 52	3. 47

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.							
	43°	44°	45°	46°	47°	48°	49°	50°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5. 59	5. 59	5. 59	5. 59	5. 59	5. 59	5. 59	5. 59
2	5. 55	5. 55	5. 55	5. 55	5. 55	5. 54	5. 54	5. 54
3	5. 52	5. 51	5. 51	5. 51	5. 50	5. 50	5. 49	5. 49
4	5. 48	5. 47	5. 47	5. 46	5. 46	5. 45	5. 45	5. 44
5	5. 44	5. 44	5. 43	5. 42	5. 42	5. 41	5. 40	5. 39
6	5. 40	5. 40	5. 39	5. 38	5. 37	5. 36	5. 35	5. 35
7	5. 37	5. 36	5. 35	5. 34	5. 33	5. 32	5. 31	5. 30
8	5. 33	5. 32	5. 31	5. 30	5. 28	5. 27	5. 26	5. 25
9	5. 29	5. 28	5. 27	5. 25	5. 24	5. 23	5. 21	5. 20
10	5. 25	5. 24	5. 22	5. 21	5. 20	5. 18	5. 17	5. 15
11	5. 21	5. 20	5. 18	5. 17	5. 15	5. 13	5. 12	5. 10
12	5. 17	5. 16	5. 14	5. 12	5. 11	5. 9	5. 7	5. 5
13	5. 13	5. 12	5. 10	5. 8	5. 6	5. 4	5. 2	5. 0
14	5. 9	5. 7	5. 5	5. 3	5. 1	4. 59	4. 57	4. 54
15	5. 5	5. 3	5. 1	4. 59	4. 57	4. 54	4. 52	4. 49
16	5. 1	4. 59	4. 57	4. 54	4. 52	4. 49	4. 46	4. 45
17	4. 57	4. 55	4. 52	4. 50	4. 47	4. 44	4. 41	4. 38
18	4. 53	4. 50	4. 47	4. 45	4. 42	4. 39	4. 36	4. 33
19	4. 48	4. 46	4. 43	4. 40	4. 37	4. 34	4. 30	4. 27
20	4. 44	4. 41	4. 38	4. 35	4. 32	4. 28	4. 25	4. 21
21	4. 39	4. 36	4. 33	4. 30	4. 26	4. 23	4. 19	4. 15
22	4. 35	4. 32	4. 28	4. 25	4. 21	4. 17	4. 13	4. 9
23	4. 30	4. 27	4. 23	4. 19	4. 15	4. 11	4. 7	4. 3
24	4. 25	4. 22	4. 18	4. 14	4. 10	4. 5	4. 1	3. 56
25	4. 20	4. 17	4. 13	4. 8	4. 4	3. 59	3. 54	3. 49
26	4. 15	4. 11	4. 7	4. 3	3. 58	3. 53	3. 48	3. 42
27	4. 10	4. 6	4. 1	3. 57	3. 52	3. 46	3. 41	3. 35
28	4. 5	4. 0	3. 55	3. 50	3. 45	3. 40	3. 34	3. 28
29	3. 59	3. 54	3. 49	3. 44	3. 38	3. 33	3. 26	3. 20
30	3. 54	3. 48	3. 43	3. 37	3. 31	3. 25	3. 18	3. 11
31	3. 48	3. 42	3. 37	3. 31	3. 24	3. 17	3. 10	3. 3
32	3. 42	3. 36	3. 30	3. 23	3. 17	3. 9	3. 2	2. 53

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.							
	51°	52°	53°	54°	55°	56°	57°	58°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5. 58	5. 58	5. 58	5. 58	5. 58	5. 58	5. 58	5. 58
2	5. 53	5. 53	5. 53	5. 53	5. 52	5. 52	5. 52	5. 51
3	5. 49	5. 48	5. 48	5. 47	5. 47	5. 46	5. 45	5. 45
4	5. 44	5. 43	5. 42	5. 42	5. 41	5. 40	5. 39	5. 38
5	5. 39	5. 38	5. 37	5. 36	5. 35	5. 34	5. 33	5. 32
6	5. 34	5. 33	5. 31	5. 30	5. 29	5. 28	5. 27	5. 25
7	5. 29	5. 27	5. 26	5. 25	5. 23	5. 22	5. 20	5. 19
8	5. 23	5. 22	5. 21	5. 19	5. 17	5. 16	5. 14	5. 12
9	5. 18	5. 17	5. 16	5. 13	5. 12	5. 10	5. 8	5. 5
10	5. 13	5. 11	5. 10	5. 8	5. 5	5. 3	5. 1	4. 59
11	5. 8	5. 6	5. 4	5. 2	4. 59	4. 57	4. 54	4. 52
12	5. 3	5. 0	4. 58	4. 56	4. 53	4. 51	4. 48	4. 45
13	4. 57	4. 55	4. 52	4. 50	4. 47	4. 44	4. 41	4. 38
14	4. 52	4. 49	4. 47	4. 44	4. 41	4. 37	4. 34	4. 30
15	4. 46	4. 44	4. 41	4. 37	4. 34	4. 31	4. 27	4. 23
16	4. 41	4. 38	4. 34	4. 31	4. 27	4. 24	4. 20	4. 15
17	4. 35	4. 32	4. 28	4. 23	4. 21	4. 17	4. 12	4. 8
18	4. 29	4. 26	4. 22	4. 18	4. 14	4. 9	4. 5	4. 0
19	4. 23	4. 19	4. 15	4. 11	4. 7	4. 2	3. 56	3. 51
20	4. 17	4. 13	4. 9	4. 4	3. 59	3. 54	3. 49	3. 43
21	4. 11	4. 6	4. 2	3. 57	3. 52	3. 46	3. 40	3. 34
22	4. 4	4. 0	3. 55	3. 50	3. 44	3. 38	3. 31	3. 24
23	3. 58	3. 53	3. 47	3. 42	3. 36	3. 29	3. 23	3. 15
24	3. 51	3. 46	3. 40	3. 34	3. 27	3. 20	3. 13	3. 5
25	3. 44	3. 38	3. 32	3. 25	3. 18	3. 11	3. 3	2. 53
26	3. 37	3. 30	3. 24	3. 17	3. 9	3. 1	2. 52	2. 42
27	3. 29	3. 22	3. 15	3. 8	2. 59	2. 50	2. 40	2. 29
28	3. 21	3. 14	3. 6	2. 58	2. 49	2. 38	2. 28	2. 15
29	3. 12	3. 5	2. 56	2. 47	2. 37	2. 26	2. 14	2. 0
30	3. 4	2. 55	2. 46	2. 36	2. 25	2. 13	1. 58	1. 41
31	2. 54	2. 45	2. 35	2. 24	2. 12	1. 57	1. 41	1. 19
32	2. 44	2. 34	2. 23	2. 11	1. 57	1. 40	1. 18	

TABLE XLIII. *Continued.*

Semi-Diurnal Arcs.

Latitude and Declination of *different* kinds.

DECLI-NATION.	LATITUDE.							
	59°	60°	61°	62°	63°	64°	65°	66°
D.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.	H. M.
1	5. 57	5. 57	5. 57	5. 57	5. 57	5. 57	5. 56	5. 56
2	5. 51	5. 50	5. 50	5. 49	5. 49	5. 48	5. 48	5. 47
3	5. 44	5. 43	5. 43	5. 42	5. 41	5. 40	5. 39	5. 38
4	5. 37	5. 36	5. 35	5. 34	5. 33	5. 32	5. 31	5. 29
5	5. 31	5. 29	5. 28	5. 27	5. 25	5. 24	5. 22	5. 20
6	5. 24	5. 22	5. 21	5. 19	5. 17	5. 15	5. 13	5. 11
7	5. 17	5. 15	5. 13	5. 11	5. 9	5. 7	5. 4	5. 1
8	5. 10	5. 8	5. 6	5. 3	5. 1	4. 58	4. 55	4. 52
9	5. 3	5. 1	4. 58	4. 55	4. 53	4. 49	4. 46	4. 42
10	4. 56	4. 53	4. 50	4. 47	4. 44	4. 40	4. 37	4. 32
11	4. 49	4. 46	4. 43	4. 39	4. 35	4. 31	4. 27	4. 22
12	4. 42	4. 38	4. 35	4. 31	4. 27	4. 22	4. 17	4. 12
13	4. 34	4. 30	4. 26	4. 22	4. 18	4. 13	4. 7	4. 1
14	4. 27	4. 23	4. 18	4. 13	4. 8	4. 3	3. 56	3. 50
15	4. 19	4. 14	4. 9	4. 4	3. 59	3. 53	3. 46	3. 39
16	4. 11	4. 6	4. 1	3. 55	3. 49	3. 42	3. 35	3. 27
17	4. 3	3. 57	3. 52	3. 45	3. 39	3. 31	3. 23	3. 14
18	3. 54	3. 48	3. 42	3. 35	3. 28	3. 20	3. 11	3. 0
19	3. 45	3. 39	3. 32	3. 25	3. 17	3. 8	2. 58	2. 46
20	3. 36	3. 29	3. 22	3. 14	3. 5	2. 55	2. 43	2. 30
21	3. 27	3. 19	3. 11	3. 2	2. 52	2. 41	2. 28	2. 12
22	3. 17	3. 9	3. 0	2. 50	2. 38	2. 25	2. 10	1. 52
23	3. 6	2. 57	2. 47	2. 36	2. 23	2. 8	1. 50	1. 27
24	2. 55	2. 45	2. 34	2. 21	2. 7	1. 49	1. 26	
25	2. 43	2. 32	2. 20	2. 5	1. 47	1. 25		
26	2. 31	2. 18	2. 3	1. 46	1. 23			
27	2. 16	2. 2	1. 45	1. 22				
28	2. 1	1. 43	1. 21					
29	1. 42	1. 21						
30	1. 20							
31								
32								

TABLE XLIV.

The Time answering to a Change of Altitude of One Degree at the Horizon.

LATI-TUDE.	DECLINATION.								
	0°	3°	6°	9°	12°	15°	18°	21°	24°
D.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.	M. S.
0	4. 0	4. 0	4. 1	4. 3	4. 6	4. 9	4. 13	4. 17	4. 22
2	4. 0	4. 0	4. 1	4. 3	4. 6	4. 9	4. 13	4. 17	4. 23
4	4. 1	4. 1	4. 2	4. 4	4. 6	4. 9	4. 13	4. 18	4. 24
6	4. 1	4. 1	4. 2	4. 4	4. 7	4. 10	4. 14	4. 18	4. 25
8	4. 2	4. 2	4. 3	4. 5	4. 8	4. 11	4. 15	4. 20	4. 26
10	4. 4	4. 4	4. 5	4. 7	4. 10	4. 12	4. 17	4. 22	4. 28
12	4. 6	4. 6	4. 7	4. 9	4. 12	4. 15	4. 19	4. 24	4. 30
14	4. 8	4. 8	4. 9	4. 11	4. 14	4. 17	4. 22	4. 27	4. 33
16	4. 10	4. 10	4. 11	4. 13	4. 16	4. 19	4. 24	4. 29	4. 36
18	4. 13	4. 13	4. 14	4. 16	4. 19	4. 22	4. 26	4. 32	4. 39
20	4. 16	4. 16	4. 17	4. 19	4. 22	4. 26	4. 30	4. 36	4. 43
22	4. 19	4. 19	4. 21	4. 23	4. 26	4. 30	4. 35	4. 41	4. 48
24	4. 23	4. 23	4. 25	4. 27	4. 30	4. 34	4. 39	4. 46	4. 53
26	4. 27	4. 27	4. 29	4. 31	4. 35	4. 39	4. 44	4. 51	4. 59
28	4. 32	4. 32	4. 34	4. 36	4. 40	4. 44	4. 50	4. 57	5. 6
30	4. 37	4. 38	4. 40	4. 42	4. 45	4. 50	4. 57	5. 5	5. 14
32	4. 43	4. 44	4. 45	4. 48	4. 52	4. 57	5. 4	5. 12	5. 23
34	4. 49	4. 50	4. 52	4. 55	4. 59	5. 5	5. 12	5. 21	5. 32
36	4. 57	4. 58	5. 0	5. 3	5. 8	5. 13	5. 21	5. 32	5. 43
38	5. 5	5. 6	5. 7	5. 11	5. 16	5. 22	5. 31	5. 42	5. 56
40	5. 13	5. 14	5. 16	5. 20	5. 26	5. 33	5. 42	5. 55	6. 10
41	5. 18	5. 19	5. 21	5. 25	5. 31	5. 39	5. 49	6. 1	6. 18
42	5. 23	5. 24	5. 26	5. 30	5. 36	5. 45	5. 55	6. 9	6. 27
43	5. 28	5. 29	5. 32	5. 36	5. 42	5. 51	6. 3	6. 18	6. 36
44	5. 33	5. 35	5. 38	5. 42	5. 49	5. 58	6. 10	6. 26	6. 45
45	5. 39	5. 41	5. 44	5. 48	5. 55	6. 5	6. 17	6. 34	6. 55
46	5. 45	5. 47	5. 50	5. 54	6. 1	6. 12	6. 26	6. 44	7. 6
47	5. 52	5. 54	5. 57	6. 1	6. 8	6. 20	6. 35	6. 55	7. 19
48	5. 59	6. 1	6. 4	6. 8	6. 16	6. 29	6. 45	7. 7	7. 34
49	6. 6	6. 8	6. 11	6. 16	6. 25	6. 38	6. 56	7. 20	7. 50
50	6. 14	6. 16	6. 20	6. 25	6. 34	6. 48	7. 8	7. 33	8. 7
51	6. 22	6. 24	6. 28	6. 34	6. 44	6. 59	7. 20	7. 48	8. 24
52	6. 31	6. 33	6. 38	6. 44	6. 55	7. 11	7. 33	8. 4	8. 43
53	6. 30	6. 42	6. 47	6. 54	7. 6	7. 23	7. 47	8. 20	9. 3
54	6. 49	6. 52	6. 58	7. 5	7. 18	7. 36	8. 1	8. 37	9. 24
55	6. 58	7. 1	7. 7	7. 16	7. 30	7. 49	8. 16	8. 55	9. 48
56	7. 8	7. 11	7. 18	7. 28	7. 43	8. 4	8. 32	9. 18	10. 18
57	7. 19	7. 23	7. 30	7. 41	7. 58	8. 21	8. 59	9. 46	10. 57
58	7. 31	7. 35	7. 43	7. 55	8. 14	8. 39	9. 20	10. 19	11. 47
59	7. 45	7. 49	7. 57	8. 10	8. 30	8. 59	9. 42	10. 55	12. 40
60	8. 0	8. 5	8. 14	8. 27	8. 48	9. 21	10. 6	11. 36	13. 45

TABLE XLV.

The Amplitudes of the Rising and Setting of the Heavenly Bodies.

LATITUDE.	DECLINATION.							
	1°	2°	3°	4°	5°	6°	7°	8°
D.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
2	1. 0	2. 0	3. 0	4. 0	5. 0	6. 0	7. 0	8. 0
4	1. 0	2. 0	3. 0	4. 1	5. 1	6. 1	7. 1	8. 2
6	1. 0	2. 1	3. 1	4. 2	5. 2	6. 2	7. 3	8. 4
8	1. 0	2. 1	3. 2	4. 3	5. 3	6. 4	7. 5	8. 6
10	1. 1	2. 2	3. 3	4. 4	5. 5	6. 6	7. 7	8. 8
12	1. 1	2. 3	3. 4	4. 5	5. 7	6. 8	7. 10	8. 11
14	1. 2	2. 4	3. 5	4. 7	5. 9	6. 11	7. 13	8. 15
16	1. 2	2. 5	3. 7	4. 10	5. 13	6. 15	7. 17	8. 19
18	1. 3	2. 6	3. 9	4. 13	5. 16	6. 19	7. 22	8. 25
20	1. 4	2. 8	3. 11	4. 16	5. 19	6. 23	7. 27	8. 31
22	1. 5	2. 10	3. 14	4. 19	5. 24	6. 29	7. 33	8. 38
24	1. 6	2. 12	3. 17	4. 23	5. 29	6. 34	7. 40	8. 46
26	1. 7	2. 14	3. 21	4. 27	5. 34	6. 41	7. 48	8. 54
28	1. 8	2. 16	3. 24	4. 32	5. 40	6. 48	7. 56	9. 4
30	1. 9	2. 19	3. 28	4. 37	5. 46	6. 55	8. 5	9. 15
32	1. 11	2. 22	3. 33	4. 43	5. 54	7. 5	8. 16	9. 27
34	1. 12	2. 25	3. 37	4. 50	6. 2	7. 15	8. 27	9. 40
36	1. 14	2. 28	3. 43	4. 57	6. 11	7. 25	8. 40	9. 54
38	1. 16	2. 32	3. 48	5. 5	6. 21	7. 37	8. 54	10. 11
40	1. 18	2. 37	3. 55	5. 14	6. 32	7. 51	9. 9	10. 28
42	1. 20	2. 41	4. 2	5. 24	6. 44	8. 5	9. 26	10. 48
43	1. 22	2. 44	4. 6	5. 29	6. 51	8. 13	9. 35	10. 58
44	1. 23	2. 47	4. 10	5. 34	6. 58	8. 21	9. 45	11. 9
45	1. 25	2. 50	4. 15	5. 40	7. 5	8. 30	9. 55	11. 21
46	1. 26	2. 53	4. 19	5. 46	7. 12	8. 39	10. 6	11. 33
47	1. 28	2. 56	4. 24	5. 52	7. 21	8. 49	10. 18	11. 46
48	1. 30	2. 59	4. 29	5. 59	7. 29	8. 59	10. 30	12. 0
49	1. 32	3. 3	4. 35	6. 6	7. 38	9. 10	10. 42	12. 15
50	1. 33	3. 7	4. 40	6. 14	7. 48	9. 21	10. 56	12. 30
51	1. 35	3. 11	4. 46	6. 22	7. 58	9. 34	11. 10	12. 47
52	1. 37	3. 15	4. 52	6. 30	8. 8	9. 47	11. 25	13. 4
53	1. 40	3. 20	4. 59	6. 40	8. 20	10. 0	11. 41	13. 22
54	1. 42	3. 24	5. 6	6. 49	8. 33	10. 15	11. 58	13. 42
55	1. 45	3. 29	5. 14	6. 59	8. 44	10. 30	12. 16	14. 3
56	1. 47	3. 35	5. 22	7. 10	8. 58	10. 46	12. 35	14. 25
57	1. 50	3. 41	5. 31	7. 22	9. 13	11. 4	12. 56	14. 48
58	1. 53	3. 47	5. 40	7. 34	9. 28	11. 23	13. 18	15. 13
59	1. 57	3. 53	5. 50	7. 47	9. 45	11. 53	13. 41	15. 41
60	2. 0	4. 0	6. 1	8. 1	10. 2	12. 4	14. 6	16. 10
61	2. 4	4. 8	6. 12	8. 16	10. 21	12. 27	14. 34	16. 41
62	2. 8	4. 16	6. 24	8. 34	10. 42	12. 52	15. 3	17. 15
63	2. 12	4. 25	6. 37	8. 50	11. 4	13. 19	15. 37	17. 51

TABLE XLV. *Continued.*

LATITUDE.	DECLINATION.							
	9°	10°	11°	12°	13°	14°	15°	16°
D.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
2	9. 0	10. 0	11. 0	12. 0	13. 0	14. 0	15. 0	16. 1
4	9. 2	10. 2	11. 2	12. 2	13. 2	14. 2	15. 2	16. 3
6	9. 4	10. 4	11. 4	12. 4	13. 4	14. 5	15. 5	16. 6
8	9. 6	10. 6	11. 7	12. 7	13. 8	14. 9	15. 9	16. 10
10	9. 8	10. 10	11. 10	12. 11	13. 12	14. 13	15. 14	16. 15
12	9. 12	10. 14	11. 14	12. 16	13. 18	14. 19	15. 21	16. 22
14	9. 17	10. 18	11. 20	12. 22	13. 24	14. 26	15. 29	16. 30
16	9. 22	10. 24	11. 27	12. 30	13. 32	14. 34	15. 37	16. 40
18	9. 28	10. 31	11. 35	12. 38	13. 41	14. 44	15. 47	16. 51
20	9. 35	10. 39	11. 43	12. 47	13. 51	14. 55	15. 59	17. 4
22	9. 43	10. 48	11. 53	12. 58	14. 2	15. 8	16. 13	17. 18
24	9. 52	10. 58	12. 3	13. 10	14. 15	15. 22	16. 28	17. 34
26	10. 1	11. 9	12. 15	13. 22	14. 30	15. 37	16. 44	17. 52
28	10. 12	11. 21	12. 29	13. 37	14. 46	15. 54	17. 8	18. 11
30	10. 24	11. 34	12. 44	13. 53	15. 3	16. 13	17. 28	18. 34
32	10. 38	11. 49	13. 0	14. 11	15. 23	16. 34	17. 46	18. 58
34	11. 52	12. 5	13. 18	14. 31	15. 45	16. 58	18. 12	19. 25
36	11. 9	12. 24	13. 39	14. 54	16. 9	17. 24	18. 40	19. 55
38	11. 27	12. 44	14. 1	15. 18	16. 35	17. 53	19. 19	20. 26
40	11. 47	13. 6	14. 25	15. 45	17. 5	18. 25	19. 45	21. 5
42	12. 9	13. 31	14. 53	16. 15	17. 37	19. 0	20. 23	21. 46
43	12. 21	13. 44	15. 7	16. 31	17. 55	19. 19	20. 43	22. 8
44	12. 34	13. 58	15. 23	16. 48	18. 13	19. 39	21. 5	22. 32
45	12. 47	14. 13	15. 39	17. 6	18. 33	20. 0	21. 28	22. 57
46	13. 1	14. 29	15. 57	17. 25	18. 54	20. 43	21. 53	23. 22
47	13. 16	14. 45	16. 15	17. 45	19. 17	20. 47	22. 18	23. 50
48	13. 28	15. 2	16. 34	18. 6	19. 39	21. 12	22. 45	24. 19
49	13. 48	15. 21	16. 55	18. 29	20. 3	21. 38	23. 15	24. 51
50	14. 5	15. 40	17. 16	18. 52	20. 29	22. 7	23. 45	25. 24
51	14. 24	16. 1	17. 39	19. 17	20. 50	22. 37	24. 17	25. 59
52	14. 43	16. 23	18. 3	19. 44	21. 16	23. 8	24. 52	26. 36
53	15. 4	16. 47	18. 29	20. 13	21. 57	23. 42	25. 28	27. 16
54	15. 26	17. 11	18. 57	20. 43	22. 30	24. 18	26. 7	27. 58
55	15. 50	17. 37	19. 26	21. 15	23. 5	24. 57	26. 49	28. 43
56	16. 15	18. 6	19. 57	21. 50	23. 43	25. 38	27. 34	29. 32
57	16. 42	18. 36	20. 30	22. 27	24. 24	26. 22	28. 29	30. 24
58	17. 10	19. 8	21. 6	23. 6	25. 7	27. 10	29. 14	31. 20
59	17. 41	19. 42	21. 45	23. 49	25. 53	28. 5	30. 10	32. 21
60	18. 14	20. 19	22. 25	24. 34	26. 46	28. 56	31. 10	33. 27
61	18. 50	20. 59	23. 11	25. 24	27. 39	29. 56	32. 16	34. 39
62	19. 28	21. 43	23. 59	26. 17	28. 38	31. 1	33. 27	35. 57
63	20. 9	22. 29	24. 51	27. 15	29. 42	32. 12	34. 46	37. 23

TABLE XLV. *Continued.*

LATITUDE.	DECLINATION.							
	17°	18°	19°	20°	21°	22°	23°	23½°
D.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.	D. M.
2	17. 1	18. 1	19. 1	20. 1	21. 1	22. 1	23. 1	23. 31
4	17. 3	18. 3	19. 3	20. 3	21. 3	22. 5	23. 4	23. 34
6	17. 6	18. 6	19. 7	20. 7	21. 7	22. 9	23. 8	23. 38
8	17. 10	18. 11	19. 12	20. 12	21. 13	22. 13	23. 14	23. 46
10	17. 16	18. 17	19. 18	20. 19	21. 20	22. 21	23. 23	23. 53
12	17. 24	18. 25	19. 27	20. 28	21. 30	22. 31	23. 33	24. 4
14	17. 32	18. 34	19. 36	20. 38	21. 41	22. 42	23. 45	24. 16
16	17. 43	18. 46	19. 48	20. 51	21. 53	22. 57	23. 59	24. 31
18	17. 54	18. 58	20. 1	21. 5	22. 8	23. 12	24. 15	24. 47
20	18. 8	19. 12	20. 16	21. 21	22. 25	23. 30	24. 34	25. 7
22	18. 23	19. 28	20. 33	21. 39	22. 44	23. 50	24. 56	25. 28
24	18. 40	19. 48	20. 53	21. 59	23. 6	24. 13	25. 21	25. 53
26	18. 59	20. 7	21. 14	22. 22	23. 30	24. 38	25. 46	26. 20
28	19. 20	20. 29	21. 38	22. 47	23. 57	25. 6	26. 16	26. 51
30	19. 44	20. 54	22. 5	23. 16	24. 27	25. 38	26. 49	27. 25
32	20. 10	21. 22	22. 35	23. 47	25. 0	26. 13	27. 26	28. 3
34	20. 38	21. 53	23. 7	24. 22	25. 37	26. 52	28. 7	28. 45
36	21. 11	22. 27	23. 44	25. 1	26. 18	27. 35	28. 53	29. 32
38	21. 47	23. 5	24. 25	25. 43	27. 3	28. 23	29. 44	30. 24
40	22. 26	23. 47	25. 9	26. 31	27. 54	29. 17	30. 40	31. 22
42	23. 10	24. 34	25. 59	27. 24	28. 50	30. 16	31. 43	32. 27
43	23. 34	25. 0	26. 26	27. 53	29. 20	30. 49	32. 18	33. 2
44	23. 59	25. 26	26. 55	28. 23	29. 53	31. 23	32. 54	33. 40
45	24. 25	25. 55	27. 25	28. 56	30. 27	31. 59	33. 33	34. 20
46	24. 53	26. 25	27. 57	29. 30	31. 3	32. 38	34. 14	35. 2
47	25. 23	26. 57	28. 31	30. 6	31. 42	33. 19	34. 57	35. 47
48	25. 55	27. 30	29. 7	30. 44	32. 23	34. 3	35. 44	36. 35
49	26. 28	28. 6	29. 45	31. 25	33. 7	34. 49	36. 33	37. 26
50	27. 7	28. 44	30. 26	32. 9	33. 53	35. 39	37. 26	38. 21
51	27. 41	29. 24	31. 9	32. 55	34. 43	36. 32	38. 23	39. 19
52	28. 21	30. 6	31. 57	33. 45	35. 36	37. 29	39. 23	40. 22
53	29. 4	30. 53	32. 45	34. 39	36. 33	38. 30	40. 49	41. 30
54	29. 50	31. 43	33. 38	35. 35	37. 34	39. 36	41. 40	42. 43
55	30. 39	32. 36	34. 35	36. 37	38. 40	40. 47	42. 56	44. 3
56	31. 32	33. 33	35. 36	37. 42	39. 51	42. 4	44. 19	45. 30
57	32. 28	34. 34	36. 43	38. 54	41. 10	43. 27	45. 50	47. 4
58	33. 29	35. 40	37. 54	40. 12	42. 33	44. 59	47. 30	48. 48
59	34. 35	36. 52	39. 12	41. 37	44. 6	46. 40	49. 21	50. 44
60	35. 47	38. 10	40. 38	43. 10	45. 47	48. 31	51. 24	52. 53
61	37. 5	39. 56	42. 11	44. 52	47. 40	50. 36	53. 42	55. 20
62	38. 31	41. 10	43. 54	46. 46	49. 46	52. 56	56. 20	58. 9
63	40. 5	42. 54	45. 49	48. 53	52. 8	55. 30	59. 24	61 26

TABLE XLVI.

To find the Enlightened Part of the Diameter of the Moon, *or* Venus, *supposing the Diameter to be divided into* 12 *equal Parts.*

Degrees.	*For the MOON*—Argument. Distance of the Moon from the Sun.						
	0°	30°	60°	90°	120°	150°	
	Parts.	Parts.	Parts.	Parts.	Parts.	Parts.	
0	0,000	0,804	3,000	6,000	9,000	11,196	[illegible]
1	0,001	0,857	3,092	6,104	9,090	11,247	[illegible]
2	0,004	0,912	3,184	6,209	9,179	11,297	28
[illegible]	0,009	0,969	3,277	6,314	9,267	11,346	27
4	0,015	1,026	3,370	6,418	9,355	11,392	26
5	0,023	1,085	3,465	6,523	9,441	11,437	25
6	0,033	1,146	3,560	6,627	9,526	11,481	24
7	0,045	1,209	3,656	6,731	9,611	11,523	23
8	0,059	1,272	3,753	6,834	9,694	11,563	22
9	0,074	1,337	3,850	6,938	9,776	11,601	21
10	0,091	1,404	3,948	7,041	9,856	11,638	20
11	0,110	1,472	4,047	7,145	9,936	11,673	19
12	0,131	1,542	4,147	7,247	10,014	11,706	18
13	0,154	1,612	4,247	7,349	10,091	11,737	17
14	0,179	1,684	4,347	7,451	10,167	11,767	16
15	0,205	1,758	4,448	7,552	10,242	11,795	15
16	0,233	1,833	4,549	7,653	10,316	11,821	14
17	0,263	1,909	4,651	7,753	10,388	11,846	13
18	0,294	1,986	4,753	7,853	10,458	11,869	12
19	0,327	2,064	4,855	7,953	10,528	11,890	11
20	0,362	2,144	4,959	8,052	10,596	11,909	10
21	0,399	2,224	5,062	8,150	10,663	11,926	9
22	0,437	2,306	5,166	8,247	10,728	11,941	8
23	0,477	2,389	5,269	8,344	10,791	11,955	7
24	0,519	2,474	5,373	8,440	10,854	11,967	6
25	0,563	2,559	5,477	8,535	10,915	11,977	5
26	0,608	2,645	5,582	8,630	10,974	11,985	4
27	0,654	2,733	5,686	8,723	11,031	11,991	3
[illegible]	0,703	2,821	5,791	8,816	11,088	11,996	2
[illegible]	0,753	2,910	5,896	8,908	11,143	11,999	1
[illegible]	0,804	3,000	6,000	9,000	11,196	12,000	0
	[illegible]	120°	90°	60°	30°	0°	Degrees.
	VENUS—Argument. Angle formed by two lines drawn from Venus to the Sun and Earth.						

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	10	11	12	13	14	15	16	17	18	19	D. M.
M. S.	600	660	720	780	840	900	960	1020	1080	1140	M. S.
0	7782	7368	6990	6642	6320	6021	5740	5477	5229	4994	0
1	7774	7361	6984	6637	6315	6016	5736	5473	5225	4990	1
2	7767	7354	6978	6631	6310	6011	5731	5469	5221	4986	2
3	7760	7348	6972	6625	6305	6006	5727	5464	5217	4983	3
4	7753	7341	6966	6620	6300	6001	5722	5460	5213	4979	4
5	7745	7335	6960	6614	6294	5997	5718	5456	5209	4975	5
6	7738	7328	6954	6609	6289	5992	5713	5452	5205	4971	6
7	7731	7322	6948	6603	6284	5987	5709	5447	5201	4967	7
8	7724	7315	6942	6598	6279	5982	5704	5443	5197	4964	8
9	7717	7309	6936	6592	6274	5977	5700	5439	5193	4960	9
10	7710	7302	6930	6587	6269	5973	5695	5435	5189	4956	10
11	7703	7296	6924	6581	6264	5968	5691	5430	5185	4952	11
12	7696	7289	6918	6576	6259	5963	5686	5426	5181	4949	12
13	7688	7283	6912	6570	6254	5958	5682	5422	5177	4945	13
14	7681	7276	6906	6565	6248	5954	5677	5418	5173	4941	14
15	7674	7270	6900	6559	6243	5949	5673	5414	5169	4937	15
16	7669	7264	6894	6554	6238	5944	5669	5409	5165	4933	16
17	7660	7257	6888	6548	6233	5939	5664	5405	5161	4930	17
18	7653	7251	6882	6543	6228	5935	5660	5401	5157	4926	18
19	7646	7244	6877	6538	6223	5930	5655	5397	5153	4922	19
20	7639	7238	6871	6532	6218	5925	5651	5393	5149	4918	20
21	7632	7232	6865	6527	6213	5920	5646	5389	5145	4915	21
22	7625	7225	6859	6521	6208	5916	5642	5384	5141	4911	22
23	7618	7219	6853	6516	6203	5911	5637	5380	5137	4907	23
24	7611	7212	6847	6510	6198	5906	5633	5376	5133	4903	24
25	7604	7206	6841	6505	6193	5902	5629	5372	5129	4900	25
26	7597	7200	6836	6500	6188	5897	5624	5368	5125	4896	26
27	7590	7193	6830	6494	6183	5892	5620	5364	5122	4892	27
28	7583	7187	6824	6489	6178	5888	5615	5359	5118	4889	28
29	7577	7181	6818	6484	6173	5883	5611	5355	5114	4885	29
30	7570	7175	6812	6478	6168	5878	5607	5351	5110	4881	30
	10	11	12	13	14	15	16	17	18	19	

TABLE XLVIII.

Showing the Hour-Angles of the Sun *with the Meridian, or Apparent Time from Noon, when his Depression is* 8° *below the Horizon, to every Half Degree of Declination, for the Latitude of Greenwich.*

Sun's Declination, *North.*				Sun's Declination, *South.*			
Declin.	Hour-Angle.	Declin.	Hour-Angle.	Declin.	Hour-Angle.	Declin.	Hour-Angle.
D. M.	H. M. S.	D. M.	H. M. S.	D. M.	H. M. S.	D. M.	H. M. S.
0. 0	6. 51. 39	12. 0	7. 58. 48	0. 0	6. 51. 39	12. 0	5. 51. 10
0. 30	6. 54. 14	12. 30	8. 1. 57	0. 30	6. 49. 5	12. 30	5. 48. 38
1. 0	6. 56. 49	13. 0	8. 5. 9	1. 0	6. 46. 31	13. 0	5. 46. 5
1. 30	6. 59. 26	13. 30	8. 8. 24	1. 30	6. 43. 58	13. 30	5. 43. 32
2. 0	7. 2. 8	14. 0	8. 11. 41	2. 0	6. 41. 25	14. 0	5. 40. 59
2. 30	7. 4. 41	14. 30	8. 15. 1	2. 30	6. 38. 53	14. 30	5. 38. 25
3. 0	7. 7. 20	15. 0	8. 18. 25	3. 0	6. 36. 21	15. 0	5. 35. 50
3. 30	7. 10. 0	15. 30	8. 21. 52	3. 30	6. 33. 49	15. 30	5. 33. 15
4. 0	7. 12. 41	16. 0	8. 25. 23	4. 0	6. 31. 18	16. 0	5. 30. 39
4. 30	7. 15. 23	16. 30	8. 28. 58	4. 30	6. 28. 47	16. 30	5. 28. 2
5. 0	7. 18. 6	17. 0	8. 32. 36	5. 0	6. 26. 17	17. 0	5. 25. 24
5. 30	7. 20. 50	17. 30	8. 36. 19	5. 30	6. 23. 47	17. 30	5. 22. 46
6. 0	7. 23. 36	18. 0	8. 40. 6	6. 0	6. 21. 16	18. 0	5. 20. 6
6. 30	7. 26. 23	18. 30	8. 43. 58	6. 30	6. 18. 46	18. 30	5. 17. 26
7. 0	7. 29. 11	19. 0	8. 47. 55	7. 0	6. 16. 16	19. 0	5. 14. 44
7. 30	7. 32. 1	19. 30	8. 51. 56	7. 30	6. 13. 46	19. 30	5. 12. 1
8. 0	7. 34. 52	20. 0	8. 56. 7	8. 0	6. 11. 16	20. 0	5. 9. 18
8. 30	7. 37. 45	20. 30	9. 0. 22	8. 30	6. 8. 46	20. 30	5. 6. 33
9. 0	7. 40. 39	21. 0	9. 4. 44	9. 0	6. 6. 15	21. 0	5. 3. 47
9. 30	7. 43. 36	21. 30	9. 9. 19	9. 30	6. 3. 45	21. 30	5. 0. 59
10. 0	7. 46. 34	22. 0	9. 13. 51	10. 0	6. 1. 14	22. 0	4. 58. 10
10. 30	7. 49. 35	22. 30	9. 18. 37	10. 30	5. 58. 44	22. 30	4. 55. 19
11. 0	7. 52. 37	23. 0	9. 23. 34	11. 0	5. 56. 18	23. 0	4. 52. 28
11. 30	7. 55. 42	23. 30	9. 28. 42	11. 30	5. 53. 42	23. 30	4. 49. 34
12. 0	7. 58. 48			12. 0	5. 51. 10		

TABLE XLIX.

Logistic Logarithms.

D. M.	0	1	2	3	4	5	[redacted]	7	8	9	D. M.
M. S.	0	60	120	180	240	[redacted]	360	420	480	540	M. S.
0		1,7782	1,4771	1,3010	1,1761	1,0792	1,0000	9331	8751	8239	0
1	3,5563	1,7710	1,4735	1,2986	1,1743	1,0777	9988	9320	874-	8231	[redacted]
2	3,2553	1,7639	1,4699	1,2962	1,1725	1,0763	9976	9310	8733	8223	2
3	3,0792	1,7570	1,4664	1,2939	1,1707	1,0749	9964	9300	8724	8215	3
4	2,9542	1,7501	1,4629	1,2915	1,1689	1 0734	9952	9289	8715	8207	4
[redacted]	2,8573	1,7434	1,4594	1,2891	1,1671	1,0720	9940	9279	8706	8199	5
6	2,7782	1,7368	1,4559	1,2868	1,1664	1,0706	9928	9269	[redacted]	8191	6
7	2,7112	1,7302	1,4525	1,2845	1,1636	1,0692	9916	9259	8688	8183	7
8	2,6532	1,7238	1,4491	1,2821	1,1619	1,0678	[redacted]	9249	8679	8175	8
9	2,6021	1,7175	1,4457	1,2798	1,1601	1,0663	9893	9238	8670	8167	9
10	2,5563	1,7112	1,4424	1,2775	1,1584	1,0649	9881	9228	8661	8159	10
11	2,5149	1,7050	1,4390	1,2753	1,1566	1,0635	9869	9218	8652	8152	11
12	2,4771	1,6990	1,4357	1,2730	1,1549	1,0621	9858	9208	8643	8144	12
13	2,4424	1,6930	1,4325	1,2707	1,1532	1,0608	9846	9198	8635	8136	13
14	2,4102	1,6871	1,4292	1,2685	1,1515	1,0594	9834	9188	8626	8128	14
15	2,3802	1,6812	1,4260	1,2663	1,1498	1,0580	9823	9178	8617	8120	15
16	2,3522	1,6755	1,4228	1,2640	1,1481	1,0566	[redacted]	9168	8608	8112	16
17	2,3259	1,6698	1,4196	1,2618	1,1464	1,0552	9800	9158	8599	8104	17
18	2,3010	1,6642	1,4165	1,2596	1,1447	1,0539	9788	9148	8591	8097	18
19	2,2775	1,6587	1,4133	1,2574	1,1430	1,0525	9777	9138	8582	8089	19
20	2,2553	1,6532	1,4102	1,2553	1,1413	1,0512	9765	9128	8573	[redacted]	[redacted]
21	2,2341	1,6478	1,4071	1,2531	1,1397	1,0498	9754	9119	8565	8073	[redacted]
22	2,2139	1,6425	1,4040	1,2510	1,1380	1,0484	9742	9109	8556	8066	22
23	2,1946	1,6372	1,4010	1,2488	1,1363	1,0471	9731	9099	8547	8058	23
24	2,1761	1,6320	1,3979	1,2467	1,1347	1,0458	9720	9089	8539	8050	24
25	2,1584	1,6269	1,3949	1,2445	1,1331	1,0444	9708	9079	8530	8043	25
26	2,1413	1,6218	1,3919	1,2424	1,1314	1,0431	9697	9070	8522	8035	26
27	2,1249	1,6168	1,3890	1,2403	1,1298	1,0418	9686	9060	8513	8027	27
[redacted]	2,1091	1,6118	1,3860	1,2382	1,1282	1,0404	9675	9050	[redacted]	8020	28
29	2,0939	1,6069	1,3831	1,2362	1,1266	1,0391	9664	9041	8496	8012	29
[redacted]	2,0792	1,6021	1,3802	1,2341	1,1249	1,0378	9652	9031	8487	8004	30
	0	1	2	3	4	5	6	7	8	9	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	0	1	2	3	4	5	[redacted]	7	[redacted]	[redacted]	D. M.
M. S.	0	60	120	180	240	300	360	420	480	540	M. S.
31	2,0649	1,5973	1,3773	1,2320	1,1233	1,0365	9641	9021	8479	7997	31
32	2,0512	1,5925	1,3745	1,2300	1,1217	1,0352	9630	9012	8470	7989	32
33	2,0378	1,5878	1,3716	1,2279	1,1201	1,0339	9619	9002	8462	7981	33
34	2,0248	1,5832	1,3688	1,2259	1,1186	1,0326	9608	8992	8453	7974	34
35	2,0122	1,5786	1,3660	1,2239	1,1170	1,0313	9597	8983	8445	7966	35
36	2,0000	1,5740	1,3632	1,2218	1,1154	1,0300	[redacted]	8973	8437	7959	36
37	1,9881	1,5695	1,3604	1,2198	1,1138	1,0287	9575	8964	8428	7951	37
38	1,9765	1,5651	1,3576	1,2178	1,1123	1,0274	9564	8954	8420	7944	38
39	1,9652	1,5607	1,3549	1,2159	1,1107	1,0261	9553	8945	8411	7936	39
40	1,9542	1,5563	1,3522	1,2139	1,1091	1,0248	9542	8935	[redacted]	7929	40
41	1,9435	1,5520	1,3495	1,2119	1,1076	1,0235	9532	8926	8395	7921	41
42	1,9331	1,5477	1,3468	1,2099	1,1061	1,0223	9521	8917	8386	7914	42
43	1,9228	1,5435	1,3441	1,2080	1,1045	1,0210	9510	8907	8378	7906	43
44	1,9128	1,5393	1,3415	1,2061	1,1030	1,0197	9499	8898	8370	7899	44
45	1,9031	1,5351	1,3388	1,2041	1,1015	1,0185	9488	[redacted]	8361	7891	45
46	1,8935	1,5310	1,3362	1,2022	1,0999	1,0172	9478	8879	8353	7884	46
47	1,8842	1,5269	1,3336	1,2003	1,0984	1,0160	9467	8870	8345	7877	47
48	1,8751	1,5229	1,3310	1,1984	1,0969	1,0147	9456	8861	8337	7869	48
49	1,8661	1,5189	1,3284	1,1965	1,0954	1,0135	9446	8851	8328	7862	49
50	1,8573	1,5149	1,3259	1,1946	1,0939	1,0122	9435	8842	8320	7855	50
51	1,8487	1,5110	1,3233	1,1927	1,0924	1,0110	9425	8833	8312	7847	51
52	1,8403	1,5071	1,3208	1,1908	1,0909	1,0098	9414	8824	8304	7840	52
53	1,8320	1,5032	1,3183	1,1889	1,0894	1,0085	9404	8814	8296	7832	53
54	1,8239	1,4994	1,3158	1,1871	1,0880	1,0073	9393	8805	8288	7825	54
55	1,8159	1,4956	1,3133	1,1852	1,0865	1,0061	9383	8796	8279	7818	55
56	1,8081	1,4918	1,3108	1,1834	1,0850	1,0049	9372	8787	8271	7811	56
57	1,8004	1,4881	1,3083	1,1816	1,0835	1,0036	9362	8778	8263	7803	57
58	1,7929	1,4844	1,3059	1,1797	1,0821	1,0024	9351	8769	8255	7796	58
59	1,7855	1,4808	1,3034	1,1779	1,0806	1,0012	9341	8760	8247	7789	59
60	1,7782	1,4771	1,3010	1,1761	1,0792	1,0000	9331	8751	8239	7782	60
	0	1	2	3	4	5	6	7	8	9	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	10	11	12	13	14	15	16	17	18	19	D. M.
M. S.	600	660	720	780	840	900	960	1020	1080	1140	M. S.
0	7782	7368	6990	6642	6320	6021	5740	5477	5229	4994	0
1	7774	7361	6984	6637	6315	6016	5736	5473	5225	4990	1
2	7767	7354	6978	6631	6310	6011	5731	5469	5221	4986	2
3	7760	7348	6972	6625	6305	6006	5727	5464	5217	4983	3
4	7753	7341	6966	6620	6300	6001	5722	5460	5213	4979	4
5	7745	7335	6960	6614	6294	5997	5718	5456	5209	4975	5
6	7738	7328	6954	6609	6289	5992	5713	5452	5205	4971	6
7	7731	7322	6948	6603	6284	5987	5709	5447	5201	4967	7
8	7724	7315	6942	6598	6279	5982	5704	5443	5197	4964	8
9	7717	7309	6936	6592	6274	5977	5700	5439	5193	4960	9
10	7710	7302	6930	6587	6269	5973	5695	5435	5189	4956	10
11	7703	7296	6924	6581	6264	5968	5691	5430	5185	4952	11
12	7696	7289	6918	6576	6259	5963	5686	5426	5181	4949	12
13	7688	7283	6912	6570	6254	5958	5682	5422	5177	4945	13
14	7681	7276	6906	6565	6248	5954	5677	5418	5173	4941	14
15	7674	7270	6900	6559	6243	5949	5673	5414	5169	4937	15
16	7669	7264	6894	6554	6238	5944	5669	5409	5165	4933	16
17	7660	7257	6888	6548	6233	5939	5664	5405	5161	4930	17
18	7653	7251	6882	6543	6228	5935	5660	5401	5157	4926	18
19	7646	7244	6877	6538	6223	5930	5655	5397	5153	4922	19
20	7639	7238	6871	6532	6218	5925	5651	5393	5149	4918	20
21	7632	7232	6865	6527	6213	5920	5646	5389	5145	4915	21
22	7625	7225	6859	6521	6208	5916	5642	5384	5141	4911	22
23	7618	7219	6853	6516	6203	5911	5637	5380	5137	4907	23
24	7611	7212	6847	6510	6198	5906	5633	5376	5133	4903	24
25	7604	7206	6841	6505	6193	5902	5629	5372	5129	4900	25
26	7597	7200	6836	6500	6188	5897	5624	5368	5125	4896	26
27	7590	7193	6830	6494	6183	5892	5620	5364	5122	4892	27
28	7583	7187	6824	6489	6178	5888	5615	5359	5118	4889	28
29	7577	7181	6818	6484	6173	5883	5611	5355	5114	4885	29
30	7570	7175	6812	6478	6168	5878	5607	5351	5110	4881	30
	10	11	12	13	14	15	16	17	18	19	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	10	11	12	13	14	15	16	17	18	19	D. M.
M. S.	600	660	720	780	840	900	960	1020	1080	1140	M. S.
31	7563	7168	6807	6473	6163	5874	5602	5347	5106	4877	31
32	7556	7162	6801	6467	6158	5869	5598	5343	5102	4874	32
33	7549	7156	6795	6462	6153	5864	5594	5339	5098	4870	33
34	7542	7149	6789	6457	6148	5860	5589	5335	5094	4866	34
35	7535	7143	6784	6451	6143	5855	5585	5331	5090	4863	35
36	7528	7137	6778	6446	6138	5850	5580	5326	5086	4859	36
37	7522	7131	6772	6441	6133	5846	5576	5322	5082	4855	37
38	7515	7124	6766	6435	6128	5841	5572	5318	5079	4852	38
39	7508	7118	6761	6430	6123	5836	5567	5314	5075	4848	39
40	7501	7112	6755	6425	6118	5832	5563	5310	5071	4844	40
41	7494	7106	6749	6420	6113	5827	5559	5306	5067	4841	41
42	7488	7100	6743	6414	6108	5823	5554	5302	5063	4837	42
43	7481	7093	6738	6409	6103	5818	5550	5298	5059	4833	43
44	7474	7087	6732	6404	6099	5813	5546	5294	5055	4830	44
45	7467	7081	6726	6398	6094	5809	5541	5290	5051	4826	45
46	7461	7075	6721	6393	6089	5804	5537	5285	5048	4822	46
47	7454	7069	6715	6388	6084	5800	5533	5281	5044	4819	47
48	7447	7063	6709	6383	6079	5795	5528	5277	5040	4815	48
49	7441	7057	6704	6377	6074	5790	5524	5273	5036	4811	49
50	7434	7050	6698	6372	6069	5786	5520	5269	5032	4808	50
51	7427	7044	6692	6367	6064	5781	5516	5265	5028	4804	51
52	7421	7038	6687	6362	6059	5777	5511	5261	5025	4800	52
53	7414	7032	6681	6357	6055	5772	5507	5257	5021	4797	53
54	7407	7026	6676	6351	6050	5768	5503	5253	5017	4793	54
55	7401	7020	6670	6346	6045	5763	5498	5249	5013	4789	55
56	7394	7014	6664	6341	6040	5758	5494	5245	5009	4786	56
57	7387	7008	6659	6336	6035	5754	5490	5241	5005	4782	57
58	7381	7002	6653	6331	6030	5749	5486	5237	5002	4778	58
59	7374	6990	6648	6325	6025	5745	5481	5233	4998	4775	59
60	7368	6990	6642	6320	6021	5740	5477	5229	4994	4771	50
	10	11	12	13	14	15	16	17	18	19	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	20	21	22	23	24	25	26	27	28	29	D. M.
M. S.	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740	M. S.
0	4771	4559	4357	4164	3979	3802	3632	3468	3310	3158	0
1	4768	4556	4354	4161	3976	3799	3629	3465	3307	3155	1
2	4764	4552	4351	4158	3973	3796	3626	3463	3305	3153	2
3	4760	4549	4347	4155	3970	3793	3623	3460	3302	3150	3
4	4757	4546	4344	4152	3967	3791	3621	3457	3300	3148	4
5	4753	4542	4341	4149	3964	3788	3618	3454	3297	3145	5
6	4750	4539	4338	4145	3961	3785	3615	3452	3294	3143	6
7	4746	4535	4334	4142	3958	3782	3612	3449	3292	3140	7
8	4742	4532	4331	4139	3955	3779	3610	3446	3289	3138	8
9	4739	4528	4328	4136	3952	3776	3607	3444	3287	3135	9
10	4735	4525	4325	4133	3949	3773	3604	3441	3284	3133	10
11	4732	4522	4321	4130	3946	3770	3601	3438	3282	3130	11
12	4728	4518	4318	4127	3943	3768	3598	3436	3279	3128	12
13	4724	4515	4315	4124	3940	3765	3596	3433	3276	3125	13
14	4721	4511	4311	4120	3937	3762	3593	3431	3274	3123	14
15	4717	4508	4308	4117	3934	3759	3590	3428	3271	3120	15
16	4714	4505	4305	4114	3931	3756	3587	3425	3269	3118	16
17	4710	4501	4302	4111	3928	3753	3585	3423	3266	3115	17
18	4707	4498	4298	4108	3925	3750	3582	3420	3264	3113	18
19	4703	4494	4295	4105	3922	3717	3579	3417	3261	3110	19
20	4699	4491	4292	4102	3919	3745	3576	3415	3259	3108	20
21	4696	4488	4289	4099	3917	3742	3574	3412	3256	3105	21
22	4692	4484	4285	4096	3914	3739	3571	3109	3253	3103	22
23	4689	4481	4282	4092	3911	3736	3568	3407	3251	3101	23
24	4685	4477	4279	4089	3908	3733	3565	3404	3248	3098	24
25	4682	4474	4276	4086	3905	3730	3563	3401	3246	3096	25
26	4678	4471	4273	4083	3902	3727	3560	3399	3243	3093	26
27	4675	4467	4269	4080	3899	3725	3557	3396	3241	3091	27
28	4671	4464	4266	4077	3896	3722	3555	3393	3238	3088	28
29	4668	4460	4263	4074	3893	3719	3552	3391	3236	3086	29
30	4664	4457	4260	4071	3890	3716	3549	3388	3233	3083	30
	20	21	22	23	24	25	26	27	28	29	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	20	21	22	23	24	25	26	27	28	29	D. M.
M. S.	1200	1260	1320	1380	1440	1500	1560	1620	1680	1740	M. S.
31	4660	4454	4256	4068	3887	3713	3546	3386	3231	3081	31
32	4657	4450	4253	4065	3884	3710	3544	3383	3228	3078	32
33	4653	4447	4250	4062	3881	3708	3541	3380	3225	3076	33
34	4650	4444	4247	4059	3878	3705	3538	3378	3223	3073	34
35	4646	4440	4244	4055	3875	3702	3535	3375	3220	3071	35
36	4643	4437	4240	4052	3872	3699	3533	3372	3218	3069	36
37	4639	4434	4237	4049	3869	3696	3530	3370	3215	3066	37
38	4636	4430	4234	4046	3866	3693	3527	3367	3213	3064	38
39	4632	4427	4231	4043	3863	3691	3525	3365	3210	3061	39
40	4629	4424	4228	4040	3860	3688	3522	3362	3208	3059	40
41	4625	4420	4224	4037	3857	3685	3519	3359	3205	3056	41
42	4622	4417	4221	4034	3855	3682	3516	3357	3203	3054	42
43	4618	4414	4218	4031	3852	3679	3514	3354	3200	3052	43
44	4615	4410	4215	4028	3849	3677	3511	3351	3198	3049	44
45	4611	4407	4212	4025	3846	3674	3508	3349	3195	3047	45
46	4608	4404	4209	4022	3843	3671	3506	3346	3193	3044	46
47	4604	4400	4205	4019	3840	3668	3503	3344	3190	3042	47
48	4601	4397	4202	4016	3837	3665	3500	3341	3188	3039	48
49	4597	4394	4199	4013	3834	3663	3497	3338	3185	3037	49
50	4594	4390	4196	4010	3831	3660	3495	3336	3183	3034	50
51	4590	4387	4193	4007	3828	3657	3492	3333	3180	3032	51
52	4587	4384	4189	4004	3825	3654	3489	3331	3178	3030	52
53	4584	4380	4186	4001	3822	3651	3487	3328	3175	3027	53
54	4580	4377	4183	3998	3820	3649	3484	3325	3173	3025	54
55	4577	4374	4180	3995	3817	3646	3481	3323	3170	3022	55
56	4573	4370	4177	3991	3814	3643	3479	3320	3168	3020	56
57	4570	4367	4174	3988	3811	3640	3476	3318	3165	3018	57
[illegible]	4566	4364	4171	3985	3808	3637	3473	3315	3163	3015	58
[illegible]	4563	4361	4167	3982	3805	3635	3471	3313	3160	3013	59
[illegible]	4559	4357	4164	3979	3802	3632	3468	3310	3158	3010	60
	20	21	22	23	24	25	26	27	28	29	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	30	31	32	33	34	35	36	37	38	39	D. M.
M. S.	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340	M. S.
0	3010	2868	2730	2596	2467	2341	2218	2099	1984	1871	0
1	3008	2866	2728	2594	2465	2339	2216	2098	1982	1869	1
2	3005	2863	2725	2592	2462	2337	2214	2096	1980	1867	2
3	3003	2861	2723	2590	2460	2335	2212	2094	1978	1865	3
4	3001	2859	2721	2588	2458	2333	2210	2092	1976	1863	4
5	2998	2856	2719	2585	2456	2331	2208	2090	1974	1862	5
6	2996	2854	2716	2583	2454	2328	2206	2088	1972	1860	6
7	2993	2852	2714	2581	2452	2326	2204	2086	1970	1858	7
8	2991	2849	2712	2579	2450	2324	2202	2084	1968	1856	8
9	2989	2847	2710	2577	2448	2322	2200	2082	1967	1854	9
10	2986	2845	2707	2574	2445	2320	2198	2080	1965	1852	10
11	2984	2842	2705	2572	2443	2318	2196	2078	1963	1850	11
12	2981	2840	2703	2570	2441	2316	2194	2076	1961	1849	12
13	2979	2838	2701	2568	2439	2314	2192	2074	1959	1847	13
14	2977	2835	2698	2566	2437	2312	2190	2072	1957	1845	14
15	2974	2833	2696	2564	2435	2310	2188	2070	1955	1843	15
16	2972	2831	2694	2561	2433	2308	2186	2068	1953	1841	16
17	2969	2828	2692	2559	2431	2306	2184	2066	1951	1839	17
18	2967	2826	2689	2557	2429	2304	2182	2064	1950	1838	18
19	2965	2824	2687	2555	2426	2302	2180	2062	1948	1836	19
20	2962	2821	2685	2553	2424	2300	2178	2061	1946	1834	20
21	2960	2819	2683	2551	2422	2298	2176	2059	1944	1832	21
22	2958	2817	2681	2548	2420	2296	2174	2057	1942	1830	22
23	2955	2815	2678	2546	2418	2294	2172	2055	1940	1828	23
24	2953	2812	2676	2544	2416	2291	2170	2053	1938	1827	24
25	2950	2810	2674	2542	2414	2289	2169	2051	1936	1825	25
26	2948	2808	2672	2540	2412	2287	2167	2049	1934	1823	26
27	2946	2805	2669	2538	2410	2285	2165	2047	1933	1821	27
28	2943	2803	2667	2535	2408	2283	2163	2045	1931	1819	28
29	2941	2801	2665	2533	2405	2281	2161	2043	1929	1817	29
30	2939	2798	2663	2531	2403	2279	2159	2041	1927	1816	30
	30	31	32	33	34	35	36	37	38	39	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	30	31	32	33	34	35	36	37	38	39	D. M.
M. S.	1800	1860	1920	1980	2040	2100	2160	2220	2280	2340	M. S.
31	2936	2796	2660	2529	2401	2277	2157	2039	1925	1814	31
32	2934	2794	2658	2527	2399	2275	2155	2037	1923	1812	32
33	2931	2792	2656	2525	2397	2273	2153	2035	1921	1810	33
34	2929	2789	2654	2522	2395	2271	2151	2033	1919	1808	34
35	2927	2787	2652	2520	2393	2269	2149	2032	1918	1806	35
36	2924	2785	2649	2518	2391	2267	2147	2030	1916	1805	36
37	2922	2782	2647	2516	2389	2265	2145	2028	1914	1803	37
38	2920	2780	2645	2514	2387	2263	2143	2026	1912	1801	38
39	2917	2778	2643	2512	2384	2261	2141	2024	1910	1799	39
40	2915	2775	2640	2510	2382	2259	2139	2022	1908	1797	40
41	2912	2773	2638	2507	2380	2257	2137	2020	1906	1795	41
42	2910	2771	2636	2505	2378	2255	2135	2018	1904	1794	42
43	2908	2779	2634	2503	2376	2253	2133	2016	1903	1792	43
44	2905	2766	2632	2501	2374	2251	2131	2014	1901	1790	44
45	2903	2764	2629	2499	2372	2249	2129	2012	1899	1788	45
46	2901	2762	2627	2497	2370	2247	2127	2010	1897	1786	46
47	2898	2760	2625	2494	2368	2245	2125	2009	1895	1785	47
48	2896	2757	2623	2492	2366	2243	2123	2007	1893	1783	48
49	2894	2755	2621	2490	2364	2241	2121	2005	1891	1781	49
50	2891	2753	2618	2488	2362	2239	2119	2003	1889	1779	50
51	2889	2750	2616	2486	2359	2237	2117	2001	1888	1777	51
52	2887	2748	2614	2484	2357	2235	2115	1999	1886	1775	52
53	2884	2746	2612	2482	2355	2233	2113	1997	1884	1774	53
54	2882	2744	2610	2480	2353	2231	2111	1995	1882	1772	54
55	2880	2741	2607	2477	2351	2229	2109	1993	1880	1770	55
56	2877	2739	2605	2475	2349	2227	2107	1991	1878	1768	56
57	2875	2737	2603	2473	2347	2225	2105	1989	1876	1766	57
58	2873	2735	2601	2471	2345	2223	2103	1987	1875	1765	58
59	2870	2732	2599	2469	2343	2220	2101	1986	1873	1763	59
60	2868	2730	2596	2467	2341	2218	2099	1984	1871	1761	60
	30	31	32	33	34	35	36	37	38	39	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	40	41	42	43	44	45	46	47	48	49	D. M.
M. S.	2400	2460	2520	2580	2640	2700	2760	2820	2880	2940	M. S.
0	1761	1654	1549	1447	1347	1249	1154	1061	0969	0880	0
1	1759	1652	1547	1445	1345	1248	1152	1059	0968	0878	1
2	1757	1650	1546	1443	1344	1246	1151	1057	0966	0877	2
3	1755	1648	1544	1442	1342	1245	1149	1056	0965	0875	3
4	1754	1647	1542	1440	1340	1243	1148	1054	0963	0874	4
5	1752	1645	1540	1438	1339	1241	1146	1053	0962	0872	5
6	1750	1643	1539	1437	1337	1240	1145	1051	0960	0871	6
7	1748	1641	1537	1435	1335	1238	1143	1050	0959	0869	7
8	1746	1640	1535	1433	1334	1237	1141	1048	0957	0868	8
9	1745	1638	1534	1432	1332	1235	1140	1047	0956	0866	9
10	1743	1636	1532	1430	1331	1233	1138	1045	0954	0865	10
11	1741	1634	1530	1428	1329	1232	1137	1044	0953	0863	11
12	1739	1633	1528	1427	1327	1230	1135	1042	0951	0862	12
13	1737	1631	1527	1425	1326	1229	1134	1041	0950	0860	13
14	1736	1629	1525	1423	1324	1227	1132	1039	0948	0859	14
15	1734	1627	1523	1422	1322	1225	1130	1037	0947	0857	15
16	1732	1626	1522	1420	1321	1224	1129	1036	0945	0856	16
17	1730	1624	1520	1418	1319	1222	1127	1034	0944	0855	17
18	1728	1622	1518	1417	1317	1221	1126	1033	0942	0853	18
19	1727	1620	1516	1415	1316	1219	1124	1031	0941	0852	19
20	1725	1619	1515	1413	1314	1217	1123	1030	0939	0850	20
21	1723	1617	1513	1412	1313	1216	1121	1028	0938	0849	21
22	1721	1615	1511	1410	1311	1214	1119	1027	0936	0847	22
23	1719	1613	1510	1408	1309	1213	1118	1025	0935	0846	23
24	1718	1612	1508	1407	1308	1211	1116	1024	0933	0844	24
25	1716	1610	1506	1405	1306	1209	1115	1022	0932	0843	25
26	1714	1608	1504	1403	1304	1208	1113	1021	0930	0841	26
27	1712	1606	1503	1402	1303	1206	1112	1019	0929	0840	27
28	1711	1605	1501	1400	1301	1205	1110	1018	0927	0838	28
29	1709	1603	1499	1398	1300	1203	1109	1016	0926	0837	29
30	1707	1601	1498	1397	1298	1201	1107	1015	0924	0835	30
	40	41	42	43	44	45	46	47	48	49	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	40	41	42	43	44	45	46	47	48	49	D. M.
M. S.	2400	2460	2520	2580	2640	2700	2760	2820	2880	2940	M. S.
31	1705	1599	1496	1395	1296	1200	1105	1013	0923	0834	31
32	1703	1598	1494	1393	1295	1198	1104	1012	0921	0833	32
33	1702	1596	1493	1392	1293	1197	1102	1010	0920	0831	33
34	1700	1594	1491	1390	1291	1195	1101	1008	0918	0830	34
35	1698	1592	1489	1388	1290	1193	1099	1007	0917	0828	35
36	1696	1591	1487	1387	1288	1192	1098	1005	0915	0827	36
37	1694	1589	1486	1385	1287	1190	1096	1004	0914	0825	37
38	1693	1587	1484	1383	1285	1189	1095	1002	0912	0824	38
39	1690	1585	1482	1382	1283	1187	1093	1001	0911	0822	39
40	1689	1584	1481	1380	1282	1186	1091	0999	0909	0821	40
41	1687	1582	1479	1378	1280	1184	1090	0998	0908	0819	41
42	1686	1580	1477	1377	1278	1182	1088	0996	0906	0818	42
43	1684	1578	1476	1375	1277	1181	1087	0995	0905	0816	43
44	1682	1577	1474	1373	1275	1179	1085	0993	0903	0815	44
45	1680	1575	1472	1372	1274	1178	1084	0992	0902	0814	45
46	1678	1573	1470	1370	1272	1176	1082	0990	0900	0812	46
47	1677	1571	1469	1368	1270	1174	1081	0989	0899	0811	47
48	1675	1570	1467	1367	1269	1173	1079	0987	0897	0809	48
49	1673	1568	1465	1365	1267	1171	1078	0986	0896	0808	49
50	1671	1566	1464	1363	1266	1170	1076	0984	0894	0806	50
51	1670	1565	1462	1362	1264	1168	1074	0983	0893	0805	51
52	1668	1563	1460	1360	1262	1167	1073	0981	0891	0803	52
53	1666	1561	1459	1359	1261	1165	1071	0980	0890	0802	53
54	1664	1559	1457	1357	1259	1163	1070	0978	0888	0801	54
55	1663	1558	1455	1355	1257	1162	1068	0977	0887	0799	55
56	1661	1556	1454	1354	1256	1160	1067	0975	0885	0798	56
57	1659	1554	1452	1352	1254	1159	1065	0974	0884	0796	57
58	1657	1552	1450	1350	1253	1157	1064	0972	0883	0795	58
59	1655	1551	1449	1349	1251	1156	1062	0971	0881	0793	59
60	1654	1549	1447	1347	1249	1154	1061	0969	0880	0792	60
	40	41	42	43	44	45	46	47	48	49	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	50	51	52	53	54	55	56	57	58	59	D. M.
M. S.	3000	3060	3120	3180	3240	3300	3360	3420	3480	3540	M. S.
0	0792	0706	0021	0539	0458	0378	0300	0223	0147	0073	0
1	0790	0704	0620	0537	0456	0377	0298	0221	0146	0072	1
2	0789	0703	0619	0536	0455	0375	0297	0220	0145	0071	2
3	0787	0702	0617	0535	0454	0374	0296	0219	0143	0069	3
4	0786	0700	0616	0533	0452	0373	0294	0218	0142	0068	4
5	0785	0699	0615	0532	0451	0371	0293	0216	0141	0067	5
6	0783	0697	0613	0531	0450	0370	0292	0215	0140	0066	6
7	0782	0696	0612	0529	0448	0369	0291	0214	0139	0064	7
8	0780	0694	0610	0528	0447	0367	0289	0213	0137	0063	8
9	0779	0693	0609	0526	0446	0366	0288	0211	0136	0062	9
10	0777	0692	0608	0525	0444	0365	0287	0210	0135	0061	10
11	0776	0690	0606	0524	0443	0363	0285	0209	0134	0060	11
12	0774	0689	0605	0522	0442	0362	0284	0208	0132	0058	12
13	0773	0687	0603	0521	0440	0361	0283	0206	0131	0057	13
14	0772	0686	0602	0520	0439	0359	0282	0205	0130	0056	14
15	0770	0685	0601	0518	0438	0358	0260	0204	0129	0055	15
16	0769	0683	0599	0517	0436	0357	0279	0202	0127	0053	16
17	0767	0682	0598	0516	0435	0356	0278	0201	0126	0052	17
18	0766	0680	0596	0514	0434	0354	0276	0200	0125	0051	18
19	0764	0679	0595	0513	0432	0353	0275	0199	0124	0050	19
20	0763	0678	0594	0512	0431	0352	0274	0197	0122	0049	20
21	0762	0676	0592	0510	0430	0350	0273	0196	0121	0047	21
22	0760	0675	0591	0509	0428	0349	0271	0195	0120	0046	22
23	0759	0673	0590	0507	0427	0348	0270	0194	0119	0045	23
24	0757	0672	0588	0506	0426	0346	0269	0192	0117	0044	24
25	0756	0670	0587	0505	0424	0345	0267	0191	0116	0042	25
26	0754	0669	0585	0503	0423	0344	0266	0190	0115	0041	26
27	0753	0668	0584	0502	0422	0342	0265	0189	0114	0040	27
28	0751	0666	0583	0501	0420	0341	0264	0187	0112	0039	28
29	0750	0665	0581	0599	0419	0340	0262	0186	0111	0038	29
30	0749	0663	0580	0598	0418	0339	0261	0185	0110	0036	30
	50	51	52	53	54	55	56	57	58	59	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	50	51	52	53	54	55	56	57	58	59	D. M.
M. S.	3000	3060	3120	3180	3240	3300	3360	3420	3480	3540	M. S.
31	0747	0662	0579	0497	0416	0337	0260	0184	0109	0035	31
32	0746	0661	0577	0495	0415	0336	0258	0182	0107	0034	32
33	0744	0659	0576	0494	0414	0335	0257	0181	0106	0033	33
34	0743	0658	0574	0493	0412	0333	0256	0180	0105	0031	34
35	0741	0656	0573	0491	0411	0332	0255	0179	0104	0030	35
36	0740	0655	0572	0490	0410	0331	0253	0177	0103	0029	36
37	0739	0654	0570	0489	0408	0329	0252	0176	0101	0028	37
38	0737	0652	0569	0487	0407	0328	0251	0175	0100	0027	38
39	0736	0651	0568	0486	0406	0327	0250	0174	0099	0025	39
40	0734	0649	0566	0484	0404	0326	0248	0172	0098	0024	40
41	0733	0648	0565	0483	0403	0324	0247	0171	0096	0023	41
42	0731	0647	0563	0482	0402	0323	0246	0170	0095	0022	42
43	0730	0645	0562	0480	0400	0322	0244	0169	0094	0021	43
44	0729	0644	0561	0479	0399	0320	0243	0167	0093	0019	44
45	0727	0642	0559	0478	0398	0319	0242	0166	0091	0018	45
46	0726	0641	0558	0476	0396	0318	0241	0165	0090	0017	46
47	0724	0640	0557	0475	0395	0316	0239	0163	0089	0016	47
48	0723	0638	0555	0474	0394	0315	0238	0162	0088	0015	48
49	0721	0637	0554	0472	0392	0314	0237	0161	0087	0013	49
50	0720	0635	0552	0471	0391	0313	0235	0160	0085	0012	50
51	0719	0634	0551	0470	0390	0311	0234	0158	0084	0011	51
52	0717	0633	0550	0468	0388	0310	0233	0157	0083	0010	52
53	0716	0631	0548	0467	0387	0309	0232	0156	0082	0008	53
54	0714	0630	0547	0466	0386	0307	0230	0155	0080	0007	54
55	0713	0628	0546	0464	0384	0306	0229	0153	0079	0006	55
56	0711	0627	0544	0463	0383	0305	0228	0152	0078	0005	56
57	0710	0626	0543	0462	0382	0304	0227	0151	0077	0004	57
58	0709	0624	0541	0460	0381	0302	0225	0150	0075	0002	58
59	0707	0623	0540	0459	0379	0301	0224	0148	0074	0001	59
60	0706	0621	0539	0458	0378	0300	0223	0147	0073	0000	60
	50	51	52	53	54	55	56	57	58	59	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	60	61	62	63	64	65	66	67	68	69	D. M.
M. S.	3600	3660	3720	3780	3840	3900	3960	4020	4080	4140	M. S.
0	1,0000	9928	9858	9788	9720	9652	9586	9521	9456	9393	0
1	9999	9927	9856	9787	9719	9651	9585	9520	9455	9392	1
2	9998	9926	9855	9786	9717	9650	9584	9519	9454	9391	2
3	9996	9925	9854	9785	9716	9649	9583	9518	9453	9390	3
4	9995	9923	9853	9784	9715	9648	9582	9516	9452	9389	4
5	9994	9922	9852	9782	9714	9647	9581	9515	9451	9388	5
6	9993	9921	9851	9781	9713	9646	9579	9514	9450	9387	6
7	9992	9920	9849	9780	9712	9645	9578	9513	9449	9386	7
8	9990	9919	9848	9779	9711	9643	9577	9512	9448	9385	8
9	9989	9918	9847	9778	9710	9642	9576	9511	9447	9384	9
10	9988	9916	9846	9777	9708	9641	9575	9510	9446	9383	10
11	9987	9915	9845	9775	9707	9640	9574	9509	9445	9381	11
12	9986	9914	9844	9774	9706	9639	9573	9508	9444	9380	12
13	9984	9913	9842	9773	9705	9638	9572	9507	9443	9379	13
14	9983	9912	9841	9772	9704	9637	9571	9506	9442	9378	14
15	9982	9910	9840	9771	9703	9636	9570	9505	9440	9377	15
16	9981	9909	9839	9770	9702	9635	9569	9504	9439	9376	16
17	9980	9908	9838	9769	9701	9633	9567	9502	9438	9375	17
18	9978	9907	9837	9767	9699	9632	9566	9501	9437	9374	18
19	9977	9906	9835	9766	9698	9631	9565	9500	9436	9373	19
20	9976	9905	9834	9765	9697	9630	9564	9499	9435	9372	20
21	9975	9903	9833	9764	9696	9629	9563	9498	9434	9371	21
22	9974	9902	9832	9763	9695	9628	9562	9497	9433	9370	22
23	9972	9901	9831	9762	9694	9627	9561	9496	9432	9369	23
24	9971	9900	9830	9761	9693	9626	9560	9495	9431	9368	24
25	9970	9899	9829	9759	9692	9625	9559	9494	9430	9367	25
26	9969	9897	9827	9758	9690	9624	9558	9493	9429	9366	26
27	9968	9896	9826	9757	9689	9622	9557	9492	9428	9365	27
28	9966	9895	9825	9756	9688	9621	9555	9491	9427	9364	28
29	9965	9894	9824	9755	9687	9620	9554	9490	9426	9363	29
30	9964	9893	9823	9754	9686	9619	9553	9488	9425	9362	30
	60	61	62	63	64	65	66	67	68	69	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	60	61	62	63	64	65	66	67	68	69	D. M.
M. S.	3600	3660	3720	3780	3840	3900	3960	4020	4080	4140	M. S.
31	9963	9892	9822	9753	9685	9618	9552	9487	9424	9361	31
32	9962	9890	9820	9751	9684	9617	9551	9486	9422	9360	32
33	9960	9889	9819	9750	9683	9616	9550	9485	9421	9359	33
34	9959	9888	9818	9749	9681	9615	9549	9484	9420	9358	34
35	9958	9887	9817	9748	9680	9614	9548	9483	9419	9356	35
36	9957	9886	9816	9747	9679	9612	9547	9482	9418	9355	36
37	9956	9885	9815	9746	9678	9611	9546	9481	9417	9354	37
38	9954	9883	9813	9745	9677	9610	9545	9480	9416	9353	38
39	9953	9882	9812	9744	9676	9609	9544	9479	9415	9352	39
40	9952	9881	9811	9742	9675	9608	9542	9478	9414	9351	40
41	9951	9880	9810	9741	9674	9607	9541	9477	9413	9350	41
42	9950	9879	9809	9740	9672	9606	9540	9476	9412	9349	42
43	9948	9877	9808	9739	9671	9605	9539	9475	9411	9348	43
44	9947	9876	9807	9738	9670	9604	9538	9473	9410	9347	44
45	9946	9875	9805	9737	9669	9603	9537	9472	9409	9346	45
46	9945	9874	9804	9736	9668	9601	9536	9471	9408	9345	46
47	9944	9873	9803	9734	9667	9600	9535	9470	9407	9344	47
48	9942	9872	9802	9733	9666	9599	9534	9469	9406	9343	48
49	9941	9870	9801	9732	9665	9598	9533	9468	9405	9342	49
50	9940	9869	9800	9731	9664	9597	9532	9467	9404	9341	50
51	9939	9868	9798	9730	9662	9596	9530	9466	9402	9340	51
52	9938	9867	9797	9729	9661	9595	9529	9465	9401	9339	52
53	9937	9866	9796	9728	9660	9594	9528	9464	9400	9338	53
54	9935	9865	9795	9727	9659	9593	9527	9463	9399	9337	54
55	9934	9863	9794	9725	9658	9592	9526	9462	9398	9336	55
56	9933	9862	9793	9724	9657	9590	9525	9461	9397	9335	56
57	9932	9861	9792	9723	9656	9589	9524	9460	9396	9334	57
58	9931	9860	9790	9722	9655	9588	9523	9459	9395	9333	58
59	9929	9859	9789	9721	9653	9587	9522	9457	9394	9332	59
60	9928	9858	9788	9720	9652	9586	9521	9456	9393	9331	60
	60	61	62	63	64	65	66	67	68	69	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	70	71	72	73	74	75	76	77	78	79	D. M.
M. S.	4200	4260	4320	4380	4440	4500	4560	4620	4680	4740	M. S.
0	9331	9269	9208	9148	9089	9031	8973	8917	8861	8805	0
1	9329	9268	9207	9147	9088	9030	8972	8916	8860	8804	1
2	9328	9267	9206	9146	9087	9029	8971	8915	8859	8803	2
3	9327	9266	9205	9145	9086	9028	8971	8914	8858	8802	3
4	9326	9265	9204	9144	9085	9027	8970	8913	8857	8802	4
5	9325	9264	9203	9143	9084	9026	8969	8912	8856	8801	5
6	9324	9263	9202	9142	9083	9025	8968	8911	8855	8800	6
7	9323	9262	9201	9141	9082	9024	8967	8910	8854	8799	7
8	9322	9261	9200	9140	9081	9023	8966	8909	8853	8798	8
9	9321	9260	9199	9139	9080	9022	8965	8908	8852	8797	9
10	9320	9259	9198	9138	9079	9021	8964	8907	8851	8796	10
11	9319	9258	9197	9137	9078	9020	8963	8906	8850	8795	11
12	9318	9257	9196	9136	9077	9019	8962	8905	8849	8794	12
13	9317	9256	9195	9135	9076	9018	8961	8904	8849	8793	13
14	9316	9255	9194	9134	9076	9017	8960	8903	8848	8792	14
15	9315	9254	9193	9133	9075	9016	8959	8903	8847	8792	15
16	9314	9253	9192	9132	9074	9015	8958	8902	8846	8791	16
17	9313	9252	9191	9131	9073	9015	8957	8901	8845	8790	17
18	9312	9251	9190	9130	9072	9014	8956	8900	8844	8789	18
19	9311	9250	9189	9129	9071	9013	8955	8899	8843	8788	19
20	9310	9249	9188	9128	9070	9012	8954	8898	8842	8787	20
21	9309	9248	9187	9128	9069	9011	8953	8897	8841	8786	21
22	9308	9247	9186	9127	9068	9010	8952	8896	8840	8785	22
23	9307	9246	9185	9126	9067	9009	8952	8895	8839	8784	23
24	9306	9245	9184	9125	9066	9008	8951	8894	8838	8783	24
25	9305	9244	9183	9124	9065	9007	8950	8893	8837	8782	25
26	9304	9243	9182	9123	9064	9006	8949	8892	8837	8781	26
27	9303	9241	9181	9122	9063	9005	8948	8891	8836	8781	27
28	9302	9240	9180	9121	9062	9004	8947	8890	8835	8780	28
29	9301	9239	9179	9120	9061	9003	8946	8889	8834	8779	29
30	9300	9238	9178	9119	9060	9002	8945	8888	8833	8778	30
	70	71	72	73	74	75	76	77	78	79	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	70	71	72	73	74	75	76	77	78	79	D. M.
M. S.	1800	4260	4320	4380	4440	4500	4560	4620	4680	4740	M. S.
31	9299	9237	9177	9118	9059	9001	8944	8888	8832	8777	31
32	9298	9236	9176	9117	9058	9000	8943	8887	8831	8776	32
33	9297	9235	9175	9116	9057	8999	8942	8886	8830	8775	33
34	9296	9234	9174	9115	9056	8998	8941	8885	8829	8774	34
35	9294	9233	9173	9114	9055	8997	8940	8884	8828	8773	35
36	9293	9232	9172	9113	9054	8996	8939	8883	8827	8772	36
37	9292	9231	9171	9112	9053	8995	8938	8882	8826	8771	37
38	9291	9230	9170	9111	9052	8994	8937	8881	8825	8771	38
39	9290	9229	9169	9110	9051	8993	8936	8880	8825	8770	39
40	9289	9228	9168	9109	9050	8992	8935	8879	8824	8769	40
41	9288	9227	9167	9108	9049	8992	8935	8878	8823	8768	41
42	9287	9226	9166	9107	9048	8991	8934	8877	8822	8767	42
43	9286	9225	9165	9106	9047	8990	8933	8876	8821	8766	43
44	9285	9224	9164	9105	9046	8989	8932	8875	8820	8765	44
45	9284	9223	9163	9104	9045	8988	8931	8875	8819	8764	45
46	9283	9222	9162	9103	9044	8987	8930	8874	8818	8763	46
47	9282	9221	9161	9102	9043	8986	8929	8873	8817	8762	47
48	9281	9220	9160	9101	9042	8985	8928	8872	8816	8761	48
49	9280	9219	9159	9100	9042	8984	8927	8871	8815	8761	49
50	9279	9218	9158	9099	9041	8983	8926	8870	8814	8760	50
51	9278	9217	9157	9098	9040	8982	8925	8869	8813	8759	51
52	9277	9216	9156	9097	9039	8981	8924	8868	8813	8758	52
53	9276	9215	9155	9096	9038	8980	8923	8867	8812	8757	53
54	9275	9214	9154	9095	9037	8979	8922	8866	8811	8756	54
55	9274	9213	9153	9094	9036	8978	8921	8865	8810	8755	55
56	9273	9212	9152	9093	9035	8977	8920	8864	8809	8754	56
57	9272	9211	9151	9092	9034	8976	8919	8863	8808	8753	57
58	9271	9210	9150	9091	9033	8975	8918	8862	8807	8752	58
59	9270	9209	9149	9090	9032	8974	8918	8861	8806	8752	59
60	9269	9208	9148	9089	9031	8973	8917	8861	8805	8751	60
	70	71	72	73	74	75	76	77	78	79	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	80	81	82	83	84	85	86	87	88	89	D. M.
M. S.	4800	4860	4920	4980	5040	5100	5160	5220	5280	5340	M. S.
0	8751	8697	8644	8591	8539	8488	8437	8387	8337	8288	0
1	8750	8696	8643	8590	8538	8487	8436	8386	8336	8287	1
2	8749	8695	8642	8589	8537	8486	8435	8385	8335	8286	2
3	8748	8694	8641	8589	8537	8485	8434	8384	8335	8286	3
4	8747	8693	8640	8588	8536	8484	8433	8383	8334	8285	4
5	8746	8692	8639	8587	8535	8484	8433	8383	8333	8284	5
6	8745	8692	8638	8586	8534	8483	8432	8382	8332	8283	6
7	8745	8691	8638	8585	8533	8482	8431	8381	8331	8282	7
8	8744	8690	8637	8584	8532	8481	8430	8380	8330	8281	8
9	8743	8689	8636	8583	8531	8480	8429	8379	8330	8281	9
10	8742	8688	8635	8582	8530	8479	8428	8379	8329	8280	10
11	8741	8687	8634	8582	8530	8478	8428	8378	8328	8279	11
12	8740	8686	8633	8581	8529	8477	8427	8377	8327	8278	12
13	8739	8685	8632	8580	8528	8477	8426	8376	8326	8278	13
14	8738	8684	8631	8579	8527	8476	8425	8375	8325	8277	14
15	8737	8684	8631	8578	8526	8475	8424	8374	8325	8276	15
16	8736	8683	8630	8577	8525	8474	8423	8373	8324	8275	16
17	8736	8682	8629	8576	8524	8473	8423	8373	8323	8274	17
18	8735	8681	8628	8575	8524	8472	8422	8372	8322	8273	18
19	8734	8680	8627	8575	8523	8472	8421	8371	8322	8273	19
20	8733	8679	8626	8574	8522	8471	8420	8370	8321	8272	20
21	8732	8678	8625	8573	8521	8470	8419	8369	8320	8271	21
22	8731	8677	8624	8572	8520	8469	8418	8368	8319	8270	22
23	8730	8677	8624	8571	8519	8468	8418	8368	8318	8269	23
24	8729	8676	8623	8570	8518	8467	8417	8367	8317	8268	24
25	8728	8675	8622	8569	8518	8467	8416	8366	8317	8268	25
26	8727	8674	8621	8568	8517	8466	8415	8365	8316	8267	26
27	8727	8673	8620	8568	8516	8465	8414	8364	8315	8266	27
28	8726	8672	8619	8567	8515	8464	8413	8363	8314	8265	28
29	8725	8671	8618	8566	8514	8463	8413	8363	8313	8265	29
30	8724	8670	8617	8565	8513	8462	8412	8362	8312	8264	30
	80	81	82	83	84	85	86	87	88	89	

TABLE XLIX. *Continued.*

Logistic Logarithms.

D. M.	80	81	82	83	84	85	86	87	88	89	D. M.
M. S.	4800	4860	4920	4980	5040	5100	5160	5220	5280	5340	M. S.
31	8723	8669	8617	8564	8513	8461	8411	8361	8312	8263	31
32	8722	8668	8616	8563	8512	8460	8410	8360	8311	8262	32
33	8721	8668	8615	8562	8511	8460	8409	8359	8310	8261	33
34	8720	8667	8614	8561	8510	8459	8408	8358	8309	8260	34
35	8719	8666	8613	8561	8509	8458	8408	8358	8308	8260	35
36	8718	8665	8612	8560	8508	8457	8407	8357	8307	8259	36
37	8718	8664	8611	8559	8507	8456	8406	8356	8307	8258	37
38	8717	8663	8610	8558	8506	8455	8405	8355	8306	8257	38
39	8716	8662	8610	8557	8506	8455	8404	8354	8305	8257	39
40	8715	8661	8609	8556	8505	8454	8403	8353	8304	8256	40
41	8714	8661	8608	8556	8504	8453	8403	8353	8304	8255	41
42	8713	8660	8607	8555	8503	8452	8402	8352	8303	8254	42
43	8712	8659	8606	8554	8502	8451	8401	8351	8302	8253	43
44	8711	8658	8605	8553	8501	8450	8400	8350	8301	8252	44
45	8710	8657	8604	8552	8501	8450	8399	8350	8300	8252	45
46	8709	8656	8603	8551	8500	8449	8398	8349	8299	8251	46
47	8709	8655	8603	8550	8499	8448	8398	8348	8299	8250	47
48	8708	8654	8602	8549	8498	8447	8397	8347	8298	8249	48
49	8707	8654	8601	8549	8497	8146	8396	8346	8297	8248	49
50	8706	8653	8600	8548	8496	8445	8395	8345	8296	8247	50
51	8705	8652	8599	8547	8495	8445	8394	8345	8295	8247	51
52	8704	8651	8598	8546	8494	8444	8393	8344	8294	8246	52
53	8703	8650	8597	8545	8494	8443	8392	8343	8294	8245	53
54	8702	8649	8596	8544	8493	8442	8392	8342	8293	8244	54
55	8702	8648	8596	8543	8492	8441	8391	8341	8292	8244	55
56	8701	8647	8595	8542	8491	8440	8390	8340	8291	8243	56
57	8700	8646	8594	8542	8490	8440	8389	8340	8291	8242	57
58	8699	8645	8593	8541	8489	8439	8388	8339	8290	8241	58
59	8698	8645	8592	8540	8489	8438	8388	8338	8289	8240	59
60	8697	8644	8591	8539	8488	8437	8387	8337	8288	8239	60
	80	81	82	83	84	85	86	87	88	89	

THE USE OF THE TABLES.

TABLE I.

For converting Degrees, Minutes and Seconds into Time.

Rule. Take the degrees, minutes and seconds from the first, third, and fifth columns, and against them you have the corresponding times, the sum of which is the time required.

Example. Reduce 74°. 39′. 57″ into time.

70°	- - -	4h. 40′. 0″
4°	- - -	0. 16. 0
30′	- - -	0. 2. 0
9′	- - -	0. 0. 36
50″	- - -	0. 0. 3,333
7″	- - -	0. 0. 0,466
Time required	-	4. 58. 39,799

Or thus. Multiply by 4; then the seconds produce thirds; the minutes produce seconds, and the degrees produce minutes.

74°. 39′. 57″
4

4h. 58′. 39″. 48″

4

TABLE II.

For converting Time into Degrees, Minutes and Seconds.

Rule. Take the time from the first, third and fifth columns, and against them you have the degrees, minutes and seconds corresponding, the sum of which is the quantity required.

Example. Reduce 17*h*. 34′. 19″ into degrees, minutes and seconds.

10*h*.	- - - -	150°. 0′. 0″
7*h*.	- - - -	105. 0. 0
30′	- - - -	7. 30. 0
4′	- - - -	1. 0. 0
10″	- - - -	0. 2. 30
9″	- - - -	0. 2. 15
Degrees required	- -	263. 34. 45

Or thus. Bring the hours into minutes, and divide by 4; then the minutes give degrees, the seconds give minutes, and the remainder give seconds.

4)1054′. 19″. 0″

263°. 34′. 45″

TABLE III.

For converting Minutes and Seconds into the Decimal of an Hour.

Rule. Take the time from the first and third columns, and against them you have the corresponding decimals, the sum of which is the decimal required.

EXAMPLE. What is the decimal of 19′. 47″?

10′	- - - - - -	,16666
9′	- - - - - -	,15000
40″	- - - - - -	,01111
7″	- - - - - -	,00194
Decimal required	- -	,32971

TABLE IV.

For finding the Length of circular Arcs to Radius Unity.

RULE. Take the degrees, minutes and seconds from the first, third and fifth columns, and against them you have the corresponding lengths, the sum of which is the length required.

EXAMPLE. What is the length of an arc of 37°. 42′. 58″?

30°	- - - - - -	0,5235988
7°	- - - - - -	0,1221730
40′	- - - - - -	0,0116355
2′	- - - - - -	0,0005818
50″	- - - - - -	0,0002424
8″	- - - - - -	0,0000388
Length required	- -	0,6582703

If the radius be not unity, the length may be found by proportion, by saying, unity : radius :: length here found : the length required.

TABLE V.

For finding the Sun's Parallax in Altitude, the apparent Altitude being given.

RULE. Find the altitude under the column *Sun's Alt.* and against it you have the parallax. If the apparent altitude be not found in the Table, the parallax must be found by proportion.

EXAMPLE. What is the parallax at the apparent altitude 47°. 27'. 20"?

At 40° app. alt. the parallax is	6",70
— 50 ————————	5, 62
10	1, 08

Hence, 10 : 7°. 27'. 20":: 1",08 : 0",8, which subtracted from 6",7 leaves 5",9 the parallax. Hence, the altitude, corrected for parallax, is 47°. 27'. 25",9.

TABLE VI.

Contains the mean Right Ascensions and North polar Distances of 36 *principal fixed Stars, for the Beginning of* 1802; *together with their annual Precessions, and proper Motions; all as settled by Dr.* MASKELYNE; *thence to deduce their places for any other Year.*

RULE. Multiply the annual precession by the number of years between the given year and 1802, and you get the annual precession for that interval. Then if the given year be *after* 1802, *add* the annual precession in right ascension for that interval to the right ascension for 1802, and you get the mean right ascension for the beginning of the given year; and apply the annual precession in north polar distance for the interval, according to the sign, to the north polar distance for 1802, and you get the north polar distance at the beginning of the given year. But if the given year be *before* 1802, *subtract* the annual precession in right ascension, and apply the annual precession of north polar distance with a *contrary* sign.

EXAMPLE. What is the mean right ascension and north polar distance of *Sirius*, at the beginning of the year 1813?

Mean right ascension for 1802 - -	6^h. 36'. 25",45
Motion in precession for 11 years - -	+ 29, 48
Mean right ascension for 1813 - -	6. 36. 54, 93

North polar distance for 1802	- -	106°. 27′. 4″,7
Motion in precession for 11 years	-	+ 34, 87
North polar distance for 1813	- -	106. 27. 39, 57

TABLE VII.

Contains the Sum of the Precession, Aberration, and Solar Inequality of Precession from the beginning of the Year, of 36 principal Stars, in sidereal Time; *being one Part of the Correction of the mean Right Ascension from the beginning of the Year.*

Rule. Against the day of the month, under the given star, you have the correction required. If the day of the month be not found in the Table, the equation must be found by proportion.

Example. What is the equation for *Sirius* on April 27?

April 20	- - -	the equation	+ 0″,23
—— 30	- - -	————	+ 0, 09
10			0, 14

Hence, 10 : 7 :: 0″,14 : 0″,1 which subtracted from + 0″,23 gives + 0″,13 the correction required.

TABLE VIII.

Contains the Equation of the Equinoxes, and Deviation of the same Stars as in the last Table, in sidereal Time; *being the other Part of the Correction in Right Ascension from the beginning of the Year.*

Rule. Enter the first column with the longitude of the moon's ascending node (Tab. XXIX, XXX.), this equation depending upon the place of the node; and against it, under the given star, you have the equation required. If the longitude be not found in the Table, the equation must be found by proportion.

Example. What is the equation of *Sirius* on April 10, 1813?

The longitude of the moon's ascending node is 4^s. 16°. 31′.

Long. of ☽'s ☊	4^s. 10°	- - -	Equation	−0″,30
———	4. 20.	- - -	———	−0, 13
	0. 10			0, 17

Hence, 10°. : 6°. 31′ :: 0″,17 : 0″,11, which subtracted from −0″,30 gives −0″,19 the correction required.

From this and the last example, we find the whole correction of *Sirius* on April 10, in the year 1813, to be +0″,13 − 0″,19 = −0″,06; but the mean right ascension of *Sirius* at the beginning of 1800, we have found to be 6*h*. 36′. 54″,93; hence the true right ascension of *Sirius* on April 10, 1813, is 6*h*. 36′. 54″,87.

TABLE IX.

Contains that Part of the Equation of the Obliquity of the Ecliptic, which arises from the unequal Force of the Sun in causing the Precession of the Equinoxes; and therefore depending upon the Sun, it must be the same every Year.

Rule. Take the day of the month, and against it you have the equation required. If the given day of the month be not found in the Table, the equation must be found by proportion.

Example. What is the equation on May 24?

May	19	- - -	Equation	−0″,4
———	26	- - -	———	−0, 5
	7			0, 1

Hence, 7 : 5 :: 0″,1 : 0″,07, which added to −0″,4 gives −0″,47 the equation.

TABLE X.

Contains that Part of the Equation of the Obliquity of the Ecliptic, which arises from the unequal Force of the Moon in causing the Precession of the Equinoxes, and which depends upon the Longitude of the Moon's ascending Node.

Rule. Enter the Table with the longitude of the moon's ascending node (Tab. XXIX. XXX.), and against it you have the equation required. If the longitude be not found in the Table, the equation must be found by proportion.

Example. What is the equation on May 24, 1798?

The longitude of the moon's node is $2^s.\ 4^\circ.\ 17'$.

Long. of ☽'s ☊	$2^s.\ 4^\circ$	Equation	$+4'',2$
———	$2.\ 5$	———	$+4,0$
	$0.\ 1$		$0,2$

Hence, $1^\circ. : 17'. :: 0'',2 : 0'',06$, which subtracted from $4'',2$ leaves $+4'',14$ the equation required.

Hence, from this and the last example, the whole equation of the obliquity of the ecliptic on May 24, 1798, is $+3'',67$.

TABLE XI.

Contains the mean *Refraction of the heavenly Bodies, corresponding to their* apparent *Zenith Distances.*

Rule. Find the apparent zenith distance, and against it you have the mean refraction. If the distance be not found in the Table, the refraction must be found by proportion.

Example. What is the mean refraction at the apparent zenith distance 71°. 15'. 48"?

At 71°. 10' zen. dist.	- -	Refraction 2'. 45",83
—— 71. 20 ——	- -	—— 2'. 47",39
—— 10		1",56

Hence, 10° : 5'. 48" :: 1",56 : 0",9, which added to 2'. 45",83 gives 2'. 46",73 the refraction required.

This Table (given by Dr. Maskelyne with his Observations, 1796) is constructed to give the refraction when the barometer stands at 29,6 inches, and the thermometer at 50 degrees; the next Table is to correct this, for any variation of the barometer and thermometer from these altitudes.

TABLE XII.

Contains Decimals, which multiplied into the mean Refraction, gives the Correction for the Variation of the Weight and Temperature of the Air.

Rule. Find, in the upper horizontal line, the height of the barometer, and in the first perpendicular line the height of the thermometer, and corresponding to them you have the decimal required; which multiplied into the mean refraction, and applied to the mean refraction according to the sign, gives the true refraction. If the altitudes be not found in the Table, the decimal must be found by proportion.

Example. What is the true refraction at the apparent zenith distance 71°. 15'. 48", the barometer standing at 30,35 inches, and the thermometer at 61,8 inches?

Against 61° under 30,3 inches	- -	Dec. —,004
—————— 30,4 ———	- -	——.—,001
0,1		,003

Hence, 0,1 : 0,05 :: ,003 : 0015, the quantity by which ,004 must be decreased because the decimals decrease.

Against 61° under 30,3 inches	- -	Dec. – ,004
——— 62 ———————	- -	Dec. – ,006
1		,002

Hence, 1 : 0,8 :: ,002 : ,0016, the quantity by which ,004 must be increased, because the decimals increase; consequently the decimal corresponding to the given altitudes of the barometer and thermometer is —,0041. Therefore the correction is –,0041 × 2′. 46″,8 (the mean refraction by the last example) = –,68; hence, the true refraction is 2′. 46″,8 – ,68 = 2′. 46″,12.

TABLE XIII.

Contains the Augmentation of the horizontal Diameter of the Moon for any apparent Altitude.

RULE. Enter the horizontal line at the top with the moon's horizontal diameter, and the first perpendicular line with the apparent altitude, and against them you have the augmentation required. If the diameter be not found in the Table, the augmentation must be found by proportion.

EXAMPLE. If the horizontal diameter of the moon be 29′. 38″, and the apparent altitude 56°. 32′, what is its diameter?

At 29′. 30″ diameter, and 56° app. alt.	- -	augment. 23″,4
29. 50 ———————————	- -	——— 23, 9
0. 20		0, 5

Hence, 20″ : 8″ :: 0″,5 : 0″,2 the increase for 8″.

At 29′. 30″ diameter, and 56° app. alt.	- -	augment. 23″,4
——————————— 57 ———	- -	——— 23, 6
1		0, 2

Hence, 1° : 32′ :: 0″,2 : 0″,1 the increase for 32′. Therefore 23″,4 + 0″,2 + 0″,1 = 23″,7 the augmentation of the diameter. Hence, 29′. 38″ + 23″,7 = 30′. 1″,7 the diameter required.

TABLE XIV.

Contains the mean *Precession of the Equinoctial Points for Years; in which you have the Number of Years in one Column, and the Precession in another against them.*

Rule. If the number of years be found in the Table, you have the precession required. But if the number of years be not in the Table, divide it into such parts as can be found.

Example. What is the mean precession in 3247 years?

Years.		
3000	Preces.	41°. 57′. 29″,0
200		2. 47. 49, 9
47		0. 39. 26, 4
Mean precession		45. 24. 45, 3

TABLE XV.

Contains the mean *Precession for every Day of the Year, including the solar Equation.*

Rule. Enter with the month above, and the day of the month on the side, and against them you have the precession; observing the rule for leap-year.

Example. The precession for January 19, in the year 1796 (being leap-year), is 3″,4.

3

TABLE XVI.

Contains the Equation of the Equinoxes in Longitude.

RULE. Enter with the longitude of the moon's ascending node (TAB. XXIX. XXX.), the signs above or below, and the degrees on one side, and corresponding thereto you have the equation, to be applied according to the sign.

EXAMPLE. What is the equation on July 19, 1796?

The place of the moon's ascending node is $3^s.\ 9^\circ.\ 59'$, against the nearest place to which you find $-17'',6$ the equation required.

The mean longitude of a star, settled to some epoch, is to be corrected by the mean precession and equation of the equinoxes; and if to this we apply the correction of aberration, we get the true longitude.

TABLE XVII.

Contains the mean *Motion of the Sun in Right Ascension to every Day in the Year.*

RULE. Enter with the month at the head, and day of the month at the side, and corresponding to them you have the mean motion.

EXAMPLE. What is the mean motion of the sun in right ascension on February 16, 1796?

This being leap-year, we must take out for the preceding day; hence, the mean motion in right ascension is $3h.\ 1'.\ 21'',5$.

TABLE XVIII.

Contains the mean *Motion of the Sun in Right Ascension, corresponding to* Sidereal *Time, for Hours and Minutes.*

RULE. Enter the columns under sidereal time with the hours and minutes, and against them you have the mean motion of the sun in right ascension.

EXAMPLE. What is the mean motion in right ascension for 17*h*. 48′, sidereal time?

17*h*.	2′. 47″,10
48′	0. 7,86
Mean motion in right ascension .	2. 54,96

TABLE XIX.

Contains the Equation of the Equinoxes, in Degrees, *in Right Ascension.*

RULE. Enter the Table with the longitude of the moon's ascending node (TAB. XXIX. XXX.), the signs at the top or bottom, and the degrees on the side, and corresponding to them you have the equation required.

EXAMPLE. What is the equation of the equinoxes in right ascension on July 19, 1796?

The place of the moon's ascending node is 3^s. 9°. 59′, against the nearest place to which you find $-16''$,1 the equation required.

TABLE XX.

Is the same as the last, only the quantity is expressed in Time.

The uniform motion of the equinoctial points being disturbed by the nutation of the earth's axis, the true equinox will differ from the equinox computed according to the mean motion, by which the true right ascension of all the stars will be affected. The two last Tables exhibit only part of this effect, except for those stars which are in the equator; the other part, called the *Deviation* (1043), is found from the next Table.

TABLE XXI.

Is to find the Deviation *of a Star in North Polar Distance, and in Right Ascension.*

For the Deviation in North Polar Distance.

RULE. Enter with the right ascension at the top, and the longitude of the moon's node in the first column on the left, and against them you have the deviation. If the right ascension be not found in the Table, the deviation must be found by proportion.

EXAMPLE. What is the deviation of a star in north polar distance, its right ascension being 2ˢ. 27°. 30′. 19″, and the longitude of the moon's node 3ˢ. 7°. 40′?

For *A.R.* 2ˢ. 25° and long. ☽'s ☊ 3ˢ. 5° the deviation is + 1″,45
———— 3. 0 ———————————— +0, 83

0. 5 — 0, 62

Hence, 5° : 2°. 30′ :: 0″,62 : 0″,31 to be subtracted, as the deviation decreases.

For *A.R.* 2ˢ. 25° and long. ☽'s ☊ 3ˢ. 5° the deviation is + 1″,45
———————————— 3. 10 ———— + 2, 26

0. 5 — 0, 81

Hence, 5° : 2°. 40′ :: 0″,81 : 0″,43 to be added, as the deviation increases; therefore the deviation in north polar distance is + 1″,45 − 0″,31 + 0″,43 = 1″,57.

For the Deviation in Right Ascension.

RULE. *Add* 3 signs to the star's right ascension, if its *declination* be *north*, or *subtract*, if *south*; and with this, as a new right ascension, find the equation from the Table as before, and multiply it by the tangent of the declination;

and if the star's right ascension, thus corrected by 3 signs, and the longitude of the moon's node, are both more, or both less than 6 signs, use the algebraic sign of the Table; but if one be more, and the other less than 6 signs, change the sign of the Table.

EXAMPLE. If the right ascension of a star be 2ˢ. 27°. 30′. 19″, and declination 37°. 28′. 30″ N. what is its deviation in right ascension, the longitude of the moon's node being 3ˢ. 7°. 40′?

The star's right ascension, increased by 3 signs, becomes 5ˢ. 27°. 30′. 19″.

For 5ˢ. 25° *A. R.* and long. ☽'s ☊ 3ˢ. 5° the deviation is		−6″,99
6. 0		−7,09
0. 5		0,10

Hence, 5° : 2°. 30′. 19″ :: 0″,1 : 0″,05, which is to be added to 6″,99, as the deviation is increasing.

For 5ˢ. 25° *A. R.* and long. ☽'s ☊	3ˢ. 5° the deviation is	−6″,99
	3. 10	−6,83
	0. 5	0,16

Hence, 5° : 2°. 40′ :: 0″,16 : 0″,09, which is to be subtracted from 6″,99; therefore −6″,99 − 0″,05 + 0″,09 = −6″,95; and this multiplied by 0,766 (tang. of dec.) gives −5″,32 the deviation in right ascension.

This Table Dr. MASKELYNE gave with his Observations for 1796.

TABLE XXII.

Contains the Time in which Light moves over any Part or Multiple of the mean Radius of the Earth's Orbit, supposed to be Unity.

RULE. Enter with the given distance in the column of *Parts of Orbis Magnus*, and against it you have the time.

Example. How long will light be in moving over 0,784 of the earth's radius?

,78	- - - -	6'. 19'',9
,004	- - -	0. 1,9
,784	- - - -	6. 21,8 Time.

TABLE XXIII.

Contains the Nonagesimal Degree of the Ecliptic, and its Altitude, for the Latitude of Greenwich, reduced (173) *to the Earth's Center, supposed to be* 51°. 14'. 7'', *for the Obliquity of the Ecliptic* 23°. 28'.

Rule. Enter with the right ascension of the meridian, and against it you have the nonagesimal degree, and its altitude. If the right ascension be not found in the Table, the required quantities must be found by proportion.

Example. If the right ascension of the meridian be 37°. 17'. 30'', what is the nonagesimal degree, and its altitude?

A. R. Mer. 37°	- -	nonag. deg.	1ˢ. 22°. 41'. 20''
——— 38°	- -	———	1. 23. 23. 20
1			42. 0

Hence, 1°. : 17'. 30'' :: 42' : 12'. 15'', which added to 1ˢ. 22°. 41'. 20'' gives 1ˢ. 22°. 53'. 35'' the nonagesimal degree.

A. R. Mer. 37°	- -	alt. nonag.	55°. 35'. 0''
38	- -	———	55. 49. 30
1			14. 30

Hence, 1° : 17′. 30″ :: 14′. 30″ : 4′. 14″, which added to 55°. 35′. 0″, gives 55°. 39′. 14″ the altitude of the nonagesimal degree.

TABLE XXIV.

Contains the Correction to be applied to the last Table, for the Latitude of one Degree north of Greenwich.

Rule. Enter with the right ascension of the mid-heaven, and you have the corrections required. If the right ascension be not in the Table, the corrections must be found by proportion. If the latitude be not one degree from that of Greenwich, change the quantities found in proportion.

Example. In the last example, what is the nonagesimal degree, and its altitude, for a place 20′ north of Greenwich?

A. R. mid-heav. 30°,	the cor. for 1° are - -	31′,4 - -	54′,3
———— 40,	———— - -	26,0 - -	55,9
10		5,4	1,6

Hence, 10° : 7°. 17′. 30″ :: $\left\{\begin{matrix}5',4\\1,6\end{matrix}\right\}$: $\left\{\begin{matrix}3',94\\1,17\end{matrix}\right\}$ the correction for the nonagesimal degree and altitude, for 1° change of latitude; therefore

$$1^\circ : 20' :: \left\{\begin{matrix}3',94\\1,17\end{matrix}\right\} : \left\{\begin{matrix}1',31\\0,39\end{matrix}\right\}$$

the respective corrections for 20′ change of latitude; hence, $31',4 - 1',31 = 30',09 = 30'.\ 5''$ the correction for the nonagesimal degree; and $54',3 + 0',39 = 54',69 = 54'.\ 41''$ the correction for the altitude. Therefore $1^s.\ 22^\circ.\ 53'.\ 35'' + 30'.\ 5'' = 1^s.\ 23^\circ.\ 23'.\ 40''$ the nonagesimal degree; and $55^\circ.\ 39'.\ 14'' - 54'.\ 41'' = 54^\circ.\ 44'.\ 33''$ its altitude.

If the place be to the south of Greenwich, apply the corrections with a contrary sign.

The calculation of parallaxes by the nonagesimal degree and its altitude, being generally used in computing solar eclipses, and occultations of the fixed stars and planets by the moon, these two Tables will be very useful for that purpose.

TABLE XXV.

Contains the Angle between the Ecliptic and a Parallel to the Equator, to the Obliquity 23°. 28′; with the Variation for 10″ Variation of the Obliquity.

Rule. Enter with the sun's declination, and you have the required angle. If the declination be not found in the Table, you must find the angle by proportion.

Example. What is the angle between the ecliptic and a parallel to the equator, when the sun's declination is 17°. 30′, and the obliquity 23°. 27′. 55″?

17°. 20′	declin.	- -	the angle is 16°. 4′. 8″
17. 40	——	- -	—— 15. 41. 58
20			22. 10

Hence, 20′ : 10′ :: 22′. 10″ : 11′. 5″, which subtracted from 16°. 4′. 8″ leaves 15°. 53′. 3″ for the angle at an obliquity 23°. 28′. Now the variation at this point for 10″ of variation of the obliquity is 15″,2; hence, the variation for 5″ is 7″,6, which subtracted from 15°. 53′. 3″ leaves 15°. 52′. 55″,4 for the angle required. If the obliquity had been taken greater than 23°. 28′, the correction must have been added.

TABLE XXVI.

Contains the Angle of Position of any Point of the Ecliptic, according to the Obliquity 23°. 28′; with the Variation for the Variation of the Ecliptic by one Minute.

Rule. Enter with the longitude of the given point of the ecliptic, and you have the angle required. If the longitude be not found in the Table, the angle must be found by proportion.

angle of position be not found in the Table, the correction must be found by proportion.

EXAMPLE. In the last example, suppose the star's longitude to be the same as the longitude of the point of the ecliptic there given, and its latitude 3°. 20′; to find the angle of position.

8° angle posit. and	3°. 0′ lat.	- correct. is	0′. 39″,7
———	3. 30 —	- ———	0. 54, 2
	30		14, 5

Hence, 30′ : 20′ :: 14″,5 : 9″,8 the first part of the correction, to be added to 0′. 39″,7.

8°	angle posit. and 3° lat.	- - correct. is	0′. 39″,7
9	———	- - ———	0. 44, 8
1			5, 1

Here, 1° : 14′. 47″,8 :: 5″,1 : 1″,3 the second part of the correction to be added to 0′. 39″,7; hence, the angle of position is 8°. 14′. 47″,8 + 0′. 39″,7 + 9″,8 + 1″,3 = 8°. 15′. 38″,6.

The four last Tables are of use in calculating the parallaxes in solar eclipses, and occultations of fixed stars and planets by the moon, by the method of the parallactic angle.

TABLE XXIX.

Contains the Epochs of the mean Longitude of the Moon's ascending Node, for the Beginning of each Year.

RULE. Enter with the epoch, and against it you have the mean longitude.

The epoch of the mean longitude of the moon's ascending node for the beginning of 1784, is 11^s. 12°. 40′; being the longitude of the node at that time, if its motion had been uniform. The epoch is put down to the nearest minute, that being sufficiently accurate for the purposes here wanted.

TABLE XXX.

Contains the mean retrograde Motion of the Node, for every Day of the Year.

Rule. Enter with the month at the top, and day of the month at the side, and against them you have the motion required; which being subtracted from the epoch for the year, gives the mean place of the node sufficiently accurate for the purpose of taking out all the equations relating to the nutation of the earth's axis.

Example. To find the mean place of the node on November 26, 1799.

The epoch for 1799 is	-	-	1^s. 22°. 35′
Ret. mot. for Nov. 26,	-	-	17. 28
Mean place of the node	-	-	1. 5. 7

TABLE XXXI.

Shows the Decrease, by Refraction, of the Diameters of the Sun or Moon which are inclined to the Horizon, upon supposition that the apparent Diameter is 30′. If the Diameter be not 30′, the Decrease will vary accordingly.

Rule. Enter with the sun's or moon's altitude at the top, and with the inclination of the diameter at the side, making proportion if you do not find the exact quantities, and you get the decrease for 30′ diameter; and the decrease for any other diameter will be in proportion to the diameter.

EXAMPLE. If the apparent diameter of the moon be 32′, what is the decrease of that diameter which is inclined 32° to the horizon, at an altitude of 17°.

At 16° alt. at an inclin.	30°,	the decrease is	1″,6
———	33	———	1, 9
	3		0, 3

Hence, 3° : 2° :: 0″,3 : 0″,2; therefore 1″,6 + 0″,2 = 1″,8 is the decrease for the inclination 32°, and altitude 16°.

At 16° alt. at an inclin.	30°,	the decrease is	1″,6
— 18 ———	,	———	1, 3
2			0, 3

Hence, 2° : 1° :: 0″,3 : 0″,15; therefore the whole decrease is 1″,8 − 0″,15 = 1″,65 the diminution for a diameter of 30′; hence, 30′ : 32′ :: 1″,65 : 1″,76 the decrease required.

TABLE XXXII.

Is for reducing sidereal to mean solar Time.

RULE. Subtract the numbers found in the Table corresponding to the given sidereal time, from that time, and it reduces it to mean solar time.

EXAMPLE. Reduce 17*h*. 19′. 23″ sidereal time, into mean solar time.

17*h*. - - - - - -	2′. 47″,10
19′ - - - - - -	0. 3, 11
20″ - - - - - -	0. 0, 05
3″ - - - - - -	0. 0, 01
	2. 50, 27
	17h. 19. 23
Mean solar time - -	17. 16. 32, 73

Hence, 1° : 20′ :: 0″,27 : 0″,09; therefore the semidiameter = 15′. 57″,23 − 0″,09 = 15′. 57″,14.

Mean anom.	9ˢ. 14°	gives the horary motion	2′. 26″,56
————	9. 15	————————	2. 26, 47
	1		0. 0, 09

Hence, 1° : 20′ :: 0″,09 : 0″,03; therefore the horary motion is 2′. 26″,56 − 0″,03 = 2′. 26″,53.

TABLE XXXV.

Contains the Reduction of the Ecliptic to the Equator, or the Quantity to be applied to the Longitude of any Point of the Ecliptic, to give the Right Ascension. This Table is for the Obliquity 23°. 28′. 15″; but there is the Correction to be applied for the Variation of the Obliquity of one Minute.

Rule. With the given longitude enter the Table, and take out the reduction; and if the longitude be not found, take the nearest less (L), with the difference, and the variation. Then proportion for the difference between that and the given longitude, the difference taken out answering to a variation of 30′ of longitude, and apply the result to L. Also, 60″ : var. :: the difference between 23°. 28′. 15″ and the given obliquity : a fourth number, which *added* to the reduction above found, or *subtracted* from it, according as the obliquity is *greater* or *less* than 23°. 28′. 15″, you have the true reduction, to be applied according to the sign.

Example. Let the sun's longitude be 1ˢ. 11°. 25′. 10″,3, and the obliquity 23°. 28′. 7″; to find the right ascension.

For 1ˢ. 11° long. - -	Reduction	2°. 25′. 56″,6
	Difference	26, 6
	Variation	12, 7 -

Hence, 30′ : 25′. 10″,3 :: 26″,6 : 22″,3, which *added* to 2°. 25′. 56″,6 (as the reduction is *increasing*) gives 2°. 26′. 18″,9.

4

Again, 60″ : 12″,7 :: 8″ : 1″,7, which subtracted from 2°. 26′. 18″,9 leaves 2°. 26′. 17″,2 the true reduction. Hence, 1ˢ. 11°. 25′. 10″,3 – 2°. 26′. 17″,2 = 1ˢ. 8°. 58′. 53″,1 the true right ascension.

By means of this Table, we may find the longitude from the right ascension, by the following

Rule. Increase the right ascension by 3 signs, and find the reduction in the very same manner as above, and apply it to the right ascension according to the sign, and you get the longitude.

Example. If the given obliquity be 23°. 28′. 7″, and the right ascension 1ˢ. 8°. 58′. 53″,1, add 3 signs to this, and it becomes 4ˢ. 8°. 58′. 53″,1; and with this the reduction is 2°. 26′. 19″, with the variation 12″,9. Hence, 60″ : 12″,9 :: 8″ : 1″,7, which subtracted from 2°. 26′. 19″ leaves 2°. 26′. 17″,3; and this added to 1ˢ. 8°. 58′. 53″,1 gives 1ˢ. 11°. 25′. 10″,4.

The reason of this operation is, that when you increase the right ascension by 3 signs, the difference of right ascension and longitude continues the same, that being now greater which before was less of the two. If therefore we consider the right ascension increased by 3 signs as the longitude, it must give the true reduction.

TABLE XXXVI.

Contains the Declination of the Points of the Ecliptic to the Equator, when the Longitude and Obliquity are given.

Rule. The operation is exactly the same as in the last Table.

Example. Let the longitude be 1ˢ. 11°. 25′. 10″,3, and the obliquity 23°. 28′. 7″; to find the declination.

For 1ˢ. 11° long. - -	Declination	15°. 8′. 49″,2
	Difference	9. 18, 6
	Variation	37, 7

Hence, 30′ : 25′. 10″,3 :: 9′. 18″,6 : 7′. 48″,7, which added to 15°. 8′. 49″,2 gives 15°. 16′. 37″,9.

Again, 60″ : 37″,7 :: 8′ : 5″, which subtracted from 15°. 16′. 37″,9 leaves 15°. 16′. 32″,9 the declination.

By means of this Table, we may find the longitude from the declination.

Rule. Enter the Table with the declination, and take out the variation; then say, 60″ : var. :: the difference between the given obliquity and 23°. 28′. 15″ : a fourth number, which *subtract* from, or *add* to the given declination, according as the obliquity is *greater* or *less* than 23°. 28′. 15″, and the result is the declination such as it would be if the obliquity was 23°. 28′. 15″. Then with this declination, enter the Table again, and, by making a proportion, you get the longitude.

Example. On May 1, 1756, the sun's declination was 15°. 16′. 34″,2 N. and the obliquity was 23°. 28′. 7″, to find the longitude.

Corresponding to this declination, the variation is 37″,7; hence, 60″ : 37″,7 :: 8″ : 5″, which added to 15°. 16′. 34″,2 gives 15°. 16′. 39″,2, the declination if the obliquity had been 23°. 28′. 15″. Now the difference of declination in the Table is 9′. 18″,6, and the difference between 15°. 16′. 39″,2 and the next less declination is 7′. 50″; hence, 9′. 18″,6 : 7′. 50″ :: 30′ : 25′. 14″,5, which added to 1s. 11°, or subtracted from 4s. 11°, each of which longitudes corresponds to the declination 15°. 8′. 49″,2 the next less than 15°. 16′. 39″,2, gives 1s. 11°. 25′. 14″,5 and 4s. 18°. 34′. 45″,5 for the longitudes corresponding to the given declination; but it being in May, the first must be the true one.

For the utmost accuracy, we ought to interpolate.

TABLE XXXVII.

Contains the Equation of Second Differences, for the Purpose of Interpolation in various Cases respecting the planetary Motions.

Rule. Take the two preceding and two following values of the given quantity, and their equations, and find the second differences, and if they be not

mean of the second differences of the moon's motion for 12 hours, whether in longitude or latitude, at the top, and with the apparent time after noon or midnight at the side, and the corresponding number is the correction; which *subtracted* from the proportional part, if the first difference *increase*, or *added*, if it *decrease*, gives the proportional part corrected; which properly applied to the equation, gives the correct equation.

To find the proportional part, reduce the minutes and seconds of time, and of degrees, into decimals, by Tables III. and XL. and add the logarithms of the second and third terms to 8,9208188, the ar. co. of log. 12, and you have the log. of the proportional part.

Example. To find the moon's longitude on July 16, 1767, at 16*h*. 22'. 16".

The given time is 4*h*. 22'. 16" after midnight; hence,

	Moon's Long.	1st Diff.	2d Diff.	Mean of 2d Diff.
July 16, noon.	11ˢ. 29°. 29'. 34"			
		7°. 10'. 51"		
midnight	0. 6. 40. 25		3'. 28"	3'. 36"
		7. 7. 23		
17, noon.	0. 13. 47. 48		3. 44	
		7. 3. 39		
midnight	0. 20. 51. 27			

Now 4*h*. 22'. 16" = 4,39111, 7°. 7'. 23" = 7,12306; hence,

12*h*.	- -	co. ar. log. 8,9208188
4,39111*h*.	- -	log. 0,6405917
7°,112306	- -	log. 0,8526666
Pro. pa. 2°,5946 = 2°. 35'. 41"	-	log. 0,4140771

Now with the second difference 3'. 36" enter the top of the Table, and with 4*h*. 22'. 16". enter the side, and the corresponding number is 24", which added to 2°. 35'. 41" gives 2°. 36'. 5" the proportional part corrected. Hence, the moon's true longitude at the given time is 0ˢ. 6°. 40'. 25" + 2°. 36'. 5" = 0ˢ. 9°. 16'. 30"; and this is as correct as the longitudes from which it is deduced.

In like manner the latitude of the moon may be found at any time.

Dr. Maskelyne has added the following remarks, respecting the use of this Table.

1. If the moon's latitude taken out of the *Ephemeris* for noon and midnight changes its denomination from north to south or from south to north, the *sum* of the two latitudes of contrary denominations, where the change happens, is to be accounted the first difference of that place.

2. If the three first differences first increase and then decrease, or *vice versâ*, first decrease and then increase, half the difference of the two second differences is to be taken for the mean second difference.

3. If the series of four latitudes taken out should first increase and then decrease about the moon's greatest latitudes, take the sum of the two first differences standing on each side of the greatest latitude for the second difference in that place; correct the moon's latitude at noon or midnight by the simple proportional part first found; and to the latitude so corrected, add always, in this case, the equation of second difference, answering to the mean second differences.

TABLE XXXIX.

Contains the Equation of Second Difference for interpolating the Moon's Distance from the Sun or Stars for every third Hour, from those computed for Noon and Midnight in the Nautical Ephemeris.

Rule. Take four distances, two before and two after the given time, and take their second differences. Enter the Table with the mean of the second differences, and *subtract* the corresponding equation at 3, 6, or 9 hours from the quarter, half, or three quarters of the change of distance in 12 hours; or *add* it to the same, according as the first difference is *increasing* or *decreasing*, and you have the moon's change of distance in 3, 6, or 9 hours; which *added* to, or *subtracted* from, the moon's distance at the preceding noon or midnight, according as the distance increases or decreases, gives the moon's true distance at 3, 6, or 9 hours.

EXAMPLE. What is the moon's distance from *α Arietis*, on Dec. 2, 1799, at 3 o'clock ?

	Distance.	1st Diff.	2d Diff.	Mean of 2d Diff.
Dec. 1, midnight	81°. 27′. 3″			
		6°. 28′. 19″		
2, noon.	74. 58. 44		6′. 45″	
		6. 21. 34		6′. 37″
midnight	68. 37. 10		6. 29	
		6. 15. 5		
3, noon.	62. 22. 5			

The quarter of 6°. 21′. 34″ is - -	1°. 35′. 23″
Equation - - - -	+37
	1. 36. 0
2d Distance at noon - -	74. 58. 44
Distance required - - - - -	73. 22. 44

TABLE XL.

Contains the Minutes and Seconds of a Degree, converted into Decimals of a Degree.

RULE. Take out the decimals corresponding to the given minutes and seconds, and add them together.

EXAMPLE. Reduce 49′. 57″ to the decimal of a degree.

	Dec.
49′ - - - - - -	,81667
57″ - - - - - -	,01583
49′. 57″ - - -	,83250

TABLE XLI.

Contains the Equation to equal Altitudes of the Sun, or the Quantity by which the middle Point of Time between the Times when the Sun had the same Altitudes on the Morning and the Afternoon, differs from the Time when the Sun was upon the Meridian.

Rule. Take the half interval of time between the two corresponding observations. With the sun's given longitude for noon, and the half interval, find, from Table I. the equation; if the longitude and half interval be not found in the Table, the equation must be found by proportion. Multiply this equation by the natural tangent of the latitude, radius being unity, and you get the first part of the equation, which is additive or subtractive, according as the sign is + or −; remembering that when the latitude is south, the sign of the Table must be changed.

With the sun's longitude, and half interval, find, in Table II. the equation, by making proportions if the given quantities be not found in the Table.

The sum of these two equations, regard being had to their signs, is the equation required; which applied, according to its sign, to the middle point of time between the two corresponding observations, gives the time by the watch when the sun was upon the meridian.

Example. Let the middle point of time between the corresponding observations be $23h.\ 59'.\ 21'',13$ by the watch; the half interval $2h.\ 34'.\ 52\frac{1}{2}''$; and the sun's longitude $6^s.\ 14^\circ.\ 36'$; to find the equation, and thence the time when the sun was on the meridian; the latitude being $33^\circ.\ 56'$ S. and longitude $18^\circ.\ 23'$ E.

Tab. I. Sun's long. $6^s.\ 10^\circ$ and half int.	$2^h.\ 30'$	equat. is	$+\ 15'',92$
———	2. 40	———	+ 16, 08
	10		0, 16

Hence, $10' : 4'.\ 52\frac{1}{2}'' :: 0'',16 : 0'',08$, which (as the equation increases) added to $15'',92$ gives $16''$ for the equation when the longitude is $6^s.\ 10^\circ$. Again,

TAB. I. Sun's long. 6ˢ. 15° and half int.	2ʰ. 30′	equat. is	+ 15″,71
	2. 40		+ 15, 87
	10		0, 16

Hence, 10′ : 4′. 52½″ :: 0″,16 : 0″,8, which added to 15″,71 gives 15″,79 for the equation when the longitude is 6ˢ. 15°. The difference between this and 16″, the equation when the sun's longitude was 6ˢ. 10°, is 0″,21.

Hence, 5° : 4°. 36′ :: 0″,21 : 0″,09; which subtracted from 16″ (because the equation decreases this way) gives 15″,81 for the equation from TABLE I. And this multiplied by 0,6728, the nat. tang. of 33°. 56′, gives 10″,64 the first part of the equation, which is *subtractive*, because the latitude is south.

TAB. II. Sun's long. 6ˢ. 10° and half int.	2ʰ. 30′	equat. is	−0″,87
	2. 40		−0, 85
	10		0, 02

Hence, 10′ : 4′. 52½″ :: 0″,02 : 0″,01, which subtracted from −0″,87 (as the equation decreases) gives −0″,86 for the equation when the longitude is 6ˢ. 10′. Again,

TAB. II. Sun's long. 6ˢ. 15° and half int.	2ʰ. 30′	equat. is	− 1″,29
	2. 40		−1, 26
	10		0, 03

Hence, 10′ : 4′. 52½″ :: 0″,03 : 0″,01, which subtracted from − 1″,29 gives − 1″,28 for the equation when the sun's longitude is 6ˢ. 15°. The difference between this and −0″,86, the equation when the sun's longitude was 6ˢ. 10°, is 0″,42.

Hence, 5° : 4°. 36′ :: 0″,42 : 0″,39, which added to − 0″,86 (because the equation increases this way) gives − 1″,25 for the second part of the equation; and the two parts added together (because they are both subtractive) gives − 11″,89 for the whole equation. This therefore subtracted from 23*h*. 59′. 21″,13, gives 23*h*. 59′. 9″,24 for the time by the watch when the sun was upon the meridian.

Hence, we may determine how much the watch was too fast, or too slow.

Equation of time at noon at Greenwich was - -	−12′. 19″,3
And 24h. : 1h. 14′ (long. in time) :: 16″,7 (daily dif. of equat.) : - - -	0, 8
Equation of time at noon at the given place - -	−12. 18, 5
	24h. 0. 0
Mean time of apparent noon - - - -	23. 47. 41, 5
Time of apparent noon by the watch - -	23. 59. 9, 2
Watch too fast for mean time - - - -	11. 27, 7

TABLE XLII.

Contains at once the Equation to equal Altitudes, sufficiently accurate for most Purposes.

Rule. With the latitude and interval between the observations at the head, and the declination at the side, you get the equation required. Whilst the sun moves from the tropic of Capricorn to the tropic of Cancer, the equation is to be *subtracted* from the middle point of time; and whilst it moves from the tropic of Cancer to the tropic of Capricorn, it is to be *added*, in order to get the time of apparent noon by the watch.

Example. In latitude 51°. 32′ N. when the sun's declination was 20° S. on January 29, the interval of equal altitudes was found to be 5h. 10′, and the middle time by the watch was 12h. 3′. 32″; to find the time of apparent noon by the watch.

With the latitude 50° N. and 5h. at the head (the nearest to the given quantities), and declination 20° S. at the side, you get the equation 13″, which is to be subtracted, because the sun was moving from the tropic of Capricorn to the tropic of Cancer; hence, 12h. 3′. 19″ is the time of apparent noon by the watch.

TABLE XLIII.

Contains the Semi-diurnal Arcs of the heavenly Bodies whose Declinations do not change, in Time; and therefore for all common Purposes, it will serve for the Sun.

For the *Sun*, the arc gives the time of its setting; and if it be subtracted from 12 o'clock, you get the time of its rising.

For a *Star*, add and subtract the equation to and from the time at which the star passes the meridian (105), and you get the time of its setting and rising.

The Table is calculated for the arcs corresponding to the time when the center of the sun appears in the horizon, the eye being at the surface of the earth; thereby taking into consideration the effect of refraction.

EXAMPLE. In latitude 52°. 12′, and declination of the sun 23°. 28′; what is the time of its rising and setting?

Lat. 52°, declin. 23°	- - - -	arc 8^h. 16′
—— 53, ——	- - - -	arc 8. 22
1		6

Hence, 1° : 12′ :: 6′ : 1′ to be added to 8*h*. 16′.

Lat. 52°, declin. 23°	- - - -	arc 8^h. 16′
—— 24	- - - -	arc 8. 24
1		8

Hence, 1° : 28′ :: 8′ : 4′ to be added also to 8*h*. 16′.

Therefore the semi-diurnal arc = 8*h*. 16′ + 1′ + 4′ = 8*h*. 21′ the time of setting; and 12*h*. − 8*h*. 21′ = 3*h*. 39′ the time of rising.

TABLE XLIV.

Is principally intended to find how long the Body of the Sun is in ascending above the Horizon, or the Time between the upper and lower Limbs of the Sun touching the Horizon.

Rule. Enter with the declination at the top, and latitude at the side, and you have the time which the sun is in ascending 1°. If the declination and latitude be not found in the Table, the time must be found by proportion. Then 1° : the diameter of the sun :: that time : the time of its rising; or by logistic logarithms, log. of sun's diameter + log. of time found from the Tables = log. of time required.

Example. How long will the sun be in rising at Cambridge on June 1, 1799?

Here the latitude is 52°. 12′,5, and declination 22°. 6′, and sun's diameter 31′. 38″.

Lat. 52°, and declin. 21°	-	-	time for 1° is 8′. 4″
—— 53 ————	-	-	———— 8. 20
1			16

Hence, 1° : 12′,5 :: 16″ : 3″ to be added, for 12′,5 of latitude.

Lat. 52° and declin. 21°	-	-	time for 1° is 8′. 4″
———— 24	-	-	———— 8. 43
3			40

Hence, 3° : 6′ :: 40″ : 1″ to be added, for 6′ of declination.

Therefore the time of rising 1° is 8′. 4″ + 3″ + 1″ = 8′. 8″. Hence,

Log. 31′. 38″	- - - - - -	2780
Log. 8. 8	- - - - - -	8679
Log. 4. 17 time of rising	- -	1,1459

TABLE XLV.

Contains the Amplitudes of the heavenly Bodies at the Time of their Rising.

Rule. Enter with the declination at the head of the Table, and the latitude at the side, and you have the amplitude. If the declination and latitude be not found in the Table, the amplitude must be found by proportion.

Example. If the declination of a star be 17°. 30′, and the latitude 57°. 20′, what is the amplitude at its rising?

Lat. 57°, and declin. 17°,	the amplitude is	32°. 28′
—— 58	——————	33. 29
1		1. 1

Hence, 1° : 20′ :: 1°. 1′ : 20′ to be added, for 20′ of latitude.

Lat. 57°, and declin. 17°,	the amplitude is	32°. 28′
—————— 18	——————	34. 34
1		2. 6

Hence, 1° : 30′ :: 2°. 6′ : 1°. 3′ to be added, for 30′ of declination.

Therefore 32°. 28′ + 20′ + 1°. 3′ = 34°. 51′ the amplitude required.

If the Table be entered with the complement of latitude instead of the latitude, you will get the sun's altitude when on the prime vertical. This follows from hence, that if for the sine of latitude in the proportion which gives the altitude, you put cos. lat. it gives the proportion for finding the amplitude. This will appear from Articles 89, 91.

TABLE XLVI.

Is to find at any Time what Proportion the enlightened Part of the Face of the Moon, *or* Venus, *bears to the Whole.*

Rule. Enter with the degrees at the top and on the first column for the moon, and at the bottom and last column for Venus, and corresponding to them you have the enlightened part, the whole face being represented by 12.

Example. If the distance of the moon from the sun be 137°. 12′, what is its enlightened part?

Dist. 137°	- - - -	enlightened part 10,388
138	- - - -	——— 10,458
1		,070

Hence, 1° : 12′ :: ,070 : ,014; therefore the enlightened part is 10,388 + ,014 = 10,402, the whole being 12.

Example. If the angle formed by two lines drawn from Venus to the earth and sun be 50°. 20′, what is its enlightened part?

Angle 50°	- - - -	enlightened part 9,856
51	- - - -	——— 9,776
1		,080

Hence, 1° : 20′ :: ,080 : ,027; therefore 9,856 − ,027 = 9,829 the enlightened part, the whole being 12.

TABLE XLVII.

Is to find the Hour-Angle of Jupiter from the Meridian, when it is 8° above the Horizon, for the Latitude of Greenwich.

Rule. Enter with Jupiter's declination in the column with that title, and against it you have the corresponding hour-angle. If the declination be not found in the Table, the hour-angle must be found by proportion.

EXAMPLE. If Jupiter's declination be 22°. 20′ N. what is its hour-angle, when it is 8° high?

Declin. 22°. 0′	- - -	hour-angle is $7^h. 1'. 50''$
——— 22. 30	- - -	——— 7. 4. 41
30		2. 51

Hence, 30° : 20′ :: 2′. 51″ : 1′. 54″; therefore the required hour-angle is $7^h. 1'. 50'' + 1'. 54'' = 7^h. 3'. 44''$.

TABLE XLVIII.

Is to find the Hour-Angle of the Sun, when it is 8° below the Horizon, for the Latitude of Greenwich.

RULE. Enter with the sun's declination in the column with that title, and against it you have the hour-angle. If the declination be not found in the Table, the hour-angle must be found by proportion.

EXAMPLE. If the sun's declination be 18°. 15′ S. what is its hour-angle, when it is 8° below the horizon?

Declin. 18°. 0′	- - -	hour-angle is $5^h. 20'. 6''$
——— 18. 30	- - -	——— 5. 17. 26
30		2. 40

Hence, 30′ : 15′ :: 2′. 40″ : 1′. 20″; therefore the required hour-angle is $5^h. 20'. 6'' - 1'. 20'' = 5^h. 18'. 46''$.

The use of the two last Tables is to find, whether an eclipse of Jupiter's satellites will be visible at Greenwich, or on that parallel of latitude. For if at the time of an eclipse, the hour-angle of Jupiter be equal to, or greater than that which the Table gives, and the hour-angle of the sun be equal to, or less than that given in the Table, the eclipse will not be visible: if the former be equal to, or less, and the latter equal to, or greater than what the Tables give, the eclipse will be visible.

TABLE XLIX.

Is a Table of Logistic Logarithms, which were first employed by Street, *and are nothing but the common Logarithms subtracted from* 3,5563, *which is the Logarithm of* 3600″, *by which means the Logarithm of* 3600″ (=60) *becomes nothing, and the Logarithm of* 360 *becomes* 1,0000. *For Numbers greater than* 3600, *the Logarithms would be negative; but instead of putting them down so, their arithmetic complements are put down.*

Rule. If the first term be 60′, or 3600″, add the logarithms of the second and third terms together; but if the sum of the two last figures in the addition, the index excepted, be equal to or greater than 10, you are not to carry 1 to the index, and the sum is the logarithm of the fourth term. If the second term be 60′, subtract the logarithm of the first term from the logarithm of the third, adding 1 to the index of the logarithm of the third, if necessary, and the remainder is the logarithm of the fourth term.

Example I. What is a fourth proportional to 60′, 65′. 25″, and 37′. 41″?

	65′. 25″ - - -	log. 9625
	37. 41 - -	log. 2020
Answer	41. 5 - - -	log. 1645

Example II. What is a fourth proportional to 60′, 1′. 36″, and 27′. 38″?

	1′. 36″ - - -	log. 1,5740
	27. 38 - - -	log. 3367
Answer	0. 44 - -	log. 1,9107

Example III. What is a fourth proportional to 16′. 47″, 60′, and 17′. 28″?

	17′. 28″ - - -	log. 5359
	16. 47 - - -	log. 5533
Answer	62. 27 - - - -	log. 9826

Here it was necessary to add 1 to the index of the logarithm of the third term, or which is the same thing, 10 was added to the last figure 5 of the upper line.

EXAMPLE IV. What is a fourth proportional to 27'. 19", 60'. and 5'. 9"?

5'. 9"	- - -	log.	1,0663
27. 19	- - -	log.	3417
Answer 11. 19	- - -		7246

If the first term be 24*h*. and the second term be hours and minutes, and the third term be given in time, or be an arc, we may find a fourth proportional, by conceiving the head of the Table to represent hours.

EXAMPLE. What is a fourth proportional to 24*h*., 13*h*. 53', and 76'. 34"?

24*h*.	- - -	ar. co.	6021
13. 53'	- -		6357
76'. 34"	- - -		8941
Answer 44. 17	- - -		1319

We reject 2 in the index, because the first and third terms are arithmetic complements, and 1 is to be rejected for each.

In like manner, whatever may be the three terms, whether hours and minutes, minutes and seconds of time; degrees and minutes, minutes and seconds of an arc, or two of one and one of the other, a fourth proportional may be found, provided the quantities fall within the limit of the Table.

We conclude this volume, with a catalogue of fixed stars by BRADLEY, DE LA CAILLE, ZACH, and MAYER.

Dr. BRADLEY's CATALOGUE OF FIXED STARS, FOR THE BEGINNING OF 1760.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension,		Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination,	
				1760.	1800.		1760.	1800.
			D. M. S.	S.	S	D. M. S.	S.	S.
1	γ Pegasi	2	0 13 35,1	45,95	45,99	76. 9. 5,3	20,01	20,01
2	ι Ceti	3	1. 47. 59,9	45,80	45,79	100. 9. 20,8	20,0[illegible]	19,99
3	* d Piscium	5	2. 4. 1,0	46,01	46,04	88. 8. 39,7	20,00	19.99
4	δ Andromedæ	3	6. 38. 13,7	47,24	47,36	60. 27. 20,0	19,88	19,85
5	α Cassiopeiæ	3	6 43 35,0	49,32	49,63	34 46. 53,5	19,87	19,65
6	β Ceti	3	7. 52. 59,0	44,97	44,92	109. 18. 27,1	19,82	19,80
7	ζ Andromedæ	4	8. 39. 58,0	47,20	47,30	67. 2. 30,5	19,78	19,75
8	* 20 Ceti	5	10. 11. 19,0	45,78	45,78	92. 27. 8,0	19,70	19,66
9	γ Cassiopeiæ	3	10 36. 19,0	52,16	52,55	30. 35. 16,2	19,67	19,63
10	* ε Piscium	4	12. 37. 43,0	46,43	46,47	83. 24. 26,6	19,53	19,48
11	* e Piscium	5	14. 0. 30,0	46,30	46,33	85. 37. 30,8	19,42	19,37
12	β Andromedæ	2	14. 6. 29,2	49,26	49,41	55. 39. 32,6	19,41	19,36
13	η Ceti	3	14. 7. 48,0	44,94	44,92	101. 27. 39,4	19,41	19,36
14	θ Cassiopeiæ	4	14 9. 48,0	52,65	52,95	36. 7. 56,0	19,40	19,35
15	* ζ Piscium	4	15. 18. 18,0	46,52	46,55	83. 42. 0,3	19,30	19,25
16	δ Cassiopeiæ	3	17. 34. 49,0	55,98	56,41	31 1. 9,5	19,08	19,01
17	θ Ceti	3	18. 0. 33,0	44,90	44,90	99. 25. 40,6	19,03	18,97
18	* μ Piscium	5	19. 24. 35,0	46,50	46,54	85. 6. 0,5	18,88	18,81
19	* ν Piscium	5	19. 40. 14,0	47,62	47,69	75. 53. 55,0	18,85	18,78
20	* π Piscium	5	21. 6. 13,0	47,32	47,38	79. 5. 39,7	18,67	18,60
21	* 105 Piscium	5	21. 41. 38,0	47,93	48,02	74. 49. 11,7	18,60	18,52
22	* ν Piscium	4	22. 14 26,0	46,49	46,53	85. 44. 7,3	18,52	18,45
23	* ο Piscium	4	23. 11. 14,0	47,03	47,08	82. 3. 31,8	18,40	18,32
24	ε Cassiopeiæ	3	24. 21. 0,0	61,76	62,32	27. 31. 26,8	18,23	18,13
25	γ Arietis	4	25. 6. 4,8	48,70	48,79	71. 53. 33,0	18,12	18,04
26	β Arietis	3	25. 21. 26,1	48,98	49,09	70. 22. 29,8	18,09	18,00
27	* ι Arietis	5	26. 4. 13,5	48,56	48,64	73. 21. 50,6	17,98	17,89
28	λ Arietis	5	26. 9 8,6	49,57	49,68	67. 35. 4,0	17,96	17,87
29	γ Andromedæ	2	27. 19. 7,0	53,96	54,18	48 50 2,0	17,78	17,68
30	α Piscium	3	27. 24. 46,0	46,18	46,23	88. 24. 18,4	17,77	17,68
31	α Arietis	2	28. 25. 26,6	49,84	49,95	67. 41. 1,4	17,60	17,50
32	* 19 Arietis	5	30. 0. 8,0	48,46	48,53	75. 51. 23,8	17,33	17,23
33	* 1 ξ Ceti	5	30. 4 35,0	47,30	47,34	82. 17. 23,2	17,32	17,22
34	* 1 δ Arietis	5	31. 12. 18,0	49,45	49,55	71. 13. 17,9	17,12	17,01
35	ο Ceti, var.	2	31. 48. 41,0	45,18	45,20	94. 4. 44,7	17,01	16,91

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	[illegible]
36	* 2 ξ Ceti	4	33. 51. 24,0	47,37	47,43	82. 37. 41,0	16,62	16,51
37	δ Ceti	3	36. 47. 59,5	45,78	45,81	90. 43. 9,3	16,03	15,91
38	θ Persei	4	36. 58. 56,0	59,39	59,68	41. 48. 13,4	15,99	15,84
39	ε Ceti	3	36. 59. 31,0	43,17	43,18	102. 54. 9,5	15,99	15,88
40	35 Arietis	4	37. 21. 28,0	52,03	52,16	63. 19. 43,2	15,91	15,78
41	γ Ceti	3	37. 43. 27,0	46,40	46,45	87. 47. 18,4	15,83	15,72
42	* μ Ceti	4	38. 0. 6,0	47,90	47,96	80. 54. 46,8	15,77	15,65
43	π Ceti	3	38. 10. 39,0	42,64	42,65	104. 53. 14,5	15,73	15,63
44	τ Persei	[illegible]	39. 20. 51,0	62,03	62,37	38. 14. 18,4	15,48	15,32
45	* 3 ρ Arietis	5	30. 43. 55,0	49,93	50,02	72. 56. 57,0	15,17	15,03
46	η Eridani	3	41. 10. 45,0	43,64	43,66	99. 51. 58,0	15,06	14,95
47	* ι Arietis	5	41. 23. 0,2	50,84	50,94	69. 38. 5,5	15,02	14,88
48	γ Persei	3	41. 53. 38,0	63 36	63,71	37. 28. 13,7	14,90	14,73
49	α Ceti	2	42. 26. 24,1	46,67	46,72	86. 51. 59,8	14,77	14,64
50	β Persei	3	43. 9. 52,0	57,42	57,62	49. 59. 15,8	14,60	14,44
51	* δ Arietis	4	44. 29. 11,0	50 70	50,80	71. 11. 53 5	14,28	14,14
52	* ζ Arietis	5	45. 17. 19,0	51,14	51,24	69. 51. 40,2	14,08	13,93
53	12 Eridani	3	45. 28. 7,0	37,73	37,70	119. 56. 51,6	14,04	13,93
54	ζ Eridani	3	46. 2. 54,0	43,46	43,48	99. 43. 32,8	13 89	13 77
55	α Persei	2	46. 49. 50,0	62,71	62,99	41. 0. 50,9	13,69	13,51
56	* 2 τ Arietis	5	47. 14. 48,0	51 24	51,33	70. 8. 2,3	13.59	13,44
57	* f Tauri	4	49. 24. 48,3	49,19	49 26	77. 54. 10,6	13,01	12.87
58	17 Eridani	4.5.	49. 41. 48,0	44,35	44,38	95. 54. 46 8	12.95	12.81
59	δ Persei	3	51. 29. 11,0	62,78	62,97	43. 0. 9,8	12,46	12,26
60	* b Pleiadum	5	52. 40. 7,0	52,79	52,89	66. 39. 38,0	12,14	11,97
61	* e Pleiadum	5	52. 44. 32,0	52.92	53,02	66. 18. 16,2	12,12	11,95
62	δ Eridani	3,4	52. 56. 33,0	42,94	42,96	100. 35. 32.4	12,06	11,92
63	* d Pleiadum	5	53. 2. 1,0	52.77	52,87	66. 49. 6,5	12,03	11,87
64	* η Tauri	3	53. 18. 55.2	52,85	52,95	66. 39. 22,1	11,96	11,79
65	γ Eridani	2	56. 42. 35,0	41,69	41,71	104. 12. 26,5	10,98	10,85
66	1 λ Persei	4	57. 12. 6,0	65,75	65 99	40. 19. 28,9	10,84	10,62
67	* A Tauri	4	57. 38. 1,0	52,55	52,64	68. 35. 37,4	10,71	10,54
68	* φ Tauri	5	61. 24. 32,0	54,79	54,88	63. 14. 45,8	9,58	9,39
69	* γ Tauri	3	61. 32. 26 2	50,65	50,71	74. 58. 20,8	9,54	9,34
70	* χ Tauri	5	62. 0. 16,0	54,18	54,27	64. 57. 30,8	9,39	9,21

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension,		Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination,	
				1760.	1800.		1760.	1800.
			D. M. S.	S.	[redacted]	D. M. S.	S.	S.
71	* 1 δ Tauri	4	62. 16. 57,0	51,32	51,39	73. 2. 28,6	9,31	9,13
72	* 2 δ Tauri	4	62. 34. 25,0	51,31	51,38	73. 7. 56,9	9,22	9,04
73	* 1 κ Tauri	5	62. 46. 34,0	53,02	53,09	68. 16. 36,0	9,16	8,97
74	* 2 κ Tauri	4	62. 47. 18,0	52,98	53,06	68. 22. 13,5	9,15	8,97
75	* 3 δ Tauri	5	62. 54. 39,0	51,50	51,56	72. 38. 31,5	9,11	8,93
76	* 1 υ Tauri	5	62. 59 43,0	53,22	53,30	67. 45. 7,8	9,09	8,90
77	* ε Tauri	2	63. 39. 28,5	51,97	52,04	71. 22. 25,3	8,88	8,70
78	* 1 θ Tauri	5	63. 43. 22,0	50,87	50,92	74. 35. 29,5	8,86	8,68
79	* 2 θ Tauri	5	63. 44 47,0	50,84	50,90	74. 40. 59,0	8,85	8,67
80	* Aldebaran	1	65. 32. 38,7	51,15	51,21	73. 59. 39,7	8,29	8,10
81	* τ Tauri	[redacted]	66. 53. 5,0	53,55	53,61	67. 31. 33,3	7,83	7,64
82	1 π Orionis	4	69. 23. 16,0	48,72	48,75	81. 31. 55,1	7,05	6,87
[redacted]	7 Camelopard.	5	69. 31. 48,0	71,12	71,31	36. 39. 46,3	7,00	6,74
84	* ι Tauri	4	72. 11. 37,7	53,33	53,38	68. 46. 31,5	6,11	5,91
85	* m Tauri	5	73. 19. 12,0	52,27	52,31	71. 42. 1,2	5,74	5,55
86	* 105 Tauri	5	73. 23. 58,0	53,45	53,55	68. 38. 12,2	5,72	5,52
87	h Eridani	3	74. 1. 5,0	44,10	44,12	95. 24. 56,9	5,51	5,34
88	Capella	1	74. 44. 59,5	65,73	65,84	44. 16. 27,5	5,26	5,02
89	Rigel	1	75. 45. 10,9	48,03	48,05	98. 29. 50,7	4,93	4,76
90	* β Tauri	2	77. 47. 7,0	56,49	56,54	61. 37. 12,0	4,23	4,02
91	γ Orionis	2	78. 4. 8,0	48,02	48,05	83. 53. 20,7	4,14	3,95
92	* e Tauri	5	78. 18. 35,0	53,73	53,86	68 17. 30,7	4,06	3,85
93	2 ψ Orionis	5	78. 34. 9,0	46,91	46,93	87. 7. 59,5	3,97	3,79
94	β Leporis	3	79. 29. 40,0	38,39	38,40	110. 58. 4,3	3,65	3,50
95	δ Orionis	2	79. 56. 22,0	45,76	45,77	90. 29. 50,1	3,50	3,32
96	α Leporis	3	80. 32. 21,0	39,51	39,52	108. 0. 43,0	3,29	3,14
97	* ζ Tauri	3	80. 49. 41,8	53,50	53,53	69. 1. 37,4	3,19	2,98
98	ι Orionis	2	81. 0. 41,0	45,45	45,47	91. 22. 33,3	3,13	2,95
99	* 125 Tauri	5	81. 13. 4,0	55,46	55,47	64. 15. 44,3	3,06	2,84
100	* 132 Tauri	4	83. 34. 30,0	54,97	55,00	65. 32. 17,3	2,24	2,03
101	* γ Leporis	3	83. 37. 3,0	37,67	37,68	112. 32. 47,0	2,22	2,08
102	* 136 Tauri	5	84. 33. 46,0	56,31	56,33	62. 28. 9,0	1,90	1,68
103	δ Aurigæ	4	84. 46. 44,3	73,60	73,65	35. 45. 53,2	1,82	1,54
104	* 1 χ Orionis	5	85. 2. 41,0	53,27	53,28	69. 47. 20,5	1,73	1,52
105	* 2 χ Orionis	5	85. 11. 7,0	53,06	53,07	70. 19. 42,7	1,68	1,47

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Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
106	α Orionis	1	85. 32. 47,2	48,50	48,51	82. 39. 34,5	1,55	1,37
107	θ Aurigæ	4	85. 50. 21,0	61,06	61,11	52. 49. 51,6	1,45	1,21
108	* H Gemino.	5	87. 23. 1,0	54,52	54,53	66. 44. 51,9	0,91	0,70
109	* κ Aurigæ	5	90. 1. 18,0	57,28	57,28	60. 26. 18,9	0,01	0,23
110	* η Gemino.	4	90. 5. 52,8	54,24	54,24	67. 26. 51,7	0,03	0,25
111	* μ Gemino.	3	92. 6. 34,1	54,26	54,25	67. 23. 13,3	0,74	0,95
112	* ν Geminorum	4	93. 40. 39,7	53,34	53,32	69. 39. 34,6	1,28	1,49
113	23 Gemino.	5	95. 32. 4,0	52,08	52,06	— — —	1,93	2,13
114	γ Gemino.	2	95. 57. 37,3	51,85	51,84	73. 25. 8,4	2,08	2,28
115	* 26 Gemino.	5	97. 6. 20,0	52,33	52,31	72. 8. 33,2	2,48	2,68
116	* ε Gemino.	3	97. 17. 20,7	55,33	55,30	64. 39. 19,7	2,54	2,75
117	* 28 Gemino.	5	97. 23. 5,0	57,04	56,99	60. 48. 41,5	2,57	2,79
118	Sirius	1	98. 38. 36,8	40,10	40,10	106. 24. 6,3	3,01	3,16
119	* ζ Gemino.	4	102. 27. 57,0	53,39	53,35	69. 6. 2,1	4,32	4,52
120	* 51 Gemino.	5	104. 53. 40,0	51,67	51,70	73. 27. 21,8	5,14	5,34
121	19 Lyncis	5	105. 48. 8,0	74,16	73,97	34. 17. 49,0	5,45	5,73
122	* λ Gemino.	5	106. 4. 22,0	51,78	51,75	73. 2. 54,7	5,54	5,73
123	* δ Gemino.	3	106. 26. 38,0	53,84	53,79	67. 35. 53,7	5,67	5,86
124	* q Gemino.	5	106. 56. 31,0	53,23	53,20	69. 7. 33,8	5,83	6,03
125	* ι Gemino.	5	107. 42. 2,0	56,17	56,11	61. 44. 54,5	6,08	6,29
126	* p Gemino.	5	108. 22. 8,0	53,57	53,51	68. 5. 12,0	6,31	6,50
127	η Canis maj.	2	108. 39. 8,0	35,48	35,48	118. 50. 57,0	6,40	6,53
128	Castor	1	109. 48. 45,7	57,87	57,39	57. 36. 36,0	6,78	6,99
129	* υ Gemino.	4	110. 16. 34,1	55,66	55,59	62. 35. 36,0	6,94	7,14
130	* f Gemino.	5	111. 24. 2,0	52,05	52,00	71. 48. 1,0	7,30	7,47
131	Procyon	1	111. 40. 56,8	47,82	47,79	84. 10. 36,0	7,39	7,56
132	* κ Gemino.	5	112. 28. 59,0	54,53	54,47	65. 2. 54,0	7,65	7,85
133	Pollux	1	112. 39. 3,7	55,99	55,91	61. 24. 56,8	7,71	7,90
134	* g Gemino.	5	113. 3. 12,0	52,30	52,24	70. 55. 32,2	7,84	8,02
135	26 Lyncis	5	114. 17. 24,0	66,30	66,11	41. 50. 18,1	8,23	8,47
136	* φ Gemino.	5	114. 41. 34,0	55,33	55,25	62. 38. 6,1	8,36	8,55
137	* 3 Cancri	5	116. 57. 4,0	52,03	51,96	71. 7. 2,8	9,07	9,25
138	μ Cancri	5	118. 0. 54,0	53,54	53,46	66. 41. 57,2	9,40	9,58
139	* 2 ψ Cancri	4	118. 59. 26,0	54,55	54,46	63. 46. 59,2	9,70	9,88
140	β Cancri	3	120. 52. 13,0	48,93	48,88	80. 5. 35,0	10,27	10,43

DR. BRADLEY's CATALOGUE OF FIXED STARS.

Number of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	An. Preces. in Right Ascension, 1800.	Mean Distance from N. Pole, Jan. 5, 1760.	An. Preces. in Declination, 1760.	An. Preces. in Declination, 1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
141	* θ Cancri	5	124. 28. 16,0	51,57	51,49	71. 6. 44,5	11,33	11,49
142	* η Cancri	5	124. 41. 53,0	52,32	52,24	68. 45. 40,5	11,39	11,56
143	* γ Cancri	4	127. 20. 30,0	52,46	52,36	67. 41. 9,1	12,14	12,30
144	* δ Cancri	4	127. 45. 17,0	51,38	48,30	70. 58. 47,9	12,25	12,41
145	ι Ursæ maj.	4	130. 40. 2,0	63,37	63,09	41. 2. 3,1	13,04	13,23
146	* 1 α Cancri	4	130. 41. 58,3	49,30	49,23	77. 28. 22,1	13,05	13,19
147	* 2 α Cancri	4	131. 20. 4,8	49,33	49,37	77. 13 43,7	13,22	13,36
148	* κ Cancri	5	133. 40. 53,0	48,90	48,84	78. 22. 52,5	13,82	14,02
149	* ξ Cancri	6	133. 52. 48,5	52,06	51,95	66. 59. 57,6	13,87	14,01
150	* ω Leonis	5	138. 58. 50,0	48,27	48,21	79. 54. 42,5	15,08	15,20
151	α Hydræ	2	138. 56. 57,0	44,17	44,15	97. 37. 48,0	15,09	15,20
152	θ Ursæ maj.	3,4	139. 10. 9,0	63,14	62,79	37. 14. 35,0	15,14	15,30
153	* ξ Leonis	4	139. 44. 49,0	48,81	48,69	77. 39. 4,3	15,27	15,39
154	10 Leon min.	4,5	139. 51. 36,6	55,82	55,63	52. 33. 6,3	15,30	15,44
155	10 Leonis	5	141. 7. 58,9	47,67	47,61	82. 5. 58,5	15,58	15,69
156	* ο Leonis	4	142. 4. 50,0	48,31	48,25	79. 1. 40,8	15,79	15,90
157	ε Leonis	3	143. 2. 44,0	51,50	51,39	65. 7. 59,5	15,99	16,11
158	* ν Leonis	5	146. 19. 20,0	48,61	48,53	76 25. 16,5	16,65	16,76
159	* π Leonis	4	146. 52. 44,0	47,70	47,64	80 48. 51,7	16,76	16,86
160	* η Leonis	4	148. 33. 18,0	49,30	49,22	72. 4. 37,5	17,07	17,17
161	* A Leonis	5	148. 47 16,0	47,97	47,91	78. 50. 10,9	17,12	17,21
162	* Regulus	1	148. 53. 32,5	48,34	48,27	76. 52. 9,1	17,14	17,23
163	ζ Leonis	3	150. 49. 28,0	50,39	50,28	65 23. 47,8	17,47	17,56
164	γ Leonis	2	151. 40. 35,0	49,58	49,48	68. 57 13,1	17,62	17,70
165	μ Ursæ maj.	3	151. 59. 9,0	54,60	54,57	47. 18. 10,5	17,67	17,76
166	* ρ Leonis	4	155. 2. 25,0	47,50	47,44	79. 27. 56,6	18,14	18,19
167	* 48 Leonis	5	155 34. 0,0	47,12	47,06	81. 49. 5,0	18,22	18,29
168	* 37 Sextantis	6	158. 23. 42,0	46,91	46,87	82. 22 6,4	18,61	18,67
169	* 38 Sextantis	6	158. 42. 21,0	46,90	46,85	82 23 40,1	18,65	18,71
170	* 55 Leonis	5	160. 50. 19,0	46,16	46,13	87. 59 20,5	18,90	18,96
171	* 56 Leonis	6	160 53. 17,0	46,79	46,74	82 32 24,0	18,91	18,96
172	β Ursæ maj.	2	161. 47. 56,0	55,80	55,40	32 20 13,9	19,01	19,07
173	* d Leonis	5	162. 2. 29,0	46,46	46,42	85 5 55,0	10,04	19,09
174	c Leonis	5	162. 4. 30,0	46,73	46,68	82 36 53,1	19,04	19,09
175	α Ursæ maj.	1,2	162. 10. 35,0	57,97	57,45	26 57 33,2	19,05	19,12

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	An. Preces. in Right Ascension, 1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	An. Preces. in Declination, 1800
			D. M. S.	S.	S.	D. M. S.	S.	S.
176	* χ Leonis	5	163. 9. 26,0	46,81	46,76	81. 22. 16,7	19,15	19,20
177	δ Leonis	3	165. 19. 38,0	47,96	47,87	68. 9. 53,3	19,36	19,40
178	θ Leonis	3	165. 24. 23,0	47,44	47,37	73. 15 46,5	19,37	19,41
179	* 75 Leonis	5	166. 14. 1,0	46,20	46,18	86. 40. 21,8	19,44	19,48
180	* 76 Leonis	5	166. 38. 57,0	46,17	46,14	87. 2. 15,5	19,47	19,51
181	* σ Leonis	5	167. 11. 17,0	46,50	46,46	82. 39. 30,3	19,52	19,55
182	* 79 Leonis	5,6	167. 55. 51,0	46,13	46,10	87. 16. 40,1	19,57	19,60
183	* τ Leonis	4	168. 53. 51,0	46,21	46,18	85. 49. 27,5	19,64	19,67
184	* e Leonis	5	169. 30 53,0	45,82	45,80	91. 40. 54,8	19,68	19,71
185	* υ Leonis	4	171. 9. 57,0	45,95	45,94	89. 30. 1,6	19,78	19,80
186	* 1 ξ Virginis	5	173. 13. 35,0	46,33	46,29	80. 24. 33,8	19,87	19,89
187	* ν Virginis	5	173. 22. 41,0	46,25	46,21	82. 7. 35,0	19,88	19,89
188	β Leonis	1,2	174. 11 59,0	46,50	46,45	74. 5. 13,0	19,91	19,92
189	* β Virginis	3	174. 32 51,3	46,08	46,02	86. 52. 56,1	19,92	19,93
190	γ Ursæ maj.	2	175. 16. 25,0	48,29	48,00	34. 58. 16,1	19,95	19,96
191	* π Virginis	5	177. 8. 28,0	46,07	46,04	82. 2. 50,3	19,99	19,99
192	δ Ursæ maj.	3	180. 51. 27,0	45,44	45,16	31. 37. 54,1	20,01	20,00
193	γ Corvi	3	180. 52. 30,0	46,01	46,07	106. 12. 27,1	20,01	20,00
194	* n Virginis	5	181. 35. 43,0	45,93	45,93	89. 27. 1,3	20,01	20,00
195	* η Virginis	3	181. 54. 30,7	45,94	45,95	89. 19. 51,9	20,00	19,99
196	* c Virginis	3	182. 2. 32,0	45,87	45,86	85. 20. 55,7	20,00	19,99
197	κ Draconis	3	185. 46. 32,0	40,04	39,67	18 53. 2,6	19,91	19,89
198	* χ Virginis	5	186. 43. 19,0	46,21	46,23	96. 40. 13,5	19,88	19,85
199	* γ Virginis	3	187. 22. 45,9	45,93	45,95	90. 7. 43,5	19,85	19,82
200	* ψ Virginis	5	190. 28. 32,0	46,45	46,50	98. 13. 47,8	19,68	19,64
201	δ Virginis	3	190. 32. 54,0	45,62	45,62	85 17. 32,5	19,65	19,61
202	ι Virginis	3	192. 33. 31,0	44,98	44,97	77. 44. 43,5	19,53	19,49
203	* g Virginis	5	193 50. 21,0	46,72	46,77	99. 27. 0,8	19,43	19,38
204	* θ Virginis	4	194. 23. 16,0	46,80	46,83	94. 15. 8,2	19,39	19,34
205	* Spica Virg.	1	198. 8. 44,1	47,01	47,07	99. 54. 3,0	19,02	18,96
206	* ι Virginis	4	198. 31. 4,0	47,22	47,27	101. 27. 1,4	18,98	18,91
207	ζ Ursæ maj.	[illegible]	198 33. 12,0	36,42	36,30	33. 48. 53,7	18,97	18,92
208	* 21 Virginis	5	199 52. 44,0	46,52	46,57	95. 0. 28,8	18,82	18,75
209	* m Virginis	5	202 15. 44,0	46,92	46,98	97. 28. 57,5	18,52	18,45
210	η Ursæ maj.	2	204 31. 1,0	35,85	35,77	39. 28. 51,7	18,21	18,15

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
211	α Draconis	2	209. 28. 28,0	24,29	24,31	24. 28. 18,2	17,42	17,33
212	* κ Virginis	4	210. 1. 50,0	47,54	47,60	99. 8. 41,8	17,33	17,23
213	Arcturus	1	211. 10. 53,0	42,07	42,06	69. 33. 27,1	17,12	17,04
214	* λ Virginis	4	211. 32. 26,0	48,20	48,27	102. 15. 12,3	17,06	16,95
215	θ Bootis	4	214. 15. 32,0	30,99	30,97	37. 1. 48,3	16,54	16,51
216	* μ Libræ	5	219. 3. 4,0	48,87	48,95	103. 8. 2,5	15,54	15,42
217	* α Libræ	2	219. 24. 40,6	49,35	49,42	105. 1. 44,7	15,46	15,35
218	* 2 ξ Libræ	5	220. 56. 46,0	48,34	48,41	100. 25. 30,5	15,12	14,99
219	* 18 Libræ	5	221. 29. 16,0	48,30	48,37	100. 9. 47,5	14,99	14,86
220	β Ursæ min.	3	222. 55. 17,0	5,43	-4,78	14. 51. 44,5	14,66	14,67
221	* 1 ν Libræ	5	223. 19. 19,0	49,69	49,76	105. 18. 33,5	14,56	14,46
222	* 1 ι Libræ	3	224. 38. 51,0	50,73	50,83	108. 51. 58,7	14,24	14,10
223	β Libræ	2	226. 1. 57,5	48,08	48,14	98. 28. 51,5	13,89	13,76
224	* 4 ζ Libræ	4	229. 50. 59,8	50,32	50,40	106. 1. 11,9	12,90	12,75
225	* γ Libræ	3,4	230. 32. 1,7	49,77	49,84	103. 58. 14,2	12,72	12,57
226	α Coron. Bor.	2	231. 8. 6,0	37,80	37,81	62. 27. 48,0	12,56	12,44
227	* 42 Libræ	5	231. 32. 12,0	52,58	52,68	113. 1. 3,0	12,45	12,29
228	* κ Libræ	4	232. 2. 34,0	51,32	51,41	108. 52. 50,6	12,31	12,15
229	α Serpentis	2	233. 7. 0,0	43,91	43,93	82. 48. 11,2	12,01	11,87
230	* 1 A Scorpii	5	234. 48. 51,0	53,41	53,54	114. 35. 22,5	11,53	11,36
231	* γ Libræ	4	234. 51. 40,0	51,70	51,78	109. 25. 48,8	11,52	11,35
232	* ϑ Libræ	4	235. 2 58,0	50,63	50,71	106. 0. 18,1	11,47	11,30
233	ε Serpentis	3	235. 11 3,0	39,38	39,40	68. 17. 9,4	11,43	11,30
234	* π Scorpii	3	236. 5 47,0	53,81	53,91	115. 23. 41,2	11,16	10,99
235	* ψ Libræ	4	236. 11 53,0	49,94	50,01	103. 34. 0,4	11,13	10,97
236	* δ Scorpii	3	236. 32. 44,0	52,65	52,73	111. 55. 1,7	11,03	10,86
237	* β Scorpii	2	237. 52. 47,0	51,81	51,88	109. 7. 39,1	10,64	10,47
238	* 1 ω Scorpii	5	238. 12. 9,0	52,13	52,20	109. 59. 53,2	10,55	10,37
239	* 2 ω Scorpii	5	238. 20. 37,0	52,20	52,27	110. 11. 57,8	10,50	10,33
240	ι Herculis	5	238. 50. 11,0	27,75	27,77	43. 17. 29,0	10,36	10,26
241	* ν Scorpii	4	239. 31. 18,0	51,80	51,88	108. 48. 58,2	10,15	9,98
242	δ Ophiuchi	3	240. 26. 54,0	46,86	46,90	93. 3. 26,9	9,87	9,71
243	* 19 Scorpii	5	241. 33. 35,0	53,60	53,69	113. 34. 2,5	9,53	9,34
244	* σ Scorpii	4	241. 39. 40,0	54,14	54,21	114. 59. 41,1	9,50	9,31
245	* ψ Ophiuchi	5	242. 31. 27,0	52,20	52,27	109. 27. 16,5	9,23	9,05

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension,		Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination,	
				1760.	1800.		1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
246	* g Ophiuchi	5	242. 48. 34,0	53,44	53,51	112. 52. 18,5	9,15	8,96
247	* Antares	1	243. 41. 0,9	54,63	54,71	115. 52. 32,5	8,87	8,68
248	* φ Ophiuchi	4	244. 21. 24,0	51,12	51,18	106. 4. 2,5	8,66	8,48
249	* ω Ophiuchi	5	244. 29. 14,0	52,84	52,90	110. 55. 51,0	8,62	8,43
250	* τ Scorpii	4	245. 14. 48,0	55,47	55,55	117. 41. 41,9	8,38	8,18
251	* 24 Scorpii	5	246. 55. 47,0	51,65	51,71	107. 15. 14,4	7,84	7,65
252	μ Draconis	4	255. 5. 46,3	18,51	18,56	35. 12. 23,0	5,15	5,08
253	* A Ophiu. dup.	5	255. 9. 24,0	55,46	55,51	116. 13. 22,5	5,12	4,92
254	α Herculis	3	255. 55. 45,0	40,85	40,85	75. 19. 5,5	4,87	4,71
255	* ρ Ophiuchi	4	256. 39. 35,0	53,40	53,37	110. 49. 49,6	4,62	4,41
256	* ϑ Ophiuchi	3	256. 49. 28,0	54,90	54,95	114. 44. 1,9	4,56	4,34
257	* 43 Ophiuchi	5	257. 4. 19,0	56,25	56,30	117. 53. 1,5	4,48	4,26
258	* B Ophiuchi	4	257. 56. 4,0	54,61	54,65	113. 55. 44,3	4,18	3,97
259	* e Ophiuchi	5	259. 11. 56,0	54,58	54,58	113. 45. 7,5	3,75	3,54
260	α Ophiuchi	2	260. 57. 4,0	41,45	41,46	77. 14. 48,9	3,14	2,99
261	μ Ophiuchi	4	261. 12. 13,0	48,69	48,71	97. 57. 5,9	3,06	2,87
262	β Draconis	3	261. 15. 28,0	20,16	20,18	37. 30. 41,2	3,04	2,96
263	* D Ophiuchi	5	262. 15. 59,0	53,76	53,78	111. 32. 29,8	2,69	2,48
264	* p Sagittarii	3	263. 7. 5,0	56,36	56,38	117. 42. 45,1	2,40	2,18
265	* b Sagittarii	5	266. 17. 18,0	54,72	54,79	113. 46. 5,5	1,30	1,08
266	γ Sagittarii	3,4	267. 36. 0,0	57,66	57,67	120. 23. 56,4	0,84	0,61
267	γ Draconis	2	267. 45. 50,0	20,76	20,76	38. 28. 23,6	0,78	0,70
268	* 1 μ Sagittarii	4	269. 51. 11,0	53,65	53,63	111. 5 47,7	0,05	0,16
269	* 2 μ Sagittarii	4	270. 13. 34,0	53,52	53,52	110. 46. 28,4	0,08	0,29
270	* δ Sagittarii	3	271. 24. 26,8	57,43	57,43	119. 54. 11,5	0,49	0,72
271	ι Sagittarii	2	272. 3. 44,0	59,66	59,65	124. 28. 12,1	0,72	0,95
272	* λ Sagittarii	4	273. 17. 32,0	55,47	55,47	115. 31. 40,8	1,15	1,36
273	α Lyræ	1	277. 12. 11,0	30,09	30,06	51. 25. 33,7	2,51	2,63
274	* φ Sagittarii	3	277. 39. 49,0	56,13	56,10	117. 12. 39,3	2,67	2,88
275	* 28 Sagittarii	5	277. 57. 59,0	54,19	54,16	112. 37. 4,5	2,77	2,98
276	c Draconis	5	279. 29. 44,0	17,42	17,40	34. 41. 45,7	3,30	3,37
277	* 1 ν Sagittarii	4	279. 55. 9,0	54,30	54,08	113. 0. 56,2	3,33	3,66
278	* σ Sagittarii	[illegible]	280. 5. 40,0	55,78	55,74	116. 34. 7,3	3,51	3,72
279	* 2 ν Sagittarii	4	280. 8. 59,0	54,27	54,24	112. 56. 47,1	3,53	3,73
280	β Lyræ	3	280. 18. 26,0	33,09	33,09	56. 53. 47,8	3,58	3,71

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb of Stars	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan 1, 1760.	An. Preces. in Right Ascension, 1760.	An. Preces. in Right Ascension, 1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	An. Preces. in Declination, 1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
316	* ρ Capricorni	5	303. 47. 15,0	51,52	51,44	108. 35. 19,0	11,19	11,29
317	* υ Capricorni	5	306. 35. 19,0	51,45	51,37	108. 57. 56,8	11,93	12,09
318	α Delphini	[illegible]	307. 7. 27,0	41,63	41,62	74. 55. 13,4	12,08	12,20
319	α Cygni	1	308. 18. 51,3	30,53	30,54	45. 33. 59,9	12,41	12,50
320	ε Aquarii	4	308. 40. 1,0	48,78	46,72	100. 21. 28,1	12,50	12,65
321	ι Cygni	3	309. 7. 39,0	35,82	35,82	56. 55. 6,6	12,63	12,73
322	μ Aquarii	4	309. 55. 21,8	48,60	48,54	99. 52. 3,3	12,84	12,98
323	* 19 Capricorni	4	310. 18. 10,0	51,13	51,04	108. 48. 58,6	12,95	13,09
324	* η Capricorni	5	312. 40. 43,0	51,51	51,42	110. 47. 13,2	13,66	13,71
325	* θ Capricorni	4	313. 6. 23,4	50,72	50,64	108. 10. 12,1	13,74	13,81
326	* 1 χ Capricorni	5	313. 41. 40,0	51,82	51,72	112. 8. 35,8	13,83	13,96
327	* ν Aquarii	5	314. 7. 30,0	49,07	49,00	102. 19. 42,8	13,93	14,07
328	* φ Capricorni	5	315. 29. 5,0	51,49	51,39	111. 37. 59,2	14,27	14,41
329	* 29 Capricorni	5	315. 36. 34,0	49,98	49,90	106. 9. 13,2	14,30	14,43
330	α Equulei	4	315. 57. 23,0	44,89	44,86	85. 43. 52,8	14,39	14,50
331	* ι Capricorni	5	317. 12. 50,0	50,30	50,22	107. 50. 30,7	14,69	14,82
332	α Cephei	3	318. 12. 28,0	21,29	21,24	28. 25. 26,7	14,92	14,97
333	ζ Capricorni	4	318. 13. 48,4	51,71	51,58	113. 26. 8,4	14,93	15,09
334	* b Capricorni	5	318. 45. 3,0	51,46	51,38	112. 50. 16,9	15,05	15,18
335	Aquarii	3	319. 43. 37,8	47,43	47,37	96. 36. 50,8	15,27	15,38
336	* ε Capricorni	4	320. 54. 7,7	50,65	50,55	110. 31. 39,5	15,53	15,65
337	* ξ Aquarii	5,6	321. 14. 24,0	47,89	47,83	98. 55. 2,7	15,61	15,72
338	ρ Cygni	4	321. 14. 34,0	33,60	33,68	45. 27. 38,1	15,61	15,72
339	β Cephei	3	321. 22. 13,0	12,49	12,29	20. 29. 22,2	15,63	15,66
340	γ Capricorni	4	321. 41. 26,0	49,89	49,81	107. 43. 58,5	15,70	15,82
341	* κ Capricorni	5	322. 18. 23,0	50,37	50,27	109. 56. 48,1	15,84	15,95
342	* λ Capricorni	5	323. 23. 57,0	48,56	48,49	102. 27. 37,7	16,07	16,18
343	* δ Capricorni	3	323. 26. 28,9	49,62	49,53	107. 12. 12,9	16,08	16,18
344	2 π Cygni	5	324. 29. 11,0	32,92	33,13	41. 47. 33,7	16,29	16,36
345	* μ Capricorni	5	325. 2. 44,4	48,93	48,85	104. 40. 7,9	16,40	16,51
346	ο Aquarii	5	327. 43. 23,5	46,54	46,50	93. 18. 12,3	16,92	17,01
347	α Aquarii	3	328. 21. 46,1	46,20	46,16	91. 28. 32,8	17,04	17,13
348	ι Aquarii	5	328. 21. 47,5	48,74	48,67	105. 1. 22,8	17,04	17,13
349	* 35 Aquarii	5	328. 56. 50,0	49,62	49,52	109. 41. 0,4	17,15	17,24
350	* θ Aquarii	4	331. 2. 16,2	47,46	47,38	98. 58. 6,0	17,51	17,60

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	1800.	Mean Distance from N. Pole, Jan. 1, 1760.	An. Preces. in Declination, 1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
351	* ε Aquarii	5	331. 53. 18,2	47,42	47,37	99. 1. 0,3	17,69	17,73
352	γ Aquarii	3	332. 18. 47,2	46,35	46,31	92. 35. 47,3	17,72	17,80
353	π Aquarii	4	333. 15. 13,1	45,90	45,87	89. 49. 56,6	17,87	17,95
354	ζ Aquarii	4	334. 6. 57,2	46,12	46,08	91. 14. 21,9	18,01	18,08
355	* σ Aquarii	5	334. 28. 51,9	47,74	47,53	101. 53. 52,2	18,06	18,14
356	7 Lacertæ	4	335. 21. 45,0	36,31	36,40	40. 56. 41,6	18,19	18,25
357	υ Aquarii	5	335. 22. 53,7	49,17	49,07	111. 15. 41,8	18,19	18,27
358	η Aquarii	4	335. 45. 17,1	46,12	46,09	91. 20. 48,2	18,25	18,31
~~359~~	~~* κ Aquarii~~	~~5~~	~~336. 19. 46,6~~	~~46,70~~	~~46,66~~	~~95. 27. 29,5~~	~~18,30~~	~~18,37~~
360	* 1 τ Aquarii	5	338. 44. 16,2	47,91	47 84	105. 18. 51,7	18,65	18,71
361	* 2 τ Aquarii	4	339. 12. 56,9	47,81	47,74	104. 51. 6,6	18,71	18,77
362	* λ Aquarii	4	340. 1. 15,9	46,99	46,94	98. 60. 49,8	18,81	18,87
363	ι Cephei	4	340. 17. 54,0	31,50	31,61	25. 3. 28,9	18,84	18,88
364	δ Aquarii	3	340. 28. 21,6	47,98	47,91	107. 8. 26,6	18,86	18,92
365	Fomalhaut	1	341. 5. 5,0	49,81	49,67	120. 53. 14,0	18,93	18,99
366	β Piscium	4	342 55. 2,0	45,67	45,60	87. 28. 1,7	19,13	19,18
367	β Pegasi	2	343. 2 39,4	42,98	43,04	63. 12. 52,0	19,14	19,19
368	* 1 h Aquarii	6	343. 9. 29,0	46,84	46,80	98. 59. 1,8	19,15	19,20
369	* 2 h Aquarii	7	343. 12. 5,5	46,85	46, 0	90. 2. 50,1	19,16	19,21
370	α Pegasi	2	343 12. 22,5	44,49	44,52	76 4. 54,0	19,16	19,21
371	* 3 h Aquarii	7	344. 20 30,6	46,86	46,81	99. 13. 25,7	19,17	19,22
372	* φ Aquarii	4	345 28 15,9	46,56	46,54	97. 20. 17,5	19,37	19,41
373	* 1 ψ Aquarii	5	345. 49. 32,6	46,83	46,78	100. 23. 26,6	19,40	19,44
374	* χ Aquarii	6	346. 5. 59,4	46,69	46,65	99. 1. 50,3	19,43	19,47
375	* 2 ψ Aquarii	5	346. 21. 15,7	46,80	46,75	100. 29. 18,5	19,45	19,49
376	* 3 ψ Aquarii	5	346. 36. 55,3	46,82	46,77	100. 55. 6,3	19,47	19,51
377	* 96 Aquarii	5	346. 44 13,0	46,45	46,41	96. 25. 55,7	19,48	19,52
378	d Cassiopeiæ	5	348. 34. 10,0	38,77	38,99	29. 1. 52,2	19,62	19,64
379	* 1 κ Piscium	5	348. 39. 34,0	45,93	45,92	90. 3. 17,9	19,62	19,65
380	1 λ Andromedæ	4	351. 28. 16,0	42,94	43,06	44. 50. 23,3	19,74	19,76
381	* λ Piscium	5	352. 27. 10,0	45,91	45,90	89. 32. 21,0	19,84	19,96
382	* 19 Piscium	5	353. 32 9,0	45,84	45,84	87. 50. 36,6	19,89	19,90
383	27 Piscium	5	356. 35. 40,0	46,03	46,01	— — —	19,98	19,98
384	* ι Piscium	4	356. 45. 4,0	45,82	45,83	84. 27. 53,0	19,98	19,99
385	* 29 Piscium	5	357. 22. 56,0	46,00	45,98	94. 21. 48,5	19,99	20,00

Dr. BRADLEY's CATALOGUE OF FIXED STARS.

Numb. of Stars.	Names of the Stars.	Magnitude.	Mean Right Ascension, Jan. 1, 1760.	An. Preces. in Right Ascension, 1760.	1800.	Mean Distance, from N. Pole, Jan. 1, 1760.	An. Preces. in Declination. 1760.	1800.
			D. M. S.	S.	S.	D. M. S.	S.	S.
386	* 30 Piscium	5	357. 24. 47,0	46,04	46,02	97. 20. 50,7	19,99	20,00
387	* 33 Piscium	5	358. 15. 47,0	46,00	45,98	97. 3. 0,5	20,00	20,00
388	α Andromedæ	2	359. 0. 25,0	45,74	45,84	62. 14. 5,9	20,01	20,01
389	β Cassiopeiæ	3	359. 7. 40,0	45,45	45,72	32. 10. 25,0	20,01	20,01

The stars marked with Asterisks, are those which may be eclipsed by the moon to any part of the globe.

Those stars whose AR is between 90° and 270° with N. Dec. and more than 270° and less than 90° with S. Dec. have their annual variation of declination *subtractive;* and those stars whose AR is more than 270° and less than 90° with N. Dec. and between 90° and 270° with S. Dec. have their annual variation of declination *additive*. This is to be understood of a time *after* Jan. 1. 1760; before that time, the variation is to be applied with a *contrary* sign.

This catalogue of the right ascensions and north-polar distances of 389 fixed stars, deduced from the observations of Dr. Bradley, is taken from the first volume of his *Observations*, published by Dr. Hornsby. It was first printed in the *Nautical Almanac* for 1773; but Dr. Hornsby re-published it with corrections; for the most important of which he has assigned the following reasons.

2. ι *Ceti*. The right ascension and polar distance of this star were both faulty. The former was deduced from the apparent right ascensions found in 1751, and carried on by precession to 1755 only; the latter was the mean polar distance for January 1, 1758, and was deduced from six observations, the difference of the extremes amounting only to 4",3.

4. λ *Andromedæ*. The polar distance of this star was, through mistake, given for 1758.

10. ι *Piscium*. The polar distance should have been one minute greater; the mistake was committed in reducing the observations of December 21, 1753, and October 28, and November 18, 1754.

θ *Andromedæ*. Having reason to believe the right ascension of this star to be faulty, I reduced the observations in 1756, and 1757, and was enabled to apply the correction.

33. 1 ξ *Ceti*. The catalogue here, at first sight, was erroneous. The polar distances were computed twice from the same observations, viz. December 8, and 26, of 1753, and January 3, and 25, of the following year, but more truly in the second instance, if we except a mistake of 3 degrees committed by the computer, in writing down the reduced zenith distances. In the former, it may be observed the precession was applied for seven years instead of six.

36. 2 ξ *Ceti*. A fault was committed in applying the precession for six years to this star, by writing 9",9 instead of 99",8 = 1'. 39",8.

37. δ *Ceti*. The observations of January 17, 1753, and of January 13, 1755, assuming the right ascension of Aldebaran, and of α Orionis, as settled by Dr. Bradley, give the same right ascension exactly, the extreme observations differing only 4",6.

47. ι *Arietis*. The right ascension of this star was corrected by three observations in 1752.

48. γ *Persei.* The precession applied was one minute too little.

57. f *Tauri.* The right ascension of this star was corrected by the observations of February 18, 1753, of January 11, and December 11, 1754, and of January 22, of the following year.

58. 17 *Eridani.* A mistake was committed in applying the aberration to the observed zenith distances; and it appears, from a loose paper, that the right ascension had been determined to be 49°. 41'. 48".

79. 2 ι *Tauri.* The ascensional difference of this and the preceding star, as given by Mr. ZACH, pouchan, must be faulty. By the observations of January 7, and December 26, 1754, of January 22, 1755, of February 8, 1756, and of January 29, 1757, the mean of the ascensional differences is 5",37 of time = 1'. 20",55, if the observation of January 22, 1755, be corrected by reading 1^h. 15'. 4",0 instead of 1^h. 15'. 5",0 and the observation of January 29, 1757, be diminished by 1". By my own observations in 1779, the ascensional difference was 5",43 of time = 1'. 21",45. By comparing these two stars with Aldebaran on October 15, 1753, and on January 1, and 3, 1754, and assuming the place of Aldebaran as used by Dr. BRADLEY himself (see the ascensional differences), I find the right ascension of the former = 63°. 43'. 18",7 and of the latter 63°. 44'. 47",3 on January 1, 1760, and by my own observations 63°. 43'. 26",6 and 63°. 44'. 49",5. It must be observed, that the observation of October 15, 1753, appears to be faulty, an error of 1" having been committed in the observation of 2 ι.

84. ι *Tauri.* I found, by several comparisons, that the right ascension of this star was to be diminished by 1'. 5",3.

104. 1 χ *Orionis.* An examination of the several observations of this and the following star, shows, that the right ascensions were very nearly determined, if that of February 1, 1752, be excepted, where the star marked 2 χ preceding α Orionis, with a difference of 2'. 2",66, is FLAMSTEAD's 1 χ, and was observed as such December 29, 1752; and the preceding star is, perhaps, the 223d of MAYER's catalogue. The error is therefore probably in the polar distance of 2 χ, and upon reducing the several observations, I find the polar distance of 2 χ, as given in the catalogue of 1773, to belong to FLAMSTEAD's 4 χ, the right ascension of which has not been calculated. The order in which the zenith distances were observed (see the observations of January 18, 1754, where the book of observations is exactly copied, notwithstanding the obvious error committed in the case of this very star, and also of February 15,

1754,) prove that the right ascension was greater than that of α Orionis; and very nearly the same with that of H. Geminorum: on these two days therefore for 2 χ we should read 3 χ Orionis.

I subjoin the right ascension of 3 χ, and also the polar distances of 3 χ and 4 χ, reduced to January 1, 1760.

3 χ Orionis	6	87°. 25'. 3",5	55",25	53",26	69°. 52'. 46",5	0",90	0",69
4 χ Orionis	6	— — —	56,08	53,09	70. 19. 26, 1	[illegible]	0,47

129. ι *Geminorum.* Having compared this star with several principal stars observed on the same day, assuming their places as determined and used by Dr. Bradley himself, I find its right ascension to be

From the observations of	1751	-	110°. 16'. 33",9
	1752	-	110. 16. 38, 9
	1754	-	110. 16. 30, 2
	1755	-	110. 16. 32, 4
	1756	-	110. 16. 35, 1
By a mean		- - - - -	110. 16. 34, 1

An error therefore of 1" was committed.

155. 10 *Leonis.* The right ascension and polar distance of this star, as given in the catalogue published in 1773, were deduced each from one observation only: and from the situation of the star, I conjectured that the right ascension belonged to 10 Leonis minoris, and would, perhaps, have been exact, if the proper corrections peculiar to that star, for precession, aberration, and nutation, had been applied. The observations of the 19th and 28th of March, and of the 12th of April, 1754, prove 10 Leonis to be the same with the first of the Sextant of Hevelius, and the same conclusion may be drawn from the British catalogue. The zenith distances both of 10 Leonis minoris, and 10 Leonis, were observed on the 12th of April; we are enabled therefore to determine the place of each star.

168. 37 *Sextantis.* In computing the polar distance, an error of 1' was committed.

194. n *Virginis.* The polar distance of this star is evidently too great, and the observations of March 9, and May 4, 1754, enable us to make the cor-

rection; for on those days the stars n and ν Virginis were both observed; and their situation is so nearly the same, that the differences of the zenith distances need only be applied to the calculated polar distance of ν.

208. 21 *Virginis.* The error of 1° in the right ascension was probably committed at the press, and also of the letter of reference.

223. 4 ξ *Libræ.* By computing the observations of June 7, and 19, 1755, and of May 16, 1756, I applied the correction.

253. μ *Draconis.* The right ascension of this star was unquestionably faulty. The observations of July 30, 1755, and of March 1, 1758, enable us to make the correction. I suspect that the right ascension first found was 255°. 5′. 49″.

333. ξ *Capricorni.* In reducing the right ascension of this star from 1755 to 1760, the precession used was 10″ too small: it should have been printed therefore 311°. 13′. 52″,0, or more exactly 51″,8.

360. 1 τ *Aquarii.* The observations of September 28, and October 3, 1753, do not justify the application of any correction to the polar distance of this or the following star.

388. α *Andromedæ.* In reducing seven observations of 1753, the refraction for 33°. 44′ was applied instead of 23°. 44′; a correction therefore for the polar distance was necessary.

3

M. DE LA CAILLE's CATALOGUE OF 515 ZODIACAL STARS,

FOR THE BEGINNING OF 1765.

Numb. of Stars.	Names of the Constellations.	BAYER's Letters.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Declination.	Annual Variation.
				D. M. S.	S.	H. M. S.	D. M. S.	S.
1	Ceti	ι	3	1. 51. 53,3	46,14	0. 7. 28	10. 7. 44,9S.	—20,06
2	Piscium	d	6	2. 7. 43,5	46,29	0. 8. 31	6. 53. 6,4N	+20,06
3	Ceti		6	3. 38. 39,4	46,17	0. 14. 35	1. 21. 9,3S.	—20,04
4	Ceti		6	5. 47. 23,2	46,03	0. 23. 10	4. 53. 24,8S.	—20,00
5	Piscium	λ	6	8. 49. 0,0	46,49	0. 35. 16	5. 27. 19,9N	+19,89
6	Piscium	δ	4	9. 7. 36,8	46,55	0. 36. 30	6. 18. 11,0N	+19,88
7	Ceti		6	10. 15. 15,8	46,06	0. 41. 1	2. 25. 24,6S.	—19,82
8	Piscium		7	11. 55. 2,8	46,56	0. 47. 40	5. 12. 47,3N	+19,72
9	Piscium	ε	4	12. 41. 25,4	46,70	0. 50. 46	6. 37. 14,5N	+19,68
10			7	12. 56. 11,7	46,20	0. 51. 45	0. 6. 15,6N	+19,66
11			6	13. 10. 52,2	46,52	0. 52. 43	4. 23. 39,1N	+19,65
12			7	13. 58. 29,0	46,26	0. 55. 54	0. 45. 39,5N	+19,58
13		e	5	14. 4. 12,6	46,54	0. 56. 17	4. 24. 13,0N	+19,57
14			7	14. 37. 18,4	46,33	0. 58. 29	1. 11. 30,0N	+19,53
15			7	15. 7. 13,6	46,29	1. 0. 29	1. 13. 27,5N	+19,49
16		ζ	4	15. 22. 1,0	46,79	1. 1. 28	6. 19. 40,7N	+19,47
17			6,7	15. 37. 41,3	46,73	1. 2. 31	5. 45. 52,8N	+19,44
18	Ceti			15. 42. 32,6	45,99	1. 2. 50	2. 14. 0,0S.	—19,44
19	Piscium	f	6	16. 25. 19,3	46,42	1. 5. 41	2. 22. 20,0N	+19,37
20	1	ρ	5	18. 24. 22,1	48,25	1. 13. 37	17. 56. 35,1N	+19,18
21	2	ρ	5	18. 30. 37,8	48,26	1. 14. 3	18. 1. 0,9N	+19,17
22				19. 1. 19,5	46,97	1. 16. 5	6. 44. 10,6N	+19,12
23			6,7	19. 12. 12,0	46,89	1. 16. 49	6. 4. 23,5N	+19,10
24		μ	5	19. 28. 20,0	46,78	1. 17. 53	4. 55. 39,2N	+19,07
25		η	4	19. 44. 4,3	47,89	1. 18. 57	14. 7. 37,4N	+19,04
26			6	20. 36. 8,4	47,61	1. 22. 25	11. 20. 55,9N	+18,94
27	1	π	5	21. 10. 1,2	47,61	1. 24. 40	10. 55. 58,3N	+18,87
28	2	π		21. 17. 52,2	47,62	1. 25. 12	10. 52. 26,5N	+18,85
29				21. 39. 18,6	46,94	1. 26. 37	5. 44. 50,0N	+18,81
30			6,7	21. 45. 32,0	48,22	1. 27. 2	15. 12. 22,4N	+18,80
31		ν	5	22. 18. 19,3	46,78	1. 29 13	4. 17. 31,7N	+18,73
32		ο	5	23. 15 7,7	47,21	1. 32. 1	7. 58. 5,1N	+18,62
33				24. 4. 16,9	4[illegible],56	1. 36. 17	2. 30. 23,4N	+18,51
34				24. 36. 28,1	48,92	1 38. 20	17. 53. 4,9N	+18,42
35	Arietis	γ	4	25. 9. 57,5	49,05	1. 40. 40	18. 8. 10,1N	+18,37

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36	Piscium		[illegible]		25. 21. 15,5	46,50	1. 41. 25	2. 1. 15,8N	+18,34
37	Arietis		[illegible]	3	25. 25. 17,9	49,29	1. 41. 41	19. 39. 6,3N	+18,32
38	Piscium		[illegible]	6	26. 8. 10,5	48,85	1. 44. 33	16. 39. 41,3N	+18,23
39	Piscium		[illegible]	3	27. 22. 40,6	46,55	1. 49. 35	1. 37. 16,4N	+18,01
40	Arietis		[illegible]	5,6	28. 21. 53,4	49,94	1. 53. 27	21. 31. 24,4N	+17,88
41			[illegible]	2	28. 29. 34,3	50,01	1. 53. 58	22. 20. 32,3N	+17,86
42	Ceti			6	29. 44. 38,4	47,51	1. 58. 58	7. 27. 36,9N	+17,64
43	Arietis		[illegible]	[illegible]	29. 55. 17,1	49,85	1. 59. 41	29. 5. 47,5N	+17,61
44				6,7	30. 4. 2,9	48,73	2. 0. 16	14. 10. 7,8N	+17,59
45	Ceti	1	[illegible]	6	30. 8. 31,7	47,57	2. 0. 34	7. 44. 8,2N	+17,58
46	Arietis		[illegible]	5,6	31. 16. 23,8	49,74	2. 5. 6	18. 48. 10,9N	+17,38
47	Ceti	2	[illegible]	4,5	33. 3. 40,3	48,05	2. 12. 15	9. 32. 13,2N	+17,07
48				7	33. 40. 50,2	48,60	2. 14. 35	9. 9. 47,1N	+16,94
49	Ceti	3	[illegible]	4,5	33. 56. 19,6	47,63	2. 15. 41	7. 24. 49,0N	+16,90
50	Arietis			6,7	34. 22. 30,8	50,04	2. 17. 30	18. 48. 1,0N	+16,82
51				6,7	35. 1. 1,2	49,07	2. 20. 4	13. 59. 6,1N	+16,69
52	Ceti		[illegible]	4,5	35. 58. 25,7	47,12	2. 23. 54	4. 32. 29,6N	+16,53
53	Arietis			5,6	35. 57. 44,2	48,60	2. 23. 51	11. 26. 43,8N	+16,51
54			[illegible]	4,5	36. 22. 41,2	50,75	2. 25. 31	20. 55. 51,5N	+16,43
55			[illegible]	6	37. 17. 19,0	52,81	2. 29. 9	23. 29. 52,3N	+16,24
56	Ceti				37. 27. 57,1	48,18	2. 29. 52	9. 13. 43,2N	+16,21
57	Ceti		[illegible]	3	37. 47. 14,6	46,80	2. 31. 9	2. 14. 6,5N	+16,14
58	Arietis		[illegible]	6	37. 54. 28,2	49,36	2. 31. 38	14. 18. 15,4N	+16,12
59				7	38. 2. 42,8	48,71	2. 32. 11	11. 26. 40,8N	+16,09
60	Ceti		[illegible]	4	38. 3. 58,5	48,19	2. 32. 16	9. 6. 37,7N	+16,09
61	Arietis			6	38. 50. 57,7	50,12	2. 35. 24	17. 17. 31,0N	+15,92
62			[illegible]	6	39. 3. 12,0	49,94	2. 36. 13	16. 28. 24,5N	+15,88
63			[illegible]	6	39. 38. 12,0	49,41	2. 38. 33	14. 6. 0,2N	+15,75
64				6	40. 26. 51,5	50,13	2. 41. 47	16. 46. 1,0N	+15,57
65		1	[illegible]	6,7	40. 39. 37,7	50,27	2. 42. 39	17. 21. 56,2N	+15,53
66		2	[illegible]	6,7	40. 48. 4,8	50,23	2. 43. 12	17. 4. 22,2N	+15,49
67				6,7	41. 10. 7,1	50,93	2. 44. 40	19. 43. 0,0N	+15,41
68			[illegible]	5	41. 27. 13,3	51,14	2. 45. 49	20. 23. 13,3N	+15,35
69	Ceti		λ	4	41. 47. 10,8	48,06	2. 47. 9	7. 57. 36,0N	+15,27
70	Arietis				41. 50. 20,0	50,31	2. 47. 22	17. 3. 33,9N	+15,26

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
71			7	43. 33. 31,0	50,43	2. 54. 14	16. 58. 33,4N	+14,87
72			6,7	43. 46. 3,8	50,67	2. 55 4	17. 52. 45,6N	+14,81
73		δ	4	44. 33. 25,1	50,73	2. 58. 14	18. 49. 23,0N	+14,63
74		ζ	5	45. 21. 28,0	51,44	3. 1. 26	20. 10. 27,8N	+14,44
75				46. 50. 48,0	51,36	3. 7. 23	19. 38. 48,6N	+14,07
76	1	τ	7	46. 55. 26,8	51,62	3. 7. 42	20. 16. 56,9N	+14,05
77	Arictis 2	τ	6	47. 19. 0,8	51,54	3. 9. 16	19. 53. 9,8N	+13,95
78				47. 37. 3,1	52,75	3. 10. 28	23. 52. 27,6N	+13,86
79	3	τ	7	47. 43. 50,2	51,59	3. 10 55	19. 57. 11,5N	+13,83
80	Tauri	ε	4	48. 2. 52,6	48,30	3. 12. 11	8. 11. 26,2N	+13,76
81	Tauri	ξ	4	48. 36. 54,2	48,85	3. 14. 28	9. 57. 4,9N	+13,61
82	Arietis		7	48. 41. 7,9	52,27	3. 14. 45	21. 58. 38,6N	+13,59
83	Tauri		6	49. 23. 54,2	48,99	3. 17. 36	10. 30. 59,6N	+13,42
84		f	5	49. 28. 55,6	49,58	3. 17. 56	12. 7. 2,2N	+13,40
85		t	6	49. 58. 36,1	48,51	3. 19. 54	8. 33. 55,8N	+13,26
86			6	50. 8. 27,4	52,83	3. 20. 34	23. 39. 30,8N	+13,21
87				51. 32. 36,0	50,45	3. 26. 8	15. 45. 28,9N	+12,85
88			6	51. 41. 36,1	53,39	3. 26. 46	24. 33. 0,3N	+12,81
89				52. 11. 56,1	51,95	3. 28. 47	19. 55. 53,9N	+12,68
90				52. 33. 32,3	51,64	3. 30. 14	18. 54. 20,4N	+12,59
91	Celeno	g	7	52. 43. 4,0	53,14	3. 30. 25	23. 31. 52,0N	+12,55
92	Electra	b	5	52. 44. 22,2	53,08	3. 30. 57	23. 21. 20,2N	+12,55
93	Asterope	m	7	52. 47. 41,9	53,38	3. 31. 14	24. 5. 0,2N	+12,54
94	Taygeta	e	5	52. 48. 50,9	53,20	3. 31. 15	23. 42. 41,0N	+12,53
95	Maia	c	6	52. 58. 14,1	53,19	3. 31. 53	23. 36. 50,9N	+12,47
96	Merope	d	5	53. 6. 19,0	53,07	3. 32. 25	23. 12. 56,5N	+12,43
97		η	3	53. 23. 6,9	53,15	3. 33. 32	23. 21. 42,6N	+12,37
98	Atlas	f	6	53. 48. 23,9	53,19	3. 35. 14	23. 18. 54,2N	+12,25
99	Pleyone	h	7,8	53. 48. 41,5	53,20	3. 35. 15	23. 24. 53,1N	+12,25
100				54. 2. 4,0	52,59	3. 36. 8	21. 30. 49,5N	+12,17
101				54. 3. 22,7	53,72	3. 36. 13	24. 51. 5,4N	+12,17
102			6	55. 45. 9,1	52,81	3. 43. 1	21. 47. 50,3N	+11,68
103		λ	4	56. 55. 18,0	49,71	3. 47. 41	11. 48. 39,8N	+11,34
104			7	57. 35. 8,1	53,52	3. 50. 21	23. 24. 16,4N	+11,15
105	1	A	5	57. 42. 29,3	52,85	3. 50. 50	21. 25. 9,7N	+11,12

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					D. M. S.	S.	H. M. S.	D. M. S.	S.
106		2	A		57. 51. 42,4	52,82	3. 51. 27	21. 21. 18,9N	+ 11,07
107			ψ	5	58. 7. 47,3	54,93	3. 52. 31	28. 10. 44,7N	+ 11,00
108		1	ω	6	58. 52. 32,4	52,11	3. 55. 30	18. 58. 10,2N	+ 10,77
109			p	6	59. 8. 26,8	54,59	3. 56. 34	25. 50. 49,4N	+ 10,69
110					59. 44. 14,2	53,11	3. 58. 57	21. 47. 20,5N	+ 10,61
111					60. 36. 42,3	50,86	4. 2. 27	14. 47. 43,8N	+ 10,26
112			μ	4	60. 41. 57,4	48,74	4. 2. 48	8. 17. 20,3N	+ 10,24
113		2	ω	6	60. 52. 49,2	52,58	4. 3. 31	19. 58. 50,6N	+ 10,19
114				7	61. 7. 34,1	52,87	4. 4. 30	20. 59. 6,5N	+ 10,11
115			φ	5	61. 28. 55,3	54,88	4. 5. 56	26. 46. 3,1N	+ 9,99
116			γ	3	61. 36. 37,0	50,92	4. 6. 26	15. 2. 36,2N	+ 9,95
117	Tauri			5	62. 4. 37,0	54,42	4. 8. 18	25. 3. 9,0N	+ 9,81
118		1	δ	4	62. 21. 7,2	51,68	4. 9. 24	16. 58. 25,1N	+ 9,73
119				7	62. 27. 44,7	54,08	4. 9. 51	23. 43. 59,4N	+ 9,70
120		2	δ	4	62. 38. 36,9	51,57	4. 10. 34	16. 52. 53,4N	+ 9,64
121		1	κ	5	62. 50. 55,2	53,83	4. 11. 24	21. 44. 5,5N	+ 9,58
122		2	κ	5	62. 51. 42,7	53,83	4. 11. 27	21. 37. 28,7N	+ 9,58
123		3	δ	6	62. 58. 46,4	51,79	4. 11. 55	17. 22. 20,2N	+ 9,54
124		1	υ	5	63. 4. 7,1	53,50	4. 12. 16	22. 15. 31,8N	+ 9,51
125				7	63. 14. 48,5	51,01	4. 12. 59	15. 4. 10,7N	+ 9,46
126		[illegible]	υ	6	63. 18. 57,7	53,53	4. 13. 16	22. 16. 41,3N	+ 9,44
127			ε	3,4	63. 43. 42,8	52,28	4. 14. 55	18. 38. 27,5N	+ 9,30
128		1	θ	5	63. 47. 32,4	51,17	4. 15. 10	15. 25. 21,3N	+ 9,28
129		2	θ	5	63. 50. 2,0	51,14	4. 15. 20	15. 19. 54,2N	+ 9,27
130				7	64. 11. 30,2	51,07	4. 16. 46	15. 7. 17,2N	+ 9,16
131				7	64. 18. 57,6	51,07	4. 17. 16	15. 9. 40,4N	+ 9,12
132				7	64. 36. 56,9	51,16	4. 18. 28	15. 19. 45,2N	+ 9,03
133			ξ	5	65. 7. 59,2	50,83	4. 20. 32	14. 19. 55,7N	+ 8,88
134			α	1	65. 36. 53,7	51,45	4. 22. 28	16. 1. 9,4N	+ 8,71
135			d	5	65. 41. 28,6	49,32	4. 22. 46	9. 39. 43,8N	+ 8,69
136	Eridani		ν	4	66. 8. 56,1	44,97	4. 24. 36	3. 50. 41,0S	— 8,55
137	Tauri	1	σ	6	66. 26. 19,1	51,14	4. 25. 45	15. 20. 7,0N	+ 8,49
138		2	σ	6	66. 27. 46,6	51,17	4. 25. 51	15. 26. 3,6N	+ 8,49
139			τ	5	67. 2. 28,2	53,85	4. 28. 10	22. 29. 3,1N	+ 8,26
140					67. 15. 22,0	53,90	4. 29. 1	23. 38. 19,7N	+ 8,19

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
176	Orionis 3	χ	5	87. 29. 24,4	53,54	5. 49. 58	20. 7. 23,2N	+ 1,35
177	Orionis	ν	4,5	88. 32. 18,3	51,48	5. 54. 9	14. 46. 37,3N	+ 0,99
178			6	89. 31. 16,0	53,44	5. 58. 5	19. 49. 25,1N	+ 0,64
179	1	f	6	89. 37. 32,4	52,02	5. 58. 30	16. 9. 55,8N	+ 0,61
180	Geminorum	η	4,5	90. 10. 15,3	54,52	6. 0. 41	22. 33. 14,2N	+ 0,41
181	Orionis 2	f	[illegible]	90. 28. 0,3	52,03	6. 1. 52	16. 11. 45,8N	+ 0,30
182	Geminorum	μ	[illegible]	92. 10. 57,4	54,51	6. 8. 44	22. 36. 47,0N	— 0,29
183				92. 48. 41,2	54,87	6. 11. 15	23. 26. 4,9N	— 0,51
184				92. 48. 51,9	54,91	6. 11. 16	23. 32. 51,1N	— 0,51
185		ι	4	93. 44. 57,7	53,62	6. 15. 0	20. 20. 27,7N	— 0,85
186			7,8	94. 38. 52,0	52,68	6. 18. 35	17. 55. 53,1N	— 1,14
187		γ	2,3	96. 1. 52,1	52,02	6. 24. 7	16. 34. 49,1N	— 1,63
188			5	97. 10. 42,3	52,59	6. 28. 43	17. 51. 18,9N	— 2,04
189		[illegible]	[illegible]	97. 21. 59,5	55,60	6. 29. 28	25. 20. 24,7N	— 2,10
190		[illegible]	6	99. 21. 41,9	54,20	6. 37. 27	22. 1. 2,0N	— 2,80
191			6	101. 13. 58,4	55,83	6. 44. 56	26. 12. 39,9N	— 3,45
192	1	ω	6	102. 1. 2,9	55,14	6. 48. 4	24. 31. 39,2N	— 3,71
193		ζ	3,4	102. 32. 12,0	53,67	6. 50. 9	20. 53. 41,8N	— 3,89
194	2	ω	6,7	102. 47. 2,0	53,47	6. 51. 8	22. 58. 12,0N	— 3,97
195		τ	5	104. 2. 9,5	57,74	6. 56. 9	30. 36. 23,5N	— 4,40
196		m	6	104. 32. 0,9	55,05	6. 58. 8	24. 29. 57,4N	— 4,57
197	Geminorum 1	λ	5,6	104. 57. 53,1	51,97	6. 59. 52	16. 32. 26,1N	— 4,66
198		n	6,7	105. 4. 28,7	55,55	7. 0. 18	25. 17. 12,9N	— 4,76
199	2	λ	5	106. 8. 31,3	51,09	7. 4. 34	16. 56. 50,9N	— 5,12
200		δ	3	106. 30. 54,7	54,12	7. 6. 4	22. 23. 42,5N	— 5,25
201		q	6,7	107. 0. 52,3	53,52	7. 8. 3	20. 52. 0,9N	— 5,42
202		A	5,6	107. 16. 47,1	55,33	7. 9. 4	25. 28. 48,6N	— 5,51
203	1	ι	6	107. 28. 32,7	56,38	7. 9. 54	28. 4. 8,2N	— 5,57
204	2	ι	4,5	107. 46. 24,7	56,45	7. 11. 6	28. 15. 31,8N	— 5,68
205		r	6	108. 16. 7,9	53,41	7. 13. 4	20. 42. 34,2N	— 5,84
[illegible]		p	6	108. 26. 17,4	53,86	7. 13. 45	21. 54. 15,9N	— 5,90
207	1	b	6	108. 39. 56,3	56,56	7. 14. 40	28. 34. 52,3N	— 5,98
208	2	b	6	108. 47. 26,2	56,45	7. 15. 10	28. 22. 40,9N	— 6,02
209		α	1	109. 53. 33,5	58,05	7. 19. 34	32. 22. 54,4N	— 6,39
210		k	6	110. 2. 35,1	51,73	7. 20. 10	16. 18. 50,6N	— 6,43

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				D. M. S.	S.	H. M. S.	D. M. S.	[illegible]
211		ν	5	110. 21. 7,0	55,94	7. 21. 24	27. 23. 48,8N	— 6,54
212		f	[illegible]	111. 28. 24,5	52,33	7. 25. 54	18. 11. 29,8N	— 6,91
213		σ	5	112. 8. 42,8	52,39	7. 28. 35	18. 25. 49,0N	— 7,12
214		c	6	112. 26. 13,1	55,37	7. 29. 45	26. 19. 24,4N	— 7,22
215		κ	4,5	112. 34. 31,6	54,80	7. 30. 18	24. 56. 28,2N	— 7,26
216		β	2	112. 43. 41,9	56,26	7. 30. 55	28. 34. 27,6N	— 7,31
217		g	6	113. 7. 21,1	52,57	7. 32. 29	19. 3. 51,8N	— 7,43
218			6	113. 37. 13,1	54,28	7. 34. 29	23. 42. 15,0N	— 7,59
219		φ	5	114. 46. 5,3	55,61	7. 39. 4	27. 21. 4,2N	— 7,96
220		l	6	115. 28. 51,6	51,90	7. 41. 55	20. 29. 14,0N	— 8,21
[illegible]				115. 54. 30,4	51,51	7. 43. 38	16. 24. 9,5N	— 8,35
222	Cancri 1	ω	6	116. 40. 9,0	54,93	7. 46. 41	26. 0. 57,5N	— 8,59
223	2	ω	6	116. 52. 44,5	54,80	7. 47. 31	25. 42. 55,7N	— 8,65
224	Geminorum	χ	5	117. 15. 34,6	55,86	7. 49. 2	28. 26. 2,8N	— 8,78
225	Cancri	μ	5	118. 28. 34,6	52,41	7. 53. 54	22. 14. 47,6N	— 9,15
[illegible]	[illegible]	ψ	[illegible]	119. 3. 51,2	54,84	7. 56. 15	26. 13. 6,5N	— 9,32
227	3	ψ	5	119. 37. 58,0	56,36	7. 58. 32	30. 20. 27,2N	— 9,50
228		ζ	5,6	119. 40. 36,7	51,96	7. 58. 42	18. 20. 27,0N	— 9,51
229		β	3,4	120. 56. 24,4	49,21	8. 3. 46	9. 53. 35,2N	— 9,91
230		χ	6	121. 26. 5,8	55,30	8. 5. 44	27. 57. 32,8N	—10,06
231		λ	6	121. 37. 50,3	54.04	8. 6. 31	24. 44. 48,5N	—10,11
232		φ	6	123. 7. 55,0	55,03	8. 12. 32	27. 41. 6,7N	—10,56
233	2	ν	6	123. 39. 26,9	53,96	8. 14. 38	24. 54. 23,0N	—10,68
234	3	ν	6,7	124. 23. 37,2	53,86	8. 17. 34	24. 51. 23,5N	—10,94
235		θ	5,6	124. 32. 28,1	51,85	8. 18. 10	18. 52. 23,2N	—10,87
236	4	ν	6,8	124. 46. 6,9	53,86	8. 19. 4	24. 52. 7,2N	—11,06
237	Cancri		6	126. 28. 16,2	52,34	8. 25. 53	20. 50. 17,9N	—11,55
238		γ	4	127. 25. 4,8	52,72	8. 29. 40	22. 17. 56,1N	—11,84
239		δ	4	127. 49. 34,3	51,75	8. 31. 18	19. 0. 19,2N	—11,94
240			7	129. 28. 37,1	50,70	8. 37. 54	16. 12. 34,8N	—12,40
[illegible]	Hydræ	ζ	4	130. 43. 29,0	47,79	8. 42. 54	6. 49. 40,7N	—12,74
[illegible]	Cancri	ο	6	131. 1. 46,0	50,61	8. 44. 7	16. 12. 37,5N	—12,81
[illegible]		[illegible]	4	131. 24. 11,0	49,61	8. 45. 37	12. 45. 18,6N	—12,91
[illegible]		[illegible]	6	132. 14. 19,3	53,22	8. 48. 57	25. 21. 44,9N	—13,13
[illegible]		τ	6,7	133. 27. 29,2	54,81	8. 53. 50	30. 34. 56,9N	—13,45

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
246	Lyncis			133. 35. 7,6	56,24	8. 54. 21	34. 49. 20,6N	—13,48
247	Cancri	ξ	5,6	133. 56. 6,0	52,32	8. 55. 44	22. 59. 2,4N	—13,56
248				135. 7. 58,1	56,33	9. 0. 32	35. 35. 15,6N	—13,88
249		π	7	135. 33. 22,8	50,20	9. 2. 13	15. 54. 10,8N	—13,98
250				136. 42. 5,4	50,86	9. 6. 48	18. 41. 22,1N	—14,27
251	Leonis	κ	4	137. 43. 43,5	53,13	9. 10. 55	27. 10. 51,3N	—14,27
252		ω	5	138. 58. 14,0	48,54	9. 15. 53	10. 4. 6,9N	—14,84
253		ξ	4	139. 48. 50,2	49,03	9. 19. 15	12. 20. 52,0N	—15,03
254		h	6	139. 50. 7,3	48,66	9. 19. 20	10. 44. 27,7N	—15,04
255			5	141. 11. 48,1	47,93	9. 24. 47	7. 52. 47,2N	—15,33
256		ο	4	142. 8. 52,0	48,60	9. 28. 36	10. 57. 6,3N	—15,55
257		ψ	6	142. 43. 35,0	49,48	9. 30. 54	15. 5. 4,2N	—15,67
258		ε	3	143. 7. 0,2	51,59	9. 32. 28	24. 50. 39,2N	—15,76
259		ν	4,5	146. 23. 22,0	48,88	9. 45. 33	13. 33. 22,4N	—16,44
260		π	4	146. 56. 41,2	47,97	9. 47. 47	9. 9. 48,3N	—16,49
261	Hydræ 2	ν	5	148. 25. 25,3	43,98	9. 53. 42	11. 55. 59,9S.	+16,84
262	Leonis	η	3	148. 37. 15,1	49,57	9. 54. 29	17. 54. 5,4N	—16,88
263		A	5	148. 51. 13,5	48,23	9. 55. 25	11. 8. 28,9N	—16,93
264	Regulus	α	1	148. 57. 31,6	48,61	9. 55. 50	13. 6. 34,9N	—16,95
265	Sextantis			149. 37. 12,0	44,91	9. 58. 29	7. 16. 4,8S.	+17,06
266	Sextantis		6	149. 51. 34,0	44,91	9. 59. 26	7. 16. 9,1S.	+17,11
267	Hydræ	λ	4	149. 55. 51,6	44,21	9. 59. 43	11. 12. 7,0S.	+17,12
268	Sextantis		6	151. 29. 31,4	45,05	10. 5. 58	6. 54. 18,3S.	+17,40
269	Leonis 1	γ	6	151. 43. 38,3	49,78	10. 6. 55	20. 39. 19,2N	—17,49
270	2	γ	3	151. 44. 30,9	49,84	10. 6. 58	21. 1. 26,2N	—17,49
271	Sextantis			153. 29. 58,5	45,28	10. 14. 0	5. 52. 56,2S.	+17,74
272	Leonis	ι	6	154. 54. 25,0	48,53	10. 19. 38	15. 20. 13,4N	—17,95
273		ρ	4	155. 6. 13,9	47,77	10. 20. 25	10. 30. 41,0N	—18,00
274			6	155. 40. 13,8	47,84	10. 22. 41	9. 51. 28,9N	—18,09
275	Hydræ	φ	5	156. 47. 14,3	43,97	10. 27. 9	15. 39. 53,6S.	+18,26
276	Leonis	k	6	158. 29. 19,5	48,26	10. 33. 57	15. 25. 49,7N	—18,49
277		l	6	159. 13. 18,3	47,61	10. 36. 53	11. 47. 7,8N	—18,59
278	Hydræ	ν	4	159. 30. 43,8	44,31	10. 38. 3	14. 58. 8,5S.	+18,62
279				159. 37. 51,5	45,26	10. 38. 31	7. 39. 35,0S.	+18,64
280	Leonis		5,6	160. 54. 15,1	46,43	10. 43. 37	1. 59. 7,7N	—18,80

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281	1	c	6,7	160. 57. 13,0	47,06	10. 43. 49	7. 26. 5,5N	—18,81
282		d	5,6	162. 6. 16,4	46,72	10. 48. 25	4. 52. 50,4N	—18,95
283	2	c	5	162. 8. 19,4	47,00	10. 48. 33	7. 21. 36,8N	—18,95
284				162. 48. 36,1	46,08	10. 51. 14	1. 14. 34,2S.	+19,02
[illegible]		g	6	162. 53. 36,1	46,32	10. 51. 35	1. 15. 39,0N	—19,03
286		χ	4,5	163. 13. 16,5	47,08	10. 52. 53	8. 36. 12,5N	—19,07
287			6	163. 43. 46,6	46,51	10. 54. 55	3. 13. 30,2N	—19,14
288			5,6	165. 26. 2,0	46,31	11. 1. 44	1. 12. 16,9N	—19,30
289		θ	3	165. 28. 15,1	47,71	11. 1. 53	16. 42. 39,9N	—19,30
290		h	6	165. 53. 17,0	47,46	11. 3. 33	14. 35. 7,0N	—19,34
291			6	166. 17. 54,9	46,47	11. 5. 12	3. 18. 2,2N	—19,38
292	Crateris	δ	4	166. 54. 7,6	45,12	11. 7. 37	13. 30. 29,5S.	+19,43
293	Leonis	σ	4,5	167. 15. 13,1	46,77	11. 9. 1	7. 19. 57,6N	—19,46
294			5,6	167. 59. 37,8	46,38	11. 11. 58	2. 41. 46,9N	—19,52
295	Crateris	ε	4	168. 11. 23,8	45,50	11. 12. 46	9. 34. 26,4S.	+19,54
296	Leonis	τ	[illegible]	168. 58. 3,5	46,47	11. 15. 52	4. 8. 57,6N	—19,60
297				170. 35. 1,7	46,45	11. 22. 20	4. 21. 53,0N	—19,71
298	Crateris	θ	4	171. 11. 42,1	45,73	11. 24. 47	8. 30. 14,0S.	+19,75
299	Virginis	κ	6	171. 35. 4,9	46,71	11. 26. 20	9. 25. 58,7N	—19,77
300	1	ξ	5	173. 17. 30,0	46,60	11. 33. 10	9. 34. 48,3N	—19,86
301		ν	5	173. 26. 54,0	46,52	11. 33. 47	7. 49. 46,0N	—19,87
302	[illegible]	ξ	6	173. 57. 33,3	46,56	11. 35. 50	9. 32. 58,9N	—19,90
303		β	3	174. 36. 37,8	46,30	11. 38. 27	3. 5. 26,1N	—19,93
304		A	6	175. 44. 50,0	46,48	11. 42. 59	9. 45. 3,8N	—19,97
305		b	5,6	176. 58. 47,0	46,29	11. 47. 55	4. 57. 58,6N	—20,01
306		π	5	177. 12. 30,8	46,34	11. 49. 14	7. 54. 35,6N	—20,01
307		o	5	178. 18. 35,4	46,37	11. 53. 14	10. 2. 25,7N	—20,04
308		r	6	179. 24. 50,8	46,22	11. 57. 39	3. 13. 12,6N	—20,05
309	Virginis	s	6	179. 31. 12,2	46,24	11. 58. 5	7. 6. 56,7N	—20,05
310		n	6	181. 39. 33,9	46,18	12. 6. 38	0. 31. 19,8N	—20,06
311		η	3,4	181. 58. 22,1	46,18	12. 7. 53	0. 38. 35,3N	—20,06
312		c	3,4	182. 6. 20,6	46,15	12. 8. 25	4. 37. 35,0N	—20,06
313	Corvi	δ	3	184. 59. 57,6	46,68	12. 20. 0	15. 12. 11,4S.	+20,02
314	Virginis	q	6	185. 25. 3,8	46,47	12. 21. 40	8. 9. 7,8S.	+20,01
315		f	6	186. 10. 39,6	46,37	12. 24. 43	4. 32. 1,6S.	+20,00

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
316		χ	5	186. 47. 10,5	46,47	12. 27. 9	6. 42. 3,7 S	+19,98
317	Virginis	γ	3	187. 26. 39,6	46,24	12. 29. 47	0. 9. 18,9 S	+19,95
318			6	190. 17. 39,1	46,33	12. 41. 11	2. 16. 18,6 S	+19,81
319		ψ	3	190. 32. 26,3	46,72	12. 42. 10	8. 15. 33.4 S	+19,80
320		δ	3	190. 56. 47,7	45,89	12. 43. 47	4. 40. 52,0N	—19,78
321	1	κ	6	191. 53. 34,0	46,37	12. 47. 34	2. 32. 28,2 S	+19,72
322	2	κ	6	192. 7. 30,0	46,35	12. 48. 30	2. 5. 58,1 S	+19,71
323		g	5	193. 54. 20,4	46.98	12. 55. 37	9. 29. 38,3 S	+19,58
324		θ	3,4	194. 27. 7,3	46,58	12. 57. 49	4. 16. 37,4 S	+19,54
325				197. 0. 18,1	47,14	13. 8. 1	9. 4. 4,0 S	+19,32
326		α	1,2	198. 12. 43,4	47,29	13. 12. 51	9. 55. 36,5 S	+19,20
327		i	4	198. 35. 7,8	47,49	13. 14. 21	11. 28. 33,3 S	+19,16
328		l	6	199. 56. 38,6	46,80	13. 19. 47	5. 0. 39,8 S	+19,12
329			6	200. 5. 10,4	47,92	13. 20. 21	14. 8. 43,2 S	+19,01
330		ζ	3	200. 41. 5,2	46,08	13. 22. 44	0. 36. 46,5N	—18,93
331		m	6	202. 19. 40,5	47,19	13. 29. 19	7. 30. 39,7 S	+18,73
332				202. 57. 47,5	48,31	13. 31. 51	14. 59. 8,2 S	+18,66
333			5,6	204. 17. 10,1	48,72	13. 37. 9	16. 58. 13,7 S	+18,48
334		p	6	205. 39. 51,2	46,35	13. 42. 39	0. 20. 7,8 S	+18,30
335			6	208. 28. 16,3	47,50	13. 53. 53	7. 45. 33,1 S	+17,85
336				209. 0. 47,1	47,58	13. 58. 3	8. 10. 52,8 S	+17,77
337		κ	4	210. 5. 56,9	47,83	14. 0. 24	9. 10. 2,1 S	+17,59
338	1	ι		210. 32. 12,6	47,05	14. 2. 9	4. 50. 43,2 S	+17,51
339	2	ι	4	210. 55. 51,3	47,06	14. 3. 43	4. 52. 9,4 S	+17,44
340		λ	4	211. 36. 28,0	48,48	14. 6. 26	12. 16. 40,3 S	+17,33
341		φ	4	214. 1. 48,3	46,43	14. 16. 7	1. 9. 38,9 S	+16,89
342				216. 7. 56,6	48,35	14. 24. 32	11. 17. 31,9 S	+16,48
343		μ	4	217. 40. 35,9	47,21	14. 30. 42	4. 37. 27,9 S	+16,16
344				218. 31. 5,5	49,55	14. 34. 4	15. 0. 8,2 S	+16,00
345	Libræ	μ	5	219. 7. 9,0	49,15	14. 36. 9	13. 9. 29,5 S	+15,87
346	1	α	6	219. 25. 59,2	49,62	14. 37. 44	14. 59. 17,7 S	+15,79
347	2	α	2,3	219. 28. 47,1	49,63	14. 37. 55	15. 3. 1,2 S	+15,78
348	1	ξ	6	220. 24. 56,7	48,73	14. 41. 40	10. 55. 37,0 S	+15,58
349	2	ξ	6	221. 0. 52,4	48,54	14. 44. 3	10. 28. 41,7 S	+15,45
350	3	ξ	7	221. 22. 53,7	48,58	14. 45. 32	10. 11. 42,0 S	+15,37

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
386	Serpentis		6	252. 1. 58,7	52,38	16. 48. 8	18. 31. 0,3S.	+ 6,64
387	Scorpionis		6	253. 2. 46,7	53,65	16. 52. 11	21. 12. 49,3S.	+ 6,30
388	Serpentis	η	2,3	254. 13. 49,0	51,52	16. 56. 55	15. 24. 49,2S.	+ 5,90
389		ρ	4	256. 44. 3,9	53,63	17. 6. 56	20. 50. 9,9S.	+ 5,06
390		θ	3	256. 54. 1,4	55,20	17. 7. 36	24. 44. 27,0S.	+ 5,06
391	Sagittarii			257. 39. 57,2	53,82	17. 10. 40	21. 11. 47,4S.	+ 4,74
392	Serpentis		6	262. 9. 42,5	54,12	17. 28. 39	21. 44. 53,0S.	+ 3,21
393		d	6	262. 20. 33,5	54,04	17. 29. 22	21. 32. 38,0S.	+ 3,05
394	Sagittarii		7	266. 56. 12,8	52,36	17. 47. 45	17. 7. 35,3S.	+ 1,89
395	1	γ	4	267. 30. 18,6	57,57	17. 50. 1	29. 33. 56,6S.	+ 1,70
396	2	γ	3,4	267. 40. 51,4	57,97	17. 50. 43	30. 24. 3,9S.	+ 1,64
397	Sagittarii 1	μ	4	269. 55. 46,4	54,02	17. 59. 43	21. 5. 55,4S.	+ 0,51
398	2	μ	6	270. 18. 5,1	53,80	18. 1. 12	20. 46. 24,5S.	+ 0,35
399		δ	3	271. 29. 1,4	57,73	18. 5. 56	29. 54. 9,0S.	+ 0, 0
400			6	272. 50. 26,4	53,76	18. 11. 22	20. 38. 53,5S.	— 0,49
401		λ	3	273. 22. 7,7	55,75	18. 13. 29	25. 31. 38,4S.	— 0,70
402	Aquilæ	m	4	275. 36. 25,2	49,15	18. 22. 26	8. 23. 15,6S.	— 1,42
403	Sagittarii			275. 57. 33,2	54,08	18. 23. 50	21. 34. 8,8S.	— 1,62
404				276. 13. 13,6	53,95	18. 24. 53	21. 13. 18,3S.	— 1,71
405			7	277. 35. 45,5	54,47	18. 30. 23	22. 36. 39,7S.	— 2,19
406		φ	3,4	277. 44. 39,0	56,41	18. 30. 59	27. 12. 30,4S.	— 2,24
407			6	278. 55. 51,6	53,63	18. 35. 43	20. 34. 4,0S.	— 2,71
408	1	ν	5	279. 59. 38,5	54,58	18. 39. 59	23. 0. 23,5S.	— 3,02
409		σ	2,3	280. 10. 19,8	56,06	18. 40. 41	26. 33. 56,5S.	— 3,08
410.	2	ν	5	280. 13. 34,1	54,55	18. 40. 54	22. 56. 19,6S.	— 3,10
411	1	ξ	5	280. 50. 36,2	53,74	18. 43. 22	20. 56. 31,5S.	— 3,31
412	2	ξ	6	280. 55. 37,3	53,92	18. 43. 42	21. 23. 28,8S.	— 3,34
413		ζ	3	281. 54. 39,5	57,63	18. 47. 39	30. 11. 31,9S.	— 3,65
414		ο	4	282. 39. 53,2	54,12	18. 50. 40	22. 3. 52,3S.	— 3,82
415		τ	4	283. 4. 47,7	56,60	18. 52. 19	27. 59. 24,3S.	— 4,07
416	Aquilæ	λ	3	283. 26. 43,7	47,97	18. 53. 47	5. 12. 59,8S.	— 4,19
417	Sagittarii	π	3	283. 56. 43,4	53,81	18. 55. 47	21. 22. 33,1S.	— 4,38
418		d	6	285. 58. 14,1	52,97	19. 3. 53	19. 20. 53,6S.	— 4,73
419	1	ρ	5	287. 0. 32,5	52,52	19. 8. 2	18. 16. 3,0S.	— 5,42
420	2	ρ	6	387. 1. 59,3	52,70	19. 8. 8	18. 43. 19,6S.	— 5,43

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				D. M. S.	S.	H. M. S.	D. M. S.	S.
456	Capricorni	γ	3	321. 45. 30,9	50,17	21. 27. 2	17. 42. 44,9 S.	—15,47
457	1	d	6	322. 11. 15,5	49,52	21. 28. 45	15. 5. 2,5 S.	—15,56
458		κ	5	322. 22. 36,3	50,63	21. 29. 30	19. 55. 32,8 S.	—15,60
459		d	6	322. 33. 20,0	49,58	21. 30. 13	15. 27. 51,7 S.	—15,64
460			6	322. 47. 22,8	49,64	21. 31. 9	15. 49. 1,4 S.	—15,68
461		c	6	323. 6. 56,0	48,38	21. 32. 28	10. 9. 5,1 S.	—15,76
462		τ	5	323. 27. 58,5	48,85	21. 33. 52	12. 26. 19,8 S.	—15,83
463		δ	3	323. 30. 39,7	49,91	21. 34. 3	17. 10. 52,6 S.	—15,84
464		μ	5	325. 6. 57,1	49,21	21. 40. 28	14. 38. 47,0 S.	—16,18
465	Aquarii	ο	5	327. 47. 23,0	46,81	21. 51. 10	3. 16. 42,2 S.	—16,71
466		α	3	328. 25. 38,6	46,47	21. 53. 43	1. 27. 6,2 S.	—16,85
467		ι	4	328. 25. 50,1	49,00	21. 53. 43	15. 0. 0,0 S.	—16,85
468		e	6	329. 30. 39,4	48,19	21. 58. 3	12. 42. 42,0 S	—17,04
469		θ	4	331. 6. 20,9	47,74	22. 4. 25	8. 56. 34,3 S.	—17,34
470		ρ	5,6	331. 57. 19,7	47,70	22. 7. 49	8. 59. 33,8 S.	—17,48
471		γ	3	332. 22. 44,6	46,62	22. 9. 31	2. 33. 47,5 S.	—17,55
472		ζ	4	334. 10. 59,1	46,39	22. 16. 44	1. 12. 48,9 S.	—17,84
473		σ	5	334. 32. 58,6	47,01	22. 18. 12	11. 52. 24,5 S.	—17,89
474		υ	5	335. 27. 6,2	49,54	22. 21. 48	21. 54. 26,2 S.	—18,06
475		η	4	335. 49. 14,7	46,39	22. 23. 17	1. 19. 13,0 S.	—18,11
476		κ	5	336. 23. 42,2	46,97	22. 25. 35	5. 26. 4,8 S.	—18,20
477	Aquarii 2	τ	5	338. 48. 15,5	48,20	22. 35. 13	15. 18. 9,1 S.	—18,48
478		τ	5,6	339. 16. 58,4	48,08	22. 37. 8	14. 49. 39,6 S.	—18,59
479		λ	4	340. 5. 8,2	47,27	22. 40. 20	8. 49. 23,7 S.	—18,70
480		δ	3	340. 32. 19,3	48,26	22. 42. 9	17. 3. 51,7 S.	—18,75
481	Piscium	β	5	342. 58. 53,0	45,94	22. 51. 56	2. 33. 37,3 N	+19,05
482	Aquarii	h	6	343. 13. 33,0	47,13	22. 52. 54	8. 57. 36,0 S.	—19,07
483			7	343. 38. 37,7	47,11	22. 54. 35	8. 57. 26,0 S.	—19,12
484			6	343. 44. 39,5	47,11	22. 54. 59	8 57. 25,4 S.	—19,13
485	Piscium	A	6	345. 9. 45,2	46,10	22. 56. 39	0. 51. 12,7 N	+19,17
486	Aquarii	φ	4,5	345. 32. 12,9	46,84	23. 2. 9	7. 18. 38,5 S.	—19,30
487	1	ψ	5	345. 53. 31,4	47,09	23. 3. 34	10. 21. 52,9 S.	—19,34
488		χ	5	346. 9. 57,0	46,98	23. 4. 40	9. 0. 13,3 S.	—19,36
489	Piscium	γ	4	346. 14. 49,3	46,03	23. 4. 59	2. 0. 6,9 N	+19,37
490	Aquarii 2	ψ	5	346. 25. 12,1	47,01	23. 5. 41	10. 27. 41,9 S.	—19,39

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS,

FOR THE BEGINNING OF THE YEAR 1750.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Right Ascension in Time.	Annual Variat. +	Declination.	Annual Variation.
			D. M. S.	H. M. S.	S.	D. M. S.	S.
1	γ Pegasi	2	0. 5. 53,5	0. 0. 24	3,08	13. 47. 36,3N	+20,04
2	β Hydri	3	3. 2. 58,3	0. 12. 12	2,72	78. 39. 50,3S.	—20,01
3	α Phœnicis	2	3. 28. 0,9	0. 13. 52	3,01	43. 39. 43,4S.	—20,00
4	δ Andromedæ	3	6. 30. 15,0	0. 26. 1	3,16	29. 29. 18,3N	+20,01
5	α Cassiopeiæ	3	6. 37. 4,8	0. 26. 28	3,30	55. 9. 37,9N	+20,00
6	β Ceti	2	7. 45. 28,6	0. 31. 2	3,01	19. 21. 51,1S.	—19,85
7	γ Cassiopeiæ	3	10. 27. 14,2	0. 41. 49	3,49	59. 21. 19,2N	+19,70
8	α Polaris	2	10. 40. 56,0	0. 42. 42	10,05	87. 58. 2,4N	+19,69
9	β Phœnicis	3	13. 43. 21,7	0. 54. 53	2,73	48. 4. 42,1S.	—19,46
10	β Andromedæ	2	13. 57. 3,6	0. 55. 48	3,29	34. 17. 15,7N	+19,43
11	η Ceti	4	14. 0. 16,1	0. 56. 1	3,00	11. 30. 48,8S.	—19,44
12	δ Cassiopeiæ	3	17. 25. 14,5	1. 9. 41	3,71	58. 55. 27,1N	+19,07
13	θ Ceti	4	17. 53. 9,1	1. 11. 33	3,01	9. 28. 49,5S.	—19,07
14	γ Phœnicis	3	19. 22. 10,7	1. 17. 29	2,67	44. 36. 18,8S.	—18,90
15	α Eridani	1	22. 5. 42,2	1. 28. 23	2,26	58. 30. 52,1S.	—18,56
16	ε Cassiopeiæ	3	24. 10. 22,9	1. 36. 42	4,13	62. 25. 20,6N	+18,27
17	α Triang. bor.	4	24. 43. 28,0	1. 38. 54	3,49	28. 20. 59,3N	+18,19
18	γ Arietis	4	24. 57. 46,3	1. 39. 51	3,27	18. 3. 34,0N	+18,16
19	β Arietis	3	25. 13. 4,7	1. 40. 52	3,28	19. 34. 34,1N	+18,12
20	γ Andromedæ	2	27. 9. 56,5	1. 48. 40	3,61	41. 7. 0,5N	+17,82
21	α Piscium	3	27. 17. 7,0	1. 49. 8	3,09	1. 32. 50,2N	+17,80
22	α Hydri	3	27. 43. 22,5	1. 50. 53	1,87	62. 47. 36,3S.	—17,73
23	α Arietis	3	28. 17. 5,0	1. 53. 8	3,34	22. 16. 9,6N	+17,63
24	β Triang. bor.	4	28. 41. 15,6	1. 54. 45	3,48	33. 47. 28,2N	+17,56
25	γ Triang. aus.	4	30. 38. 0,7	2. 2. 32	3,52	32. 40. 37,7N	+17,22
26	ο Ceti	4	31. 40. 53,8	2. 6. 44	3,03	4. 7. 28,0S.	—17,05
27	δ Ceti	3	36. 40. 35,4	2. 26. 42	3,08	0. 45. 50,2S.	—16,07
28	ε Ceti	3	36. 52. 22,9	2. 27. 30	2,90	12. 56. 51,9S.	—16,03
29	γ Ceti	3	37. 35. 34,0	2. 30. 22	3,12	2. 10. 10,5N	+15,87
30	Lilii bor.	4	38. 16. 0,0	2. 33. 4	3,53	28. 11. 26,3N	+15,73
31	Lilii aus.	4	38. 49. 39,5	2. 35. 19	3,50	26. 12. 44,4N	+15,61
32	γ Persei	3	41. 42. 55,6	2. 46. 52	4,24	52. 30. 13,8N	+14,96
33	θ Eridani	3	42. 11. 52,7	2. 48. 48	2,30	41. 19. 4,0S.	—14,85
34	α Ceti	2	42. 18. 33,6	2. 49. 14	3,13	3. 5. 32,6N	+14,81
35	β Medusæ	2	43. 0. 4,6	2. 52. 0	3,84	39. 58. 17,9N	+14,64

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	Declination. D. M. S.	Annual Variation. S.
71	α Orionis	1	85. 24. 46,2	5. 41. 39	3,25	7. 20. 10,6N	+ 1,60
72	β Columbæ	3	85. 32. 30,4	5. 42. 10	2,11	35. 52. 40,3S.	— 1,65
73	θ Aurigæ	3	85. 40. 8,5	5. 42. 41	4,08	37. 9. 51,6N	+ 1,51
74	η Castoris	4	89. 56. 36,3	5. 59. 48	3,63	22. 33. 15,5N	+ 0,02
75	μ Pollucis	4	91. 58. 25,0	6. 7. 54	3,63	22. 30. 59,2N	— 0,07
76	ζ Canis maj.	3	92. 40. 58,9	6. 10. 44	2,31	29. 58. 11,8S.	+ 0,94
77	β Canis maj.	3	92. 55 25,3	6. 11. 42	2,63	17. 51. 11,4S.	+ 1,02
78	Canopus	1	94. 36. 7,6	6. 18. 25	1,34	52. 34. 8,9S.	+ 1,60
79	γ Pollucis	3	95. 48. 54,7	6. 23. 16	3,48	16. 35. 16,9N	— 2,02
80	ι Castoris	3	97. 8. 9,0	6. 28. 33	3,71	25. 21. 9,4N	— 2,49
81	ν Navis	3	97. 31. 44,4	6. 30. 7	1,84	42. 59. 25,2S.	+ 2,62
82	Sirius	1	98. 32. 0,3	6. 34. 8	2,69	16. 23. 36,6S.	+ 2,97
83	ε Canis maj.	3	102. 12. 7,2	6. 48. 48	2,36	28. 38. 59,6S.	+ 4,23
84	ζ Pollucis	3	102. 18. 49,4	6. 49. 15	3,58	20. 54. 45,1N	— 4,27
85	δ Canis maj.	2	104. 33. 26,8	6. 58. 14	2,45	26. 0. 56,1S.	+ 5,03
86	δ Pollucis	3	106. 17. 26,4	7. 5. 10	3,61	22. 25. 8,5N	— 5,61
87	π Navis	3	107. 4. 44,0	7. 8. 19	2,13	36. 39. 44,4S.	+ 5,88
88	β Canis min.	3	108. 23. 46,5	7. 13. 35	3,27	8. 46. 20,2N	— 6,32
89	η Canis maj.	2	108. 33. 3,6	7. 14. 12	2,39	28. 49. 59,7S.	+ 6,37
90	α Castoris	2	109. 39. 2,7	7. 18. 36	3,87	32. 24. 34,5N	— 6,73
91	σ Navis	3	110. 19. 40,5	7. 21. 19	1,94	42. 48. 25,6S.	+ 6,96
92	Procyon	1	111. 32. 59,4	7. 26. 12	3,21	5. 50. 42,5N	— 7,37
93	β Pollucis	2	112. 29. 45,6	7. 29. 59	3,75	28. 36. 22,5N	— 7,66
94	ζ Navis	2	118. 42. 4,5	7. 54. 48	2,12	39. 18. 39,5S.	+ 9,62
95	γ Navis	2	120. 27. 48,0	8. 1. 51	1,85	46. 36. 34,9S.	+10,16
96	β Cancri	4	120. 44. 4,3	8. 2. 56	3,15	9. 56. 7,0N	—10,23
97	ε Navis	3	124. 20. 10,7	8. 17. 21	1,26	58. 42. 52,5S.	+11,30
98	γ Asini	4	127. 11. 42,2	8. 28. 47	3,52	22. 20. 59,3N	—12,11
99	δ Asini	4	127. 36. 43,4	8. 30. 27	3,44	19. 3. 22,0N	—12,22
100	δ Navis	2	129. 27. 5,0	8. 37. 48	1,61	53. 48. 2,1S.	+12,73
101	ι Ursæ maj.	3	130. 29. 7,9	8. 41. 57	4,25	48. 59. 58,7N	—13,00
102	α Cancri	3	130. 32. 21,3	8. 42. 9	3,31	12. 48. 33,5N	—13,01
103	κ Ursæ maj.	4	131. 36. 28,0	8. 46. 26	4,20	48. 7. 16,6N	—13,30
104	λ Navis	3	134. 42. 17,7	8. 57. 49	2,21	42. 26. 4,1S.	+14,09
105	β Navis	1	137. 35. 26,0	9. 10. 22	0,75	68. 41. 26,5S.	+14,79

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	Declination. D. M. S.	Annual Variation. S.
106	ι Navis	2	137. 36. 3,3	9. 10. 24	1,65	58. 14. 8,3S.	+15,79
107	α Luc. Boreal.	3	138. 36. 52,9	9. 14. 24	1,86	53. 57. 0,1S.	+15,06
108	α Hydræ	2	138. 49. 40,0	9. 15. 19	2,96	7. 35. 13,1S.	+15,08
109	θ Ursæ maj.	3	138. 59. 39,2	9. 15. 59	4,22	52. 47. 42,3N	—15,11
110	κ Leonis	4	141. 56. 46,0	9. 27. 47	3,24	10. 40. 56,5N	—15,79
111	ι Leonis	3	142. 54. 6,0	9. 31. 36	3,45	24. 24. 35,2N	—15,99
112	μ Leonis	3	144. 37. 21,0	9. 38. 29	3,47	27. 10. 8,5N	—16,33
113	υ Navis	3	145. 12. 41,2	9. 40. 51	1,50	63. 55. 6,7S.	+16,45
114	ο Leonis	3	148. 24. 53,8	9. 53. 40	3,31	17. 58. 18,4N	—17,07
115	Regulus	1	148. 45. 26,0	9. 55. 2	3,24	13. 10. 43,7N	—17,13
116	ζ Leonis	3	150. 40. 55,0	10. 2. 44	3,34	24. 39. 4,0N	—17,47
117	γ Leonis	3	151. 32. 6,1	10. 6. 0	3,31	21. 5. 49,3N	—17,61
118	ρ Leonis	4	154. 54. 20,0	10. 19. 37	3,18	10. 35. 6,4N	—18,14
119	ϑ Navis	3	158. 31. 34,9	10. 34. 6	2,12	63. 5. 24,9S.	+18,64
120	ο Navis	2	158. 51. 29,3	10. 35. 26	2,27	58. 22. 39,2S.	+18,68
121	μ Navis	3	159. 1. 11,4	10. 36. 5	2,55	48. 6. 15,0S.	+18,81
122	β Ursæ maj.	2	161. 38. 26,5	10. 46. 34	3,75	57. 42. 51,0N	—19,01
123	α Crateris	4	161. 54. 13,7	10. 47. 37	2,95	16. 58. 29,0S.	+19,05
124	α Ursæ maj.	2	162. 0. 45,1	10. 48. 3	3,89	63. 5. 33,9N	—19,05
125	δ Leonis	3	165. 11. 27,2	11. 0. 46	3,22	21. 53. 28,6N	—19,37
126	θ Leonis	3	165. 16. 13,1	11. 1. 5	3,18	16. 47. 32,5N	—19,38
127	β Leonis	2	174. 4. 17,4	11. 36. 17	3,12	15. 58. 7,2N	—19,93
128	β Virginis	3	174. 25. 5,0	11. 37. 40	3,08	3. 10. 20,5N	—19,94
129	γ Ursæ maj.	2	175. 8. 7,1	11. 40. 32	3,24	55. 5. 0,7N	—19,98
130	δ Centauri	3	178. 51. 51,4	11. 55. 27	3,05	49. 19. 33,8S.	+20,03
131	α Corvi	4	178. 53. 31,4	11. 55. 34	3,07	23. 20. 6,0S.	+20,03
132	ε Corvi	4	179. 19. 46,1	11. 57. 19	3,06	21. 13. 44,6S.	+20,04
133	δ Crucis	3	180. 30. 34,7	12. 2. 2	3,10	57. 21. 30,0S.	+20,04
134	δ Ursæ maj.	2	180. 43. 47,3	12. 2. 55	3,05	58. 25. 19,3N	—20,04
135	γ Corvi	3	180. 44. 44,5	12. 2. 59	3,09	16. 9. 32,5S.	+20,04
136	η Virginis	3	181. 46. 48,1	12. 7. 7	3,07	0. 43. 31,1N	—20,02
137	α Crucis	1	183. 13. 53,7	12. 12. 56	3,22	61. 42. 47,4S.	+20,01
138	δ Corvi	4	184. 14. 29,6	12. 16. 58	3,09	15. 7. 18,2S.	+19,98
139	γ Crucis	2	184. 28. 35,0	12. 17. 51	3,36	70. 44. 54,3S.	+19,97
140	β Corvi	3	185. 19. 30,9	12. 21. 18	3,14	22. 0. 40,0S	+19,95

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS.

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			D.	M.	S.	H.	M.	S.	S.	D.	M.	S.	S.
141	κ Corvi	4	185.	38.	41,6	12.	22.	35	3,42	67.	46.	17,2S.	+19,94
142	γ Centauri	2	186.	57.	58,8	12.	27.	32	3,27	47.	34.	52,9S	+19,89
143	γ Virginis	3	187.	15.	7,2	12.	29.	0	3,08	0.	4.	22,3S.	+19,88
144	β Virginis	4	187.	48.	58,4	22.	31.	16	3,52	66.	44.	6,1S	- 19,84
145	β Crucis	2	188.	19.	40,1	12.	33.	19	3,42	58.	19.	8,3S.	+19,83
146	ε Ursæ maj.	2	190.	44.	5,6	12.	42.	56	2,69	57.	19.	15,1N	—19,69
147	δ Virginis	3	190.	45.	18,1	12.	43.	1	3,06	4.	45.	43,2N	—19,69
148	Luc. subUrs. m	3	191.	4.	20,0	12.	44.	17	2,86	39.	40.	25,8N	—19,66
149	ε Virginis	3	192.	25.	51,8	12.	49.	43	3,01	12.	18.	34,3N	—19,57
150	θ Virginis	4	194.	15.	26,2	12.	57.	2	3,09	4.	11.	47,2S	+19,41
151	γ Hydræ	3	196.	20.	46,6	13.	5.	23	3,22	21.	50.	43,8S	+19,22
152	ι Centauri	3	196.	39.	27,6	13.	6.	38	3,34	35.	23.	6,6S.	+19,20
153	α Virginis	1	198.	0.	51,4	13.	12.	3	3,15	9.	50.	51,1S	+19,06
154	ζ Ursæ maj.	2	198.	26.	53,8	13.	13.	48	2,45	56.	14.	12,2N	—19,01
155	ζ Virginis	3	200.	29.	37,0	13	21.	58	3,08	0.	41.	27,5N	—18,77
[illegible]	ε Centauri	3	201.	3.	30,4	13.	24.	14	3,69	52.	10.	55,2S.	+18,70
157	η Ursæ maj.	2	204.	24.	58,6	13.	37.	40	2,41	50.	34.	5,4N	—18,24
158	ζ Centauri	[illegible]	205.	1.	14,6	13.	40.	5	3,66	46.	2.	38,3S.	+18,15
159	η Bootis	3	205.	41.	34,1	13.	42.	46	2,88	19.	39.	45,0N	—18,06
160	β Centauri	[illegible]	206.	36.	41,8	13.	46.	27	4,09	59.	9.	1,8S.	+17,91
161	θ Centauri	3	207.	59.	57,5	13.	52.	0	3,52	35.	7.	34,9S.	+17,69
162	α Draconis	3	209.	24.	28,7	13.	57.	38	1,50	65.	34.	32,5N	—17,45
163	κ Virginis	[illegible]	209.	53.	57,8	13.	59.	36	3,19	9.	5.	48,3S.	+17,37
164	Arcturus	1	211.	4.	1,0	14.	4.	16	2,82	20	29.	39,2N	—17,16
165	λ Virginis	4	211.	24.	18,4	14.	5.	37	3,22	12.	12.	29,0S.	+17,20
166	η Centauri	3	214.	36.	9,1	14.	19.	45	3,75	41.	2.	28,8S.	+16,43
167	γ Trionum	3	215.	29.	55,5	14.	22.	0	2,43	38.	24.	19,0N	—16,31
168	α Circini	3	215.	39.	45,4	14.	22.	39	4,68	63.	51.	48,2S.	+16,26
169	α Centauri	1	215.	42.	31,1	14.	22.	50	4,41	59.	47.	9,4S.	+16,26
170	α Lupi	3	216.	21.	48,0	14.	25	27	3,89	46.	17.	40,1S	+16,14
171	ζ Trionum	3	217.	18.	14,6	14.	29.	13	2,86	14.	48.	52,2N	—15,93
172	ι Trionum	3	218.	32.	15,8	14.	34.	9	2,63	28.	8	27,0N	—15,67
173	α Libræ	2	219.	16.	20,9	14.	37.	5	3,30	14.	59.	9,7S.	+15,50
174	β Lupi	3	220,	34.	6,6	14.	42	16	3,86	42.	6.	13,9S.	+15,22
175	κ Centauri	3	220	45.	12,7	14.	43.	1	3,84	41.	4.	45,6S.	+15,17
176	γ Scorpionis	[illegible]	222	22.	28,5	14.	49.	30	3,48	24.	16.	59,5S.	+14,70
177	β Ursæ min.	3	222.	55.	42,3	14.	51.	43	2,28	75.	10.	51,2N	—14,66

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214	ι Herculis	3	252. 40. 52,2	16. 50. 43	2,29	31. 18 41,6N	— 5,96
215	η Scorpionis	3	253. 34. 34,1	16. 54. 18	4,28	42 52 39,2S	+ 5,66
216	η Ophiuchi	2	254. 0. 53,9	16. 56. 4	3,44	15. 23. 28,4S	+ 5,52
217	α Herculis	2	255. 48. 49,2	17. 3. 15	2,74	14. 41. 44,5N	— 4,91
218	δ Herculis	3	256. 26. 36,4	17. 5. 46	2,47	25. 9. 4,7N	— 4,69
219	θ Ophiuchi	3	256. 40. 11,2	17. 6. 21	3,67	24. 43. 20,6S	+ 4,61
220	α Aræ	3	258. 8. 27,0	17. 12. 34	4,61	49. 38. 31,8S	+ 4,12
221	υ Scorpionis	4	258. 27. 0,8	17. 13. 48	4,09	37. 3. 55,9S	+ 4,01
222	λ Scorpionis	3	259. 9. 59,4	17. 16. 40	4,08	36. 53. 28,6S	+ 3,77
223	θ Scorpionis	3	259. 50. 56,1	17. 19. 24	4,30	42 48. 23,2S	+ 3,53
224	α Ophiuchi	2	260. 50. 5,3	17. 23. 20	2,78	12. 45. 47,9N	— 3,17
225	β Draconis	3	261. 12. 6,8	17. 24. 48	1,36	52. 29. 44,5N	— 3,06
226	κ Scorpionis	3	261. 18. 19,3	17. 25. 13	4,13	38. 52. 14,6S.	+ 3,03
227	ι Scorpionis	3	262. 31. 53,5	17. 30. 8	4,18	39. 59. 49,0S.	+ 2,60
228	β Ophiuchi	3	262. 46. 55,0	17. 31. 8	2,97	4. 41. 40,6N	— 2,52
229	γ Ophiuchi	3	263. 50. 35,2	17. 35. 22	3,01	2. 49. 28,3N	— 2,15
230	μ Herculis	4	264. 10. 9,5	17. 36. 41	2,38	27. 53. 3,9N	— 2,03
231	ξ Serpentis	4	266. 49. 26,9	17. 47. 18	3,16	3. 38. 56,3S.	+ 1,11
232	θ Herculis	3	266. 55. 25,6	17. 47. 42	2,06	37. 17. 56,4N	— 1,08
233	* Sagittarii	4	267. 26. 21,5	17. 49. 45	3,87	30. 53. 23,7S.	+ 0,39
234	γ Draconis	3	267. 42. 3,0	17. 50. 48	2,21	51. 31. 37,9N	— 0,80
235	μ Sagittarii	4	269. 42. 12,7	17. 58. 49	3,60	21. 5. 56,9S.	— 0,10
236	δ Sagittarii	3	271. 14. 31,8	18. 4. 58	3,85	29. 54. 18,4S.	— 0,43
237	ε Sagittarii	3	271. 53. 39,7	18. 7. 35	4,00	34. 28. 17,4S.	— 0,67
238	λ Sagittarii	3	273. 8. 5,2	18. 12. 32	3,72	25. 31. 57,4S.	— 1,09
239	α Lyræ	1	277. 7. 7,0	18. 28. 28	2,06	38. 34. 0,0N	+ 2,48
240	φ Sagittarii	4	277. 30. 26,7	18. 30. 2	3,77	27. 13. 13,0S.	— 2,61
241	σ Sagittarii	3	279. 56. 13,4	18. 39. 45	3,74	26. 34. 51,0S.	— 3,45
242	β Lyræ	3	280. 12. 47,3	18. 40. 51	2,22	33. 5. 23,4N	+ 3,55
243	θ Serpentis	4	280. 56. 51,2	18. 43. 47	3,00	3. 54. 6,0N	+ 3,80
244	δ Lyræ	3	281. 26. 36,3	18. 45. 46	2,11	36. 35. 48,6N	+ 3,97
245	ζ Sagittarii	3	281. 40. 16,6	18. 46. 41	3,84	30. 12. 36,0S.	— 4,06
246	ε Aquilæ	4	282. 4. 17,5	18. 48. 17	2,73	14. 44. 56,1N	+ 4,18
247	γ Lyræ	3.	282. 23. 54,3	18. 49. 36	2,26	32. 21. 49,0N	+ 4,29
[illegible]	ο Sagittarii	4	282. 25. 19,1	18. 49. 41	3,60	22. 5. 0,8S.	— 4,31
[illegible]	[illegible] Sagittarii	4	282. 49. 38,3	18. 51. 19	3,77	28. 0. 30,6S.	— 4,45

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STA

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	Declination. D. M. S.	Annual Variation. S.
250	ζ Aquilæ	4	283. 28. 50,4	18. 53. 55	2,69	13. 30. 42,5N	+ 4,6
251	π Sagittarii	3	283 35. 45,4	18. 54. 23	3,59	21. 23. 48,8S	— 4,7
252	β Sagittarii	4	286. 9. 8,7	19. 4. 37	4,34	44. 53. 42,4S.	— [illegible]
253	α Sagittarii	[redacted]	286. 37 43,3	19. 6. 31	4,20	41. 3. 21,5S	— [illegible]
254	δ Draconis	3	288. 6. 23,0	19. 12. 26	0,06	67. 18. 17,1N	+ [illegible]
255	δ Aquilæ	3	288. 13. 24,7	19. 12. 54	3,02	2. 38. 18,2N	+ [illegible]
256	β Cygni	3	290. 9. 25,3	19. 20. 38	2,43	27. 27. 4,0N	+ [illegible]
257	ι Antinoi	4	290. 58. 30,6	19. 23. 54	4,01	1. 49. 12,2S.	— [illegible]
258	α Sagittæ	4	292. 14. 1,0	19. 28. 56	2,69	17. 27. 28,9N	+ [illegible]
259	γ Aquilæ	3	293. 35. 30,2	19. 34. 22	2,28	10. 1. 22,0N	+ [illegible]
260	δ Cygni	3	294. 17. 28,4	19. 37. 10	1,86	44. 31. 58,5N	+ [illegible]
261	α Aquilæ	2	294. 38. 46,7	19. 38. 35	2,91	8. 13. 44,3N	+ [illegible]
262	η Antinoi	4	294 55. 55,0	19. 39. 44	3,07	0. 23. 5,4N	+ [illegible]
263	β Aquilæ	3	295. 45. 31,0	19. 43. 2	3,02	5. 48. 10,6N	+ [illegible]
264	δ Pavonis	4	295. 58. 55,0	19. 43. 56	5,92	66. 46. 50,8S.	— [illegible]
265	α Capricorni	3	301. 2. 24,3	20. 3. 46	3,35	13. 18. 2,8S.	— [illegible]
266	α Pavonis	2	301. 25. 20,0	20. 5. 41	4,89	57. 30. 27,4S.	— [illegible]
267	β Capricorni	3	301. 44. 3,0	20. 6. 56	3,39	15 33 2,7S	— [illegible]
268	γ Cygni	3	303. 18. 50,0	20. 13 15	2,16	39. 8. 12,8N	— [illegible]
269	α Indi	3	304 57. 56,9	20 19. 52	4,29	48. 8. 14,0S.	— [illegible]
270	ε Delphini	4	305. 18. 56,4	20. 21. 16	2,88	10 28. 20,0N.	+ [illegible]
271	β Pavonis	3	305. 31. 18,6	20. 22. 5	5,65	67 4. 8,5S.	— [illegible]
272	ζ Delphini	4	305 54. 17,8	20. 23. 37	2,82	13 49 52,2N	— [illegible]
273	β Delphini	3	306. 27. 34,3	20. 25. 50	2,82	13. 44. 32,6N	— [illegible]
274	α Delphini	[redacted]	307. 0. 22,2	20. 28. 0	2,79	15. 2 51,5N	+ [illegible]
275	δ Delphini	4	307. 56 44,9	20. 31 47	2,81	14 11. 36,6N	— [illegible]
276	α Cygni	2	308 13. 38,5	20 32. 55	2,05	44 23. 54,5N	— [illegible]
277	γ Delphini	4	308. 46. 3,0	20. 35. 4	2,79	15. 14. 22,5N	— [illegible]
278	ε Cygni	3	309. 1. 19,6	20. 36. 5	2,40	33 2. 50,4N	— [illegible]
279	ζ Cygni	4	315. 34. 26,4	21. 2. 18	2,55	29 12 50,7N	+ [illegible]
280	α Equulei	4	315. 49. 36,0	21. 3. 18	3,01	4. 14. 8,1N	— [illegible]
281	Pavonis	4	316 21. 5,2	21. 5 24	5,28	66 28. 19,8S	— [illegible]
282	ε Pegasi	4	317 37. 25,3	21 10. 30	2,79	18. 44 53,7N	— [illegible]
283	α Cephei	3	318 8 41,8	21. 12 34	1,43	61 31. 55,7N	— [illegible]
284	β Aquarii	3	319 35. 53,7	21. 18. 24	3,18	6. 39. 22,4S	— [illegible]

M. DE LA CAILLE's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Right Ascension in Time.	Annual Variat. +	Declination.	Annual Variation.
			D. M. S.	H. M. S.	S.	D. M. S.	S.
285	β Cephei	4	321. 19. 34,4	21. 25. 18	0,81	69. 27. 54,8N	+ 15,64
286	γ Capricorni	3	321. 27. 12,5	21. 25. 49	3,35	17. 46. 42,0S	— 15,66
287	ι Pegasi	3	322. 58. 21,6	21. 31. 53	2,95	8. 44. 29,8N	+ 16,00
288	μ Cygni	4	323. 14. 41,1	21. 32. 59	2,64	27. 37. 24,1N	+ 16,05
289	δ Capricorni	3	323. 18. 8,6	21. 33. 13	3,33	17. 14. 48,3S.	— 16,06
290	γ Gruis	3	324. 40. 33,5	21. 38. 42	3,70	38. 31. 28,7S.	— 16,35
291	α Gruis	2	328. 5. 7,7	21. 52. 21	3,98	48. 9. 21,4S.	— 17,00
292	α Aquarii	3	328. 14. 0,6	21. 52. 56	3,10	1. 31. 25,9S.	— 17,03
293	α Tucanæ	3	330. 16. 46,0	22. 1. 7	4,31	61. 29. 34,0S.	— 17,40
294	γ Aquarii	3	332. 11. 5,6	22. 8. 44	3,09	2. 38. 13,5S.	— 17,71
295	β Gruis	3	336. 54. 3,2	22. 27. 36	3,68	48. 10. 51,0S	— 18,43
296	ζ Pegasi	3	337. 14. 43,9	22. 28. 59	2,99	9. 32. 4,3N	+ 18,46
297	η Pegasi	3	337. 49. 37,7	22. 31. 18	2,80	28. 55. 14,5N	+ 18,55
298	λ Aquarii	4	339. 53. 27,8	22. 39. 34	3,16	8. 54. 8,2S.	— 18,82
299	δ Aquarii	3	340. 20. 14,6	22. 41. 21	3,22	17. 8. 39,9S	— 18,87
300	Fomalhaut	1	340. 56. 41,7	22. 43. 47	3,34	30. 56. 24,8S	— 18,94
301	ο Andromedæ	4	342. 36. 49,0	22. 50. 27	2,72	40. 59. 22,0N	+ 19,12
302	β Pegasi	2	342. 55. 14,5	22. 51. 41	2,89	26. 43. 52,7N	+ 19,15
303	α Pegasi	2	343. 4. 57,3	22. 52. 20	2,99	13. 51. 56,0N	+ 19,17
304	φ Aquarii	4	345. 20. 23,1	23. 1. 22	3,13	7. 23. 31,2S	— 19,38
305	γ Cephei	4	352. 19. 22,1	23. 29. 17	2,31	76. 14. 11,0N	+ 19,86
306	α Andromedæ	2	358. 52. 44,1	23. 55. 31	3,06	27. 42. 32,1N	+ 20,04
307	β Cassiopeiæ	3	358. 59. 41,4	23. 55. 59	3,04	56. 46. 7,0N	+ 20,04

THESE Catalogues of M. de la CAILLE were taken from the *Ephemerides des Mouvemens Célestes*, from 1765 to 1775. The right ascensions in the second Catalogue were determined by taking equal altitudes with a quadrant of three feet radius; but this method of determining the right ascensions is less exact than that by the transit instrument. The declinations in each Catalogue were deduced from the meridian zenith distances observed with a quadrant of six feet radius. The right ascensions in the first Catalogue were settled with a transit instrument, by comparison with the stars in the *Fundamenta Astronomiæ;* but the right ascensions of the stars in the *Fundamenta Astronomiæ* having been settled by equal altitudes, the right ascensions of the stars compared with them must be subject to the same inaccuracy.

ZACH's CATALOGUE OF 381 PRINCIPAL STARS,

FOR THE BEGINNING OF 1800.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Time.	Annual Variat. +	Right Ascension in Degrees.	Annual Variat. +	Declination.
			H. M. S.	S.	D. M. S.	S.	D. M.
1	88 γ *Pegasi*	2	0. 2. 56,79	3,063	0. 44. 11,85	45,95	14. 4 N.
2	8 ι Ceti	3	0. 9. 13,51	3,059	2. 18. 22,66	45,89	9. 57 S.
3	15 κ Cassiopeiæ	4	0. 21. 45,12	3,301	5. 26. 16,75	49,51	61. 50 N.
4	17 ζ Cassiopeiæ	4	0. 25. 53,93	3,262	6. 28. 29,01	48,93	52. 49 N.
5	17 ζ Andromedæ	3	0. 28. 39,02	3,161	7. 9. 45,31	47,42	29. 45 N.
6	18 α Cassiopeiæ	3	0. 29. 14,40	3,311	7. 18. 35,95	49,66	55. 26 N.
7	16 β Ceti	2,3	0. 33. 31,83	3,001	8. 22. 57,40	45,01	19. 5 S.
8	24 η Cassiopeiæ	4	0. 37. 1,44	3,389	9. 15. 21,64	50,83	56. 46 N.
9	63 δ Piscium	4	0. 38. 19,08	3,093	9. 34. 46,15	46,39	6. 30 N.
10	27 γ Cassiopeiæ	3	0. 44. 44,75	3,505	11. 11. 11,29	52,58	59. 38 N.
11	71 ε Piscium	4	0. 52. 33,95	3,103	13. 8. 29,20	46,55	6. 49 N.
12	43 β Andromedæ	2	0. 58. 34,23	3,297	14. 38. 33,38	49,46	34. 33 N.
13	33 θ Cassiopeiæ	4	0. 59. 0,22	3,531	14. 45. 3,33	52,96	53. 35 N.
14	86 ζ Piscium	4	1. 3. 16,99	3,109	15. 49. 14,80	46,63	6. 31 N.
15	37 δ Cassiopeiæ	3	1. 12. 50,58	3,761	18. 12. 38,70	56,42	59. 11 N.
16	98 μ Piscium	5	1. 19. 42,07	3,108	19. 55. 31,07	46,62	5. 7 N.
17	102 π Piscium	5	1. 30. 30,70	3,164	22. 37. 40,56	47,46	11. 7 N.
18	106 ν Piscium	4,5	1. 31. 1,77	3,107	22. 45. 26,62	46,61	4. 28 N.
19	110 ο Piscium	4,5	1. 34. 50,72	3,144	23. 42. 40,84	47,16	8. 9 N.
20	45 ε Cassiopeiæ	3	1. 40. 10,01	4,155	25. 2. 30,13	62,33	62. 41 N.
21	55 ζ Ceti	3	1. 41. 36,69	2,953	25. 24. 10,33	44,30	11. 20 S.
22	2 α Triang. bor	3,4	1. 41. 42,74	3,379	25. 25. 41,15	50,68	28. 36 N.
23	5 γ 1 Arietis	4	1. 42. 34,52	3,258	25. 38. 37,73	48,87	18. 19 N.
24	6 β Arietis	3	1. 43. 36,77	3,277	25. 54. 11,48	49,15	19. 50 N.
25	9 λ Arietis	5	1. 46. 48,86	3,315	26. 42. 12,83	49,73	22. 37 N.
26	57 γ Andromedæ	2	1. 51. 41,05	3,615	27. 55. 15,76	54,23	41. 22 N.
27	* praec. α ♈		1. 50. 26,15		27. 36. 32,25		
28	13 α *Arietis*	2	1. 55. 55,27	3,335	28. 58. 49,05	50,02	22. 31 N.
29	* seq. α ♈		1. 59. 38,13		29. 54. 31,95		
30	22 θ 1 Arietis	5,6	2. 7. 1,64	3,308	31. 45. 24,55	49,62	18. 58 N.
31	68 ο Ceti (var.)	0,2	2. 9. 14,70	3,019	32. 18. 40,50	45,29	3. 54 S
32	42 π Arietis	6	2. 38. 9,29	3,321	39. 32. 19,32	49,81	16. 38 N.
33	43 σ Arietis	6	2. 40. 28,09	3,285	40. 7. 1,39	49,28	14. 15 N.
34	82 δ Ceti	3	2. 29. 14,17	3,060	37. 18. 32,54	45,90	0. 33 S.
35	83 ε Ceti	3	2. 29. 53,42	2,884	37. 28. 21,37	43,27	12. 44 S.

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Time.	Annual Variat. +	Right Ascension in Degrees.	Annual Variat. +	Declination.
			H. M. S.	S.	D. M. S.	S.	D. M.
36	86 γ Ceti	3	2. 32. 57,18	3,102	38. 14. 17,70	46,53	2. 23 N.
37	89 π Ceti	3	2. 34. 36,02	2,849	38. 39. 0,29	42,74	14. 43 S.
38	39 Υ Lilii bor.	4	2. 35. 57,70	3,521	38. 59. 25,49	52,81	28. 25 N.
39	41 Υ Lilii aust.	4	2. 38. 14,43	3,489	39. 33. 36,49	52,34	26. 26 N.
40	40 ρ 2 Arietis	6	2. 44. 35,65	3,344	41. 8. 54,72	50,16	17. 31 N.
41	46 ρ 3 Arietis	5,6	2. 45. 9,35	3,340	41. 17. 20,19	50,10	17. 13 N.
42	3 η Eridani	3	2. 46. 39,72	2,917	41. 39. 55,78	43,75	2. 42 S.
43	48 ε Arietis	5	2. 47. 48,12	3,401	41. 57. 1,80	51,01	20. 32 N.
44	23 γ Persei	3	2. 50. 24,42	4,250	42. 36. 6,25	63,75	52. 43 N.
45	92 α *Ceti*	2	2. 51. 50,07	3,119	42. 57. 31,06	46,66	3. 18 N.
46	* seq. α Ceti		2. 51. 54,61		42. 58. 39.15		
47	26 β Persei	2,3	2. 55. 12,07	3,846	43. 48. 1,04	57,69	40. 11 N.
48	57 δ Arietis	4	2. 0. 12,71	3,393	45. 3. 10,59	50,89	18. 58 N.
49	58 ζ Arietis	5	3. 3. 25,98	3,422	45. 51. 29,77	51,33	20. 18 N.
50	13 ζ Eridani	3	3. 6. 7,48	2,904	46. 31. 52,20	43,56	9. 34 S.
51	61 τ 1 Arietis	7	3. 9. 42,36	3,433	47. 25. 35,39	51,49	20. 25 N.
52	33 α Persei	2	3. 10. 6,85	4,203	47. 31. 42,77	63,05	49. 8 N.
53	63 τ 2 Arietis	6	3. 11. 16,37	3,428	47. 49. 5,48	51,42	20. 1 N.
54	65 Arietis	7	3. 12. 55,52	3,430	48. 13. 52,74	51,45	20. 5 N.
55	5 f Tauri	5	3. 19. 50,65	3,289	49. 57. 39,78	49,33	12. 15 N.
56	18 ε Eridani	3,4	3. 23. 31,52	2,883	50. 52. 52,84	43,24	10. 9 N.
57	39 δ Persei	3	3. 28. 44,93	4,203	52. 11. 13,91	63,05	47. 8 N.
58	25 η Luc. Pleiad	3	3. 35. 37,17	3,535	53. 54. 17,56	53,03	23. 29 N.
59	44 ζ Persei	3	3. 41. 35,20	3,734	55. 23. 48,00	56,01	31. 17 N.
60	45 ε Persei	3	3. 44. 29,19	3,977	56. 7. 17,87	59,66	39. 25 N.
61	34 γ Eridani	2,3	3. 48. 42,25	2,786	57. 10. 33 77	41,79	14. 5 N.
62	37 A Tauri	5	3. 52. 53,46	3,515	58. 13. 21,83	52,72	21. 32 N.
63	54 γ Tauri	3	4. 8. 25,25	3,387	62. 6. 18,48	50 80	15. 8 N.
64	61 δ 1 Tauri	3,4	4. 11. 24,68	3,432	62. 51. 10,26	51,48	17. 4 N.
65	64 δ 2 Tauri	4	4. 12. 34.93	3,431	63. 8. 43,89	51,46	16. 58 N.
66	65 κ 1 Tauri	5	4. 13. 27,60	3,545	63. 21. 54.93	53,17	21. 50 N.
67	67 κ 2 Tauri	4,5	4. 13. 31,13	3,543	63. 22. 47,02	53,14	21. 44 N.
68	74 ε Tauri	3,4	4. 16. 56,98	3,475	64. 14. 14,70	52,12	18. 44 N.
69	77 ϑ Tauri	5	4. 17. 9,30	3,401	64. 17. 19,53	51,02	15. 31 N.
70	78 c Tauri	5	4. 17. 19,55	3,399	64. 19. 52,91	50,99	15. 25 N.

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Time.	Annual Variat. +	Right Ascension in Degrees.	Annual Variat. +	Declination.
			H. M. S.	S.	D M. S.	S.	D. M.
71	* præc. α ♉		4. 22 12,56		65. 33. 8,40		
72	87 *Aldebaran*	1	4. 24. 2[illegible],29	3,421	66. 6. 49,38	51,31	16. 6 N.
73	* seq. α ♉		4 26. 43,44		66. 40. 51,60		
74	91 σ 1 Tauri	6	4. 27. 44,78	3,406	66. 56. 11,77	51,09	15. 24 N.
75	52 υ 2 Eridani	3,4	4. 27. 47,26	2,329	66. 56. 48,8[illegible]	34,94	30. 59 S.
76	92 σ 2 Tauri	6	4 27. 50,67	3,409	66. 57. 40,07	51,13	15. 31 N.
77	54 Eridani	3,4	4. 31. 43,15	2,615	67. 55. 47 21	39,23	20. 4 S.
78	102 ι Tauri	4	4. 51. 9,38	3,565	72. 47. 20,75	53,47	21. 18 N.
79	67 β Eridani	3	4. 58. 2,38	2,948	74. 30. 35,74	44,22	5 21 S.
80	69 λ Eridani	4	4. 59 34,73	2,863	74. 53. 40,95	42,95	9. 1 S.
81	* præc. α Aurig.		5. 1. 39,44		75. 24. 51,60		
82	13 *Capella*	1	5 1. 56,16	4,414	75. 29. 2,40	66,21	45. 47 N.
83	* seq. α Aurig.		5. 3. 14,28		75. 48. 34,20		
84	* præc. β Orion.		5. 3. 56,39		75. 59. 5,85		
85	19 *Rigel*	1	5. 4. 55,54	2,867	76. 13. 53,10	43,01	8. 27 S.
86	* seq. β Orionis		5. 8. 24,57		77. 6. 8,55		
87	120 β *Tauri*	2	5 13. 39,38	3,778	78. 24. 50,70	56,67	28. 26 N.
88	24 γ Orionis	2	5. 14. 24,54	3,209	78. 36. 8,16	48,13	6. 9 N.
89	9 β Leporis	3,4	5. 19. 41,09	2 565	79. 55. 16 40	38,47	20. 56 S.
90	34 δ Orionis	2	5. 21. 47,58	3,057	80. 26. 53,69	45,86	0. 28 S.
91	11 α Leporis	3	5. 23. 54,94	2,639	80. 58. 44,13	39,59	17. 59 S.
92	123 ζ Tauri	3	5. 25. 42,41	3,575	81. 25. 36,14	53,62	21. 0 N.
93	46 ε Orionis	2	5. 26. 4,15	3,037	81. 31. 2,19	45,55	1. 20 S.
94	50 ζ Orionis	2	5. 30. 40,57	3,020	82. 40. 8,57	45,30	2. 4 S.
95	κ Columbæ	2	5. 32. 25,03	2,167	83. 6. 15,45	32,50	34. 11 S.
96	13 γ Leporis	3,4	5. 36. 9,02	2,517	84. 2. 15,34	37,75	22. 31 S.
97	53 κ Orionis	4	5 38. 16,28	2,839	84. 34. 4,21	42 59	9. 45 S.
98	* præc. α Orion.		5. 41. 27,74		85. 21. 56,10		
99	58 α *Orionis*	1	5. 44 20,57	3,239	86. 5. 8,55	48,59	7. 21 N.
100	* seq. α Orionis		5. 47. 53,59		86. 58. 23,85		
101	34 β Aurigæ	2,3	5. 44. 51,68	4,398	86. 12. 55,22	69,97	44. 55 N.
102	1 H Geminorum	4,5	5. 51. 57,72	3,642	87. 59. 25,77	54 63	23. 16 N.
103	7 η Geminorum	3,4	6. 2. 48,29	3,623	90. 42. 4,34	54,34	22. 33 N.
104	13 μ Geminorum	3	6. 10. 51,44	3,624	92. 42. 51,64	54,36	22. 36 N.
105	1 ζ Canis maj.	3	6. 12. 39,0[illegible]	2,29	93. 9. 46,24	34,47	29. 59 S.

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Declination. D. M.
106	2 β Canis maj.	2,3	6. 13. 53,76	2,638	93. 28. 26,39	39,57	17. 52 S.
107	18 ν Geminorum	[illegible]	6. 17. 5,48	3,562	94. 16. 22,22	53 43	20. 20 N.
108	24 γ Geminorum	2,3	6. 26. 9,35	3,469	96. 32. 20,32	51,95	16. 34 N.
109	27 ε Geminorum	3,4	6. 31. 37,34	3,696	97. 54. 20,05	55,42	25. 19 N.
110	* præc. α Can. maj.		6. 29. 41,80		97. 25. 27,00		
111	9 *Sirius*	1	6. 36. 19,91	2,647	99. 4. 58,65	39,71	16. 28 S.
112	* seq. α Can. maj.		6. 41. 26,68		100. 21. 40,20		
113	21 ε Canis maj.	2,3	6. 50. 46,21	2,354	102. 41. 33,20	35,31	28. 43 S.
114	43 ζ Geminorum	3,4	6. 52. 14,55	3,567	103. 3. 38,24	[illegible]	20. 51 N.
115	25 δ Canis maj.	[illegible]	7. 0. 15,59	[illegible]	105. 3. 53,85	36,54	26. 5 S.
116	55 δ Geminorum	3	7. 8. 10,06	[illegible]	107. 2. 30,94	52,91	22. 20 N.
117	3 β Canis min.	3	7. 16. 18,01	3,261	109. 4. 30,21	46,92	8. 41 N.
118	* præc. α Gemin.		7. 16. 13,66		109. 3. 24,90		
119	66 *Castor*	1,2	7. 21. 48,81	3,855	110. 27. 12,15	57,83	32. 19 N.
120	* seq. α Gemin.		7. 27. 4,24		111. 46. 12,60		
121	69 υ Geminorum	4,5	7. 23. 34,54	3,715	110. 53. 38,04	55,72	27. 21 N.
122	* pr. α Can. min.		7. 26. 40,27		111. 40. 4,05		
123	10 *Procyon*	1,2	7. 28. 49,10	3,137	112. 12. 16,50	47,06	5. 44 N.
124	* seq. α Can. min.		7. 30. 27,12		112. 36. 46,80		
125	78 *Pollux*	2	7. 33. 3,18	3,687	113. 15. 47,70	55,31	28. 30 N.
126	* seq. β Gemin.		7. 35. 28,78		113. 52. 11,70		
127	10 μ 2 Cancri	5	7. 55. 57,64	3,545	118. 59. 24,55	53,18	22. 9 N.
128	14 ψ 2 Cancri	4	7. 58. 23,25	3,639	119. 35. 48,73	54,58	26. 7 N.
129	17 β Cancri	3,4	8. 5. 39,37	3,266	121. 24. 50,61	48,99	9. 48 N.
130	31 θ Cancri	5,6	8. 20. 16,35	3,441	125. 2. 35,31	51,61	18. 46 N.
131	33 η Cancri	6,7	8. 21. 7,79	3,491	125. 16. 56,87	52,36	21. 6 N.
132	4 δ Hydræ	4	8. 27. 3,04	3,189	126. 45. 45,59	47,83	6. 23 N.
133	43 γ Cancri	4	8. 31. 41,86	3,499	127. 55. 27,86	52,49	22. 10 N.
134	47 δ Cancri	4	8. 33. 18,11	3,428	128. 19. 31,70	51,42	18. 53 N.
135	11 ε Hydræ	[illegible]	8. 36. 10,14	3,199	129. 2. 32,08	[illegible]	7. 8 N.
136	16 ζ Hydræ	4,5	8. 44. 48,86	3,187	131. 12. 12,84	47,81	6. 42 N.
137	60 α 1 Cancri	4,5	8. 44. 59,39	3,290	131. 14. 50,81	[illegible]	12. 24 N.
138	65 α 2 Cancri	3,4	8. 47. 31,82	3,292	131. 52. 57,26	[illegible]	12. 37 N.
139	76 κ Cancri	4,5	8. 56. 54,33	3,263	134. 13. 34,92	48,95	11. 26 N.
140	66 ξ 1 Cancri	5,6	8. 57. 50,37	3,472	134. 27. 35,48	52,06	22. 51 N.

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			H. M. S.	S.	D. M. S.	S.	D. M.
141	22 θ Hydræ	4	9. 3. 54,80	3,120	135. 58. 42,03	46,80	3. 10 N.
142	1 κ Leonis	4	9. 12. 58,32	3,524	138. 14. 34,77	52,86	27. 2 N.
143	30 *Alphard*	2	9. 17. 44,97	2,935	139. 26. 14,55	44,03	7. 48 S.
144	* seq. α Hydræ		9. 23. 9,19		140. 47. 17,85		
145	5 ξ Leonis	4	9. 21. 9,26	3,253	140. 17. 18,97	48,80	12. 11 N.
146	14 ο Leonis	4	9. 30. 27,65	3,224	142. 37. 4,82	48,36	10. 48 N.
147	17 ε Leonis	3	9. 34. 28,29	3,434	143. 37. 4,31	51,51	24. 41 N.
148	24 μ Leonis	3	9. 41. 21,84	3,457	145. 20. 27,54	51,85	26. 57 N.
149	27 ν Leonis	4	9. 47. 26,92	3,243	146. 51. 43,76	48,65	13. 24 N.
150	29 π Leonis	[illegible]	9. 49. 37,99	3,183	147. 24. 29,89	47,75	9. 0 N.
151	30 η Leonis	3,4	9. 56. 24,60	3,289	149. 6. 9,04	49,33	17. 44 N.
152	32 *Regulus*	1	9. 57. 42,02	3,204	149. 25. 30,30	48,06	12. 56 N.
153	* seq. α Leonis		10. 4. 28,58		151. 7. 8,70		
154	36 ζ Leonis	[illegible]	10. 5. 32,34	3,361	151. 23. 5,16	50,42	24. 25 N.
155	41 γ 2 Leonis	2,3	10. 8. 55,22	3,306	152. 13. 48,23	49,60	20. 51 N.
156	34 μ Ursæ maj.	[illegible]	10. 10. 21,55	3,635	152. 35. 23,32	54,52	42. 30 N.
157	47 ρ Leonis	4	10. 22. 15,77	3,170	153. 33. 56,49	47,55	10. 20 N.
158	48 β Ursæ maj.	2	10. 49. 39,53	3,709	162. 24. 52,93	55,63	57. 27 N.
159	7 α Crateris	4	10. 50. 4,55	2,943	162. 31. 8,25	44,14	17. 14 S.
160	50 α Ursæ maj.	1,2	10. 51. 15,84	3,847	162. 48. 55,61	57,70	62. 50 N.
161	11 β Crateris	3,4	11. 1. 50,06	2,933	165. 27. 30,97	44,02	31. 44 S.
162	68 δ Leonis	2,3	11. 3. 26,39	3,199	165. 51. 35,91	47,98	21. 37 N.
163	70 θ Leonis	3	11. 3. 44,23	3,165	165. 56. 3,49	47,48	16. 31 N.
164	13 λ Crateris	5,6	11. 13. 28,15	2,981	168. 22. 2,21	44,72	17. 17 S.
165	78 ι Leonis	4	11. 13. 28,32	3,125	168. 22. 4,85	46,87	11. 38 N.
166	84 τ Leonis	4	11. 17. 39,49	3,085	169. 24. 52,34	46,28	3. 57 N.
167	91 υ Leonis	4	11. 26. 42,82	3,069	171. 40. 42,29	46,04	0. 17 N.
168	3 ν Virginis	5	11. 35. 34,45	3,087	173. 53. 51,70	46,31	7. 39 N.
169	* præc. β Leonis		11. 38. 19,46		174. 34. 51,90		
170	94 *Denebola*	1,2	11. 38. 50,49	3,062	174. 42. 37,35	45,93	15. 41 N.
171	5 β *Virginis*	3	11. 40. 16,38	3,122	175. 4. 5,70	46,93	2. 54 N.
172	64 γ Ursæ maj.	2	11. 43. 14,22	3,212	175. 48. 33,33	48,18	54. 48 N.
173	1 α Corvi	4	11. 58. 6,94	3,062	179. 31. 44,10	45,93	23. 37 S.
174	2 ε Corvi	4	11. 59. 51,63	3,067	179. 57. 54,47	46,20	21. 30 N.
175	69 δ Ursæ maj.	3	12. 5. 27,23	3,021	181. 21. 48,42	45,32	58. 9 N.

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176	4 γ Corvi	3	12. 5. 32,31	3,077	181. 23. 4,68	46,16	16. 26 S.
177	15 η Virginis	3	12. 9. 40,74	3,067	182. 25. 11,15	46,01	0. 27 N.
178	9 β Corvi	[illegible]	12. 23. 54,39	3,124	183. 58. 35,92	46,86	22. 17 S.
179	5 κ Draconis	3	12. 24. 47,65	2,661	186. 11. 54,72	39,91	70. 53 N.
180	29 γ 1 Virginis	3	12. 31. 38,85	3,069	187. 53. 27,72	46,03	0. 21 S.
181	77 ε Ursæ maj.	2,3	12. 45. 12,58	2,746	191. 18. 8,67	41,19	57. 3 N.
182	43 δ Virginis	3	12. 45. 33,66	3,047	191. 23. 24,93	45,71	4. 29 N.
183	47 ε Virginis	3	12. 52. 13,38	[illegible]	193. 3. 20,01	45,06	12. 2 N.
184	51 θ Virginis	3,4	12. 59. 36,46	3,095	194. 54. 6,97	46,42	4. 28 S.
185	2 γ Hydræ	3	13. 8. 4,31	3,225	197. 1. 4,64	48,38	22. 7 S.
186	* præc. α Virg.		13. 9. 13,00		197. 18. 15,00		
187	67 *Spica*	1	13. 14. 40,11	3,137	198. 40. 1,66	47,06	10. 7 S.
188	79 ζ Ursæ maj.	3	13. 15. 49,62	2,425	198. 57. 24,26	36,37	55. 59 N.
189	99 ι Virginis	4	13. 16. 10,79	3,129	199. 2. 41,83	46,93	11. 40 S.
190	79 ζ Virginis	3	13. 24. 30,65	3,064	201. 7. 39,68	45,96	0. 26 N.
191	4 τ Bootis	4	13. 37. 46,36	2,884	204. 26. 35,35	43,26	18. 27 N.
192	85 η Ursæ maj.	2,3	13. 39. 38,85	2,355	204. 54. 42,80	35,88	50. 19 N.
193	8 η Bootis	3	13. 45. 9,21	2,860	206. 17. 18,12	42,90	19. 25 N.
194	11 α Draconis	2,3	13. 58. 58,88	1,628	209. 44. 43,24	24,42	65. 20 N.
195	98 κ Virginis	4	14. 2. 14,87	3,179	210. 33. 43,04	47,68	9. 20 S.
196	16 *Arcturus*	1	14. 6. 32,21	2,722	211. 38. 3,16	40,83	20. 15 N.
197	* seq. α Bootis		14. 6. 36,46		211. 39. 6,90		
198	100 λ Virginis	4	14. 8. 19,14	3,223	212. 4. 47,16	48,35	12. 27 S.
199	24 γ Bootis	3	14. 24. 1,50	2,428	216. 0. 22,54	36,42	39. 11 N.
[illegible]	30 ζ Bootis	3	14. 31. 35,56	2,854	217. 53. 53,42	42,81	14. 36 S.
201	36 ε Bootis	3	14. 36. 14,99	2,622	219. 3 44,80	39,33	27. 56 N.
202	7 μ Libræ	5	14. 38. 22,95	3,268	219. 35. 44,22	49,02	13. 18 S.
203	8 α 1 *Libræ*	6	14. 39. 38,74	3,299	219. 54. 41,10	49,49	15. 9 S.
204	* præc. α 2 ♎		14. 39. 38,77		219. 54. 41,55		
205	9 α 2 *Libræ*	2,3	14. 39. 49,97	3,289	219. 57. 29,55	49,34	15. 12 S.
206	7 β Ursæ min.	3	14. 51. 27,55	-0,329	222. 51. 53,19	-4 94	74. 59 N.
207	20 γ Scorpii	3	14. 52. 24,35	3,482	223. 6. 5,22	52,23	24. 29 S.
208	42 β Bootis	3	14. 54. 24,99	2,262	223. 36. 14,85	33,93	41. 11 N.
209	43 ψ Bootis	5	14. 55. 52,50	2,580	223. 58. 7,57	38,70	27. 44 N.
210	27 β Libræ	2,3	15. 6. 15,61	3,215	226. 33. 54,21	48,22	8. 38 S.

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211	49 δ Bootis	3	15. 7. 26,62	2,409	226. 51. 39,28	36,13	34. 4 N.
212	1 ο Coron. Bor.	6	15. 11. 51,97	2,487	227. 57. 59,55	37,30	30. 21 N.
213	2 η Coron. Bor	5	15. 14. 56,05	2,465	228. 44. 0,78	36,97	31. 1 N.
214	3 β Coron. Bor.	4	15. 19. 34,88	2,483	229. 53. 43,13	37,24	29. 48 N.
215	13 γ 2 Ursæ min	2,3	15. 21. 11,76	-0,209	230. 17. 56,39	—3,14	72. 33 N.
216	35 ζ 4 Libræ	[illegible]	15. 21. 38,42	3,365	230. 24. 36,26	50,48	16. 10 S.
217	38 γ Libræ	3,4	15. 24. 21,82	3,328	231. 5. 18,34	49,92	14. 7 S.
218	13 δ Serpentis	3	15. 25. 15,84	2,861	231. 18. 57,61	42,91	11. 13 N.
219	5 *Gemma*	2	15. 26. 13,29	2,543	231. 33. 19,35	38,15	27. 24 N.
220	43 κ Libræ	4	15. 30. 27,12	3,433	232. 36. 46,77	51,49	19. 1 S.
221	24 α *Serpentis*	2	15. 34. 25,21	2,936	233. 36. 18,00	44,04	7. 4 N.
222	* seq. α Serpent		15. 36. 23,53		234. 5. 53,05		
223	28 β Serpentis	[illegible]	15. 36. 57,70	2,756	234. 14. 25,47	41,34	16. 4 N.
224	32 μ Serpentis	4	15. 39. 10,30	3,028	234. 47. 34,55	45,35	2. 48 S.
225	37 ι Serpentis	3,4	15. 40. 50,97	2,969	235. 12. 44,51	44,54	5. 6 N.
226	10 δ Coron. Bor.	4	15. 41. 12,45	2,515	235. 18. 6,71	37,73	26. 42 N.
227	45 λ Libræ	4	15. 41. 44,86	3,457	235. 26. 12,93	51,86	19. 33 S.
228	5 ρ Scorpii	3,4	15. 44. 38,34	3,671	236. 8. 20,06	55,06	28. 37 S.
229	6 π Scorpii	3	15. 46. 46,43	3,600	236. 41. 36,48	54,00	25. 31 S.
230	48 ψ Libræ	4	15. 47. 0,91	3,339	236. 45. 13,63	50,09	13. 41 S.
231	41 γ Serpentis	3	15. 47. 12,93	2,740	236. 48. 13,98	41,10	16. 21 N.
232	7 δ Scorpii	3	15. 48. 31,89	3,521	237. 7. 58,38	52,82	22. 2 S.
233	13 ε Coron. Bor.	4,5	15. 49. 18,54	2,483	237. 19. 38,04	37,24	27. 28 N.
234	44 π Serpentis	4	15. 53. 41,18	2,576	238. 25. 17,72	38,64	23. 21 N.
235	8 β Scorpii	2	15. 53. 49,71	3,465	238. 27. 25,65	51,97	19. 15 S.
236	13 θ Draconis	3,4	15. 58. 8,28	1,142	239. 32. 4,27	17,13	59. 6 N.
237	14 ν Scorpii	4	16. 0. 23,31	3,465	240. 5. 49,60	51,96	18. 56 S.
238	1 δ Ophiuchi	3	16. 3. 52,80	3,132	240. 58. 11,95	46,98	3 10 S.
239	2 ε Ophiuchi	3,4	16. 7. 45,09	3,154	241. 56. 16,31	47,30	4. 12 S.
240	20 γ Herculis	3	16. 13. 5,83	2,642	243. 16. 27,41	39,63	19. 38 N.
241	21 *Antares*	1	16. 17. 9,69	3,645	244. 17. 25,35	54,68	25. 58 S.
242	* α Scorpii		16. 19. 6,66		244. 46. 39,90		
243	8 φ Ophiuchi	4,5	16. 19. 43,03	3,418	244. 55. 45,42	51,27	16. 10 S
244	14 η Draconis	3,4	16. 21. 18,33	0,785	245. 19. 34,92	11,78	61. 58 N.
245	27 β Herculis	3	16. 21. 37,87	2,579	245. 21. 23,08	38,68	21. 56 N

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246	23 τ Scorpii	4	16. 23. 26,96	3,709	245. 51. 44,38	55,64	27. 47 S.
247	13 ζ Ophiuchi	2,3	16. 26. 9,55	3,287	246. 32. 23,24	49,30	10. 9 S.
248	40 ζ Herculis	3,4	16. 33. 45,64	2,292	248. 26. 24,67	34,38	32. 1 N.
249	44 η Herculis	3,4	16. 36. 3,14	2,046	249. 0. 47,15	30,69	39. 19 N.
250	58 ε Herculis	3	16. 52. 38,68	2,292	253. 9. 40,16	34,38	31. 16 N.
251	35 η Ophiuchi	2,3	16. 58. 55,15	3,424	254. 43. 47,26	51,36	15. 28 S.
252	* præc. α Herc.		17. 5. 12,70		256. 18. 10,50		
253	64 α *Herculis*	2,3	17. 5. 31,76	2,726	256. 22. 56,40	40,89	14. 38 N.
254	65 δ Herculis	3,4	17. 6. 49,41	2,459	256. 42. 21,17	36,88	25. 5 N.
255	42 θ Ophiuchi	3	17. 9. 44,25	3,669	257. 26. 3,66	[illegible]	24. 47 S.
256	35 λ Scorpii	3	17. 20. 2,64	4,057	260. 0. 39,55	60,85	36. 57 S.
257	* præc. α Oph.		17. 24. 45,90		261. 11. 28,50		
258	55 α *Ophiuchi*	2	17. 25. 38,97	2,768	261. 24. 44,55	[illegible]	12. 43 N.
259	* seq. α Ophiuchi		17. 29. 11,02		262. 17. 45,32		
260	23 β Draconis	3	17. 25. 55,99	1,346	261. 28. 59,82	20,22	52. [illegible] N.
261	60 β Ophiuchi	3	17. 33. 35,77	2,959	263. 23. 56,54	44,39	4. 40 N.
262	62 γ Ophiuchi	3	17. 37. 52,04	3,003	264. 28. 0,56	45,05	2. 48 N.
263	57 ζ Serpentis	3,4	17. 49. 54,59	3,153	267. 28. 38,87	47,30	3. 40 S.
264	67 o Oph. Tau. Pon	4	17. 50. 37,47	2,999	267. 39. 21,99	44,99	2. 57 N.
265	33 γ Draconis	2,3	17. 51. 57,79	1,389	267. 59. 26,85	20,83	51. 31 N.
266	10 γ Sagittarii	3,4	17. 52. 58,05	3,851	268. 14. 30,76	57,77	30. 25 S.
267	b Taur. Poniat.		18. 0. 41,10	2,993	270. 10. 16,50	[illegible]	3. 19 N.
268	30 μ 1 Sagittarii	[illegible]	18. 1. 48,37	3,584	270. 27. 5,63	53,76	21. 6 S.
269	15 μ 2 Sagittarii	4,6	18. 3. 16,90	3,575	270. 49. 13,57	53,62	20. 46 S.
270	20 ε Sagittarii	2,3	18. 10. 53,67	3,984	272. 43. 25,11	59,76	34. 28 S.
271	22 λ Sagittarii	4	18. 15. 37,66	3,705	273. 54. 24,92	55,57	25. 31 S.
272	* præc. α Lyræ		18. 28. 40,12		277. 10. 1,88		
273	3 *Wega*	1	18. 30. 9,89	2,028	277. 32. 28,35	30,43	38. 36 N.
274	* seq. α Lyræ		18. 31. 40,00		277. 54. 50,00		
275	27 φ Sagittarii	3,4	18. 33. 9,39	3,747	278. 17. 20,84	56,21	27. 11 S.
276	4 ε Lyræ	5	18. 37. 42,87	1,963	279. 25. 43,03	29,74	39. 28 N.
277	32 ν 1 Sagittarii	4,5	18. 42. 5,41	3,625	280. 31. 21,22	54,38	22. 59 S.
278	10 β Lyræ	3	18. 42. 41,86	2,211	280. 40. 27,89	33,16	33. 9 N.
279	34 σ Sagittarii	3	18. 42. 51,40	3,724	280. 42. 50,99	[illegible]	26. 32 S.
280	35 ν 2 Sagittarii	4,5	18. 43. 1,13	3,623	280. 45. 16,99	[illegible]	22. 54 S.

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281	63 ϑ Serpent. dup.	3	18. 46. [illegible]	2,977	281. 34. [illegible]	44,66	3. 57 N
282	12 ϑ Lyræ	3,4	18. 47. 31,16	2,095	281. 52. 47,43	31,42	36. 39 N
283	47 ο Draconis	4	18. 48. 14,15	0,880	282. 3. 32,20	13,21	59. 9 N
284	14 γ Lyræ	3	18. 51. 28,04	2,241	282. 52. 0,60	33,61	32. 26 N
285	39 ο Sagittarii	4	18. 52. 41,18	3,595	283. 10. 17,72	53,92	22. 1 S.
286	40 τ Sagittarii	4	18. 54. 26,45	3,758	283. 36. 36,82	56,37	27. 57 S.
287	16 λ Antinoi	3,4	18. 55. 38,10	3,186	283. 54. 31,55	47,79	5. 10 S
288	17 ζ Aquilæ	3	18. 56. 12,69	2,755	284. 3. 10,37	41,33	13. 35 N
289	41 π Sagittarii	3,4	18. 57. 51,37	3,574	284. 27. 50,57	53,61	21. 20 S.
290	42 ψ Sagittarii	4,5	19. 3. 15,27	3,685	285. 48. 49,04	55,27	25. 35 S.
291	43 d Sagittarii	4,6	19. 5. 55,86	3,517	286. 28. 57,90	52,76	19. 18 S
292	57 δ Draconis	3	19. 12. 27,95	0,033	288. 6. 59,21	0,49	67. 19 N
293	1 κ Cygni	4	19. 12. 28,28	1,383	288. 7. 4,19	20,73	52. 58 N
294	30 δ Aquilæ	3	19. 15. 24,12	3,008	288. 51. 1,79	45,12	2. 44 N
295	6 β Cygni	3	19. 22. 38,53	2,415	290. 39. 37,97	36,23	27. 33 N
296	10 ι Cygni	4,6	19. 24. 39,61	1,511	291. 9. 54,19	22,67	51. 19 N
297	41 ι Antinoi	3,4	19. 26. 22,16	3,106	291. 35. 32,37	46,59	1. 43 S.
298	13 ϑ Cygni	4	19. 31. 5,16	1,645	292. 46. 17,40	24,68	49. 46 N
299	5 α Sagittæ	4	19. 31. 9,25	2,678	292. 47. 18,74	40,17	17. 34 N
300	56 f Sagittarii	6	19. 34. 41,43	3,520	293. 40. 21,39	52,80	20. 14 S.
301	* præc. γ Aquilæ		19. 35. 12,50		293. 48. 7,50		
302	50 γ *Aquilæ*	3	19. 36. 44,50	2,837	294. 11. 7,50	42,59	10. 8 N
303	* seq. γ Aquilæ		19. 39. 0,81		294. 45. 12,16		
304	18 δ Cygni	[illegible]	19. 38. 43,09	1,869	294. 40. 46,34	28,02	44. 39 N
305	* præc. α Aquilæ		19. 38. 35,87		294. 38. 58,05		
306	53 *Atair*	1,2	19. 41. 1,02	2,918	295. 15. 15,30	43,78	8. 21 N
307	* seq. α Aquilæ		19. 42. 52,37		295. 43. 5,55		
308	55 η Antinoi	3,4	19. 42. 17,14	3,058	295. 34. 17,08	45,87	0. 30 N
309	59 b Sagittarii	4,5	19. 44. 39,66	3,699	296. 9. 53,46	55,48	27. 41 S
310	60 β *Aquilæ*	3,4	19. 45. 28,97	2,939	296. 22. 14,55	44,08	5. 55 N
311	65 ϑ Aquilæ	[illegible]	20. 0. 58,52	3,097	300. 14. 37,75	46,45	1. 24 S.
312	5 α 1 *Capricorni*	3,4	20. 6. 32,79	3,330	301. 38. 11,88	49,95	13. 7 S.
313	* præc. α 2 Capr.		20. 5. 17,48		301. 19. 22,20		
314	6 α 2 *Capricorni*	3	20. 6. 56,48	3,331	301. 44. 7,20	49,96	13. 9 S.
315	* seq. α 2 Capr.		20. 9. 33,32		302. 23. 19,80		

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascensions in Time. H. M. S.	Annual Variat. + S.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Declination. D. M.
316	pr. β ♑ 827 Mayeri		20. 9. 31,35	3,380	302. 22. 50,19	50,70	15. 24 S.
317	8 ν Capricorni	6	20. 9. 33,69	3,337	302. 23. 25,39	50,06	15. 23 S.
318	9 β Capricorni	3	20. 9. 45,50	3,380	302. 26. 22,54	50,70	15. 24 S.
319	37 γ Cygni	3	20. 15. 2,63	2,148	303. 45. 39,39	32,22	39. 38 N.
320	11 ρ Capricorni	6	20. 17. 26,45	3,438	304. 21. 36,72	51,57	18. 28 S.
321	4 ζ Delphini	4,5	20. 25. 57,50	2,801	306. 29. 22,44	42,01	14. 0 N.
322	6 β Delphini	3	20. 28. 10,32	2,804	307. 2. 34,74	42,06	13. 55 N.
323	9 α Delphini	3	20. 30. 20,76	2,780	307. 35. 11,37	41,70	15. 13 N.
324	50 *Deneb.*	1,2	20. 34. 36,68	2,034	308. 39. 10,20	30,51	44. 34 N.
325	* seq. α Cygni		20. 40. 28,55		310. 7. 8,25		
326	2 ε Aquarii	4,5	20. 36. 50,38	3,255	309. 12. 35,72	48,86	10. 13 S.
327	* praec. γ Delph.		20. 37. 21,94		309. 20. 29,08		
328	12 γ Delphini	3	20. 37. 22,96	2,783	309. 20. 44,34	41,75	15. 25 N.
329	53 ε Cygni	3	20. 38. 6,70	2,393	309. 31. 40,56	35,89	33. 13 N.
330	6 μ Aquarii	4,5	20. 41. 51,17	3,243	310. 27. 47,61	48,65	9. 43 S.
331	7 Aquarii	6	20. 46. 4,59	3,255	311. 31. 8,80	48,82	10. 28 S.
332	23 θ Capricorni	5,4	20. 54. 39,73	3,384	313. 39. 50,25	50,76	18. 1 S.
333	13 ν Aquarii	5	20. 58. 41,24	3,274	314. 40. 18,64	49,11	12. 10 S.
334	8 α Equulei	4	21. 5. 48,78	2,997	316. 27. 11,73	44,96	4. 26 N.
335	32 ι Capricorni	5	21. 11. 5,67	3,355	317. 46. 25,00	50,33	17. 41 S.
336	4 β Equulei	6	21. 12. 57,71	2,981	318. 14. 25,62	44,72	5. 58 N.
337	18 Aquarii	6	21. 13. 14,77	3,286	318. 18. 41,53	49,29	13. 44 S.
338	5 α Cephei	3	21. 13. 46,83	1,427	318. 26. 42,64	21,40	61. 45 N.
339	22 β Aquarii	3	21. 21. 1,12	3,165	320. 15. 16,74	47,48	6. 27 S.
340	39 ε Capricorni	4	21. 25. 52,29	3,379	321. 28. 4,34	50,68	20. 21 S.
341	8 β Cephei	3	21. 26. 1,18	0,821	321. 30. 17,70	12,32	69. 41 N.
342	40 γ Capricorni	3,4	21. 28. 59,14	3,329	322. 14. 47,15	49,93	17. 33 S.
343	43 κ Capricorni	5	21. 31. 27,98	3,360	322. 51. 59,74	50,40	19. 46 S.
344	8 ε Pegasi	3	21. 34. 21,53	2,943	323. 35. 22,99	44,15	8. 58 N.
345	80 π 1 Cygni	4	21. 34. 59,63	2,116	323. 44. 54,38	31,74	50. 17 N.
346	49 δ Capricorni	3	21. 35. 58,68	3,310	323. 59. 40,25	49,65	17. 1 S.
347	* praec. α Aquar.		21. 55. 8,21		328. 47. 3,15		
348	34 α *Aquarii*	3	21. 55. 29,75	3,067	328. 52. 26,25	46,00	1. 17 S.
349	48 γ Aquarii	3	22. 11. 18,89	3,094	332. 49. 43,39	46,41	2. 23 S.
350	52 π Aquarii	4,5	22. 15. 4,00	3,065	333. 45. 59,98	45,97	0. 22 N.

ZACH's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Declination. D. M.
351	55 ζ Aquarii	4	22. 18. 31,68	3,079	334. 37. 55,26	46,18	1. 2 S.
352	57 σ Aquarii	5	22. 20. 2,99	3,186	335. 0. 44,84	47,79	11. 42 S.
353	7 Lacertæ	4	22. 23. 7,61	2,431	335. 46. 54,14	36,46	49. 16 N.
354	62 η Aquarii	4	22. 25. 4,59	3,079	336. 16. 8,78	46,19	1. 9 S.
355	63 κ Aquarii	5	22. 27. 23,38	3,117	336. 50. 50,67	46,76	5. 14 S.
356	42 ζ Pegasi	3	22. 31. 29,06	2,981	337. 52. 15,89	44,72	9. 48 N.
357	44 η Pegasi	3	22. 33. 37,87	2,792	338. 24. 27,91	41,88	29. 11 N.
358	69 τ 1 Aquarii	5	22. 37. 4,70	3,197	339. 16. 10,57	47,96	15. 7 S.
359	71 τ 2 Aquarii	5,6	22. 38. 59,29	3,190	339. 44. 49,41	47,85	14. 39 S.
360	73 λ Aquarii	4	22. 42. 10,52	3,137	340. 32. 37,87	47,05	8. 38 S.
361	32 ι Cephei	4	22. 42. 35,33	2,109	340. 38. 49,95	31,63	65. 9 N.
362	76 δ Aquarii	3	22. 44. 1,80	3,201	341. 0. 27,06	48,02	16. 53 S.
363	* præc. α Pisc. aus.		22. 40. 17,35		340. 4. 20,25		
364	24 *Fomalhaut*	1	22. 46. 33,60	3,330	341. 38. 24,00	49,95	30. 41 S.
365	* seq. α Pisc. aus.		22. 48. 38,04		342. 9. 30,60		
366	53 β Pegasi	2	22. 54. 5,50	2,874	343. 31. 22,47	43,11	27. 0 N.
367	54 *Markab*	2	22. 54. 47,99	2,964	343. 41. 59,85	44,46	14. 8 N.
368	* seq. α Pegasi		22. 55. 35,64		343. 53. 54,60		
369	90 φ Aquarii	4,5	23. 3. 57,39	3,109	345. 59. 20,79	46,64	7. 7 S.
370	91 ψ 1 Aquarii	5	23. 5. 23,05	3,125	346. 20. 45,68	46,88	10. 10 S.
371	6 γ Piscium	5	23. 6. 46,42	3,057	346. 41. 36,37	45,85	2. 12 N.
372	95 ψ 3 Aquarii	3	23. 8. 32,63	3,125	347. 8. 9,48	46,88	10. 42 S.
373	16 Piscium	6	23. 26. 11,40	3,065	351. 32. 50,96	45,97	1. 0 N.
374	18 λ Piscium	5	23. 31. 51,01	3,066	352. 57. 45,08	45,99	0. 40 N.
375	19 Piscium	5,6	23. 36. 10,88	3,062	354. 2. 43,18	45,93	2. 23 N.
376	28 ω Piscium	5	23. 49. 2,88	3,061	357. 15. 43,16	45,92	5. 46 N.
377	* præc. α Androm		23. 55. 45,47		358. 56. 22,05		
378	* præc. α Androm		23. 56. 15,28	3,060	359. 3. 49,19	45,90	
379	21 α *Andromedæ*	2	23. 58. 4,32	3,065	359. 31. 4,95	45,97	27. 59 N.
380	* seq. α Androm.		0. 1. 32,98		0. 23. 14,70		
381	11 β Cassiopeiæ	2,3	23. 58. 34,32	3,051	359. 38. 34,75	45,76	58. 3 N.

M. ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS, WITH THEIR ANNUAL VARIATIONS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800. D. M. S.	Annual Variation. S.
1	Polaris	88. 14. 25 N.	+19,57
2	*Polaris*	88. 14. 26 N.	
3	η Ursæ majoris	50. 19. 4 N.	—18,20
4	α Persei	49. 8. 10 N.	+13,59
5	ι Ursæ majoris	48. 49. 9 N.	—18,21
6	γ Persei	47. 8. 14 N.	+12,35
7	*Capella*	45. 46. 50 N.	+ 5,09
8	α Cygni	44. 34. 20 N.	+12,52
9	α *Cygni*	44. 34. 19 N.	
10	β Bootis	41. 11. 11 N.	—14,54
11	α Lyræ	38. 36. 15 N.	+ 2,59
12	α *Lyræ*	38. 36. 10 N.	
13	ζ Herculis	32. 58. 19 N.	— 7,40
14	Castor	32. 18. 54 N.	— 6,95
15	*Castor*	32. 18. 41 N.	
16	*Pollux*	28. 29. 47 N.	— 7,46
17	β Tauri	28. 25. 25 N.	+ 4,08
18	β *Tauri*	28. 25. 30 N.	
19	ε Bootis	27. 55. 32 N.	—15,59
20	α Andromedæ	27. 59. 15 N.	+20,25
21	α *Andromedæ*	27. 59. 11 N.	
22	β Cygni	27. 32. 51 N.	+ 7,04
23	Gemma	27. 23. 48 N.	—12,50
24	*Gemma*	27. 23. 49 N.	
25	μ Leonis	26. 56. 37 N.	—16,46
26	β Pegasi	26. 59. 58 N.	+19,21
27	ε Geminorum	25. 18. 55 N.	— 2,72
28	ε *Geminorum*	25. 18. 56 N.	
29	δ Herculis	25. 5. 4 N.	— 4,56
30	ε Leonis	24. 41. 17 N.	—16,10
31	ζ Leonis	24. 24. 27 N.	—17,56
32	Alcione	23. 28. 34 N.	+11,88
33	Electra	23. 28. 27 N.	+12,04
34	Atlas	23. 25. 54 N.	+11,74
35	*Propus*	23. 15. 40 N.	+ 0,75

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800. D. M. S.	Annual Variation. S.
36	τ Pegasi	22. 38. 51 N.	+19,57
37	μ *Geminorum*	22. 36. 12 N.	— 0,89
38	η Geminorum	22 32. 59 N.	— 0,19
39	η *Geminorum*	22. 33. 5 N.	— 0,19
40	α Arietis	22. 30. 37 N.	+17,55
41	α *Arietis*	22. 30. 40 N.	
42	δ Geminorum	22. 20. 19 N.	— 5,83
43	γ Cancri	22. 10. 43 N.	—12,28
44	μ Cancri	22. 9. 9 N.	— 9,67
45	β Herculis	21. 56. 2 N.	— 8,88
46	δ Leonis	21. 37. 1 N.	—19,43
47	ζ Tauri	21. 0. 32 N.	+ 3,05
48	γ Leonis	20. 50. 54 N.	—17,72
49	ζ Geminorum	20. 51. 0 N.	— 4,48
50	ζ *Geminorum*	20. 51. 5 N.	
51	ν *Geminorum*	20. 19. 34 N.	— 1,44
52	Arcturus	20. 13. 45 N.	—19,10
53	*Arcturus*	20. 13. 45 N.	—19,10
54	γ Herculis	19. 37. 52 N.	— 9,05
55	η Bootis	19. 24. 19 N.	—18,00
56	δ Cancri	18. 52. 52 N.	—12,40
57	e Pegasi	18 57. 14 N.	+14,91
58	β Arietis	18. 49. 30 N.	+18,03
59	γ Arietis	18. 18. 29 N.	+18,09
60	δ Sagittæ	18. 3. 15 N.	+ 7,73
61	η Leonis	17. 43. 57 N.	—17,18
62	α Sagittæ	17. 33. 48 N.	+ 7,73
63	1 δ Tauri	17. 3. 38 N.	+ 9,19
64	θ Leonis	16. 31. 13 N.	—19,43
65	γ Geminorum	16. 33. 27 N.	— 2,22
66	γ *Geminorum*	16. 33. 28 N.	
67	γ Sepentis	16. 19. 40 N.	—11,01
68	β Serpentis	16. 3 21 N.	—11,75
69	Aldebaran	16. 5. 43 N.	+ 8,16
70	*Aldebaran*	16. 5. 45 N.	

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800.	Annual Variation.
		D. M. S.	S.
71	β Leonis	15. 41. 30 N.	—19,96
72	β *Leonis*	15. 41. 27 N.	
73	γ Delphini	15. 24 40 N.	+12,68
74	α Delphini	15. 12. 48 N.	+12,21
75	γ Tauri	15. 8. 1 N.	+ 9,42
76	ζ Bootis	14. 35. 34 N.	—15,85
77	α Herculis	14. 37. 38 N.	— 4,75
78	α Pegasi	14. 7. 49 N.	+19,22
79	α *Pegasi*	14. 7. 57 N.	+19,22
80	γ Pegasi	14. 4. 16 N.	+20,04
81	γ *Pegasi*	14. 4. 15 N.	
82	β Delphini	13. 54. 31 N.	+12,05
83	ζ Aquilæ	13. 34. 32 N.	+ 4,83
84	Regulus	12. 56. 23 N.	—17,24
85	*Regulus*	12. 56. 20 N.	
86	α Cancri	12. 37. 30 N.	—13,18
87	α Ophiuchi	12. 42. 55 N.	— 3,05
88	α *Ophiuchi*	12. 43. 7 N.	
89	ε Virginis	12. 2. 11 N.	—19,54
90	δ Serpentis	11. 12. 56 N.	—12,57
91	ο Leonis	10. 47. 40 N.	—15,94
92	ε Delphini	10. 37. 50 N.	+11,73
93	ζ Leonis	10. 19. 52 N.	—18,24
94	γ Aquilæ	10. 8. 6 N.	+ 8,17
95	γ [illegible]	10. 8. 10 N.	
96	[illegible]	8. 57. 43 N.	+16,10
[illegible]	[illegible]	8. 40. 51 N.	— 6,51
[illegible]	[illegible]	8. 20. 58 N.	[illegible]
[illegible]	[illegible]	8. 20. 48 N.	
[illegible]	[illegible]	7. 21. 27 N.	[illegible]
[illegible]	[illegible]	7. 21. 27 N.	
[illegible]	[illegible]	7. 8. 37 N	[illegible]
[illegible]	[illegible]	7. 3. 55 N.	[illegible]
[illegible]	[illegible]	7. 3. 50 N.	
[illegible]	[illegible]	6. 23. 22 N.	[illegible]

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800.	Annual Variation.
		D. M. s.	s.
106	β Aquilæ	5. 55. 4 N.	+ 8,86
107	β *Aquilæ*	5. 55. 19 N.	
108	*Procyon*	5. 44. 11 N.	— 7,51
109	β Ophiuchi	4. 39. 41 N.	— 2,35
110	δ Virginis	4. 29. 13 N.	—19,66
111	θ Serpentis	3. 57. 12 N.	+ 3,97
112	α Ceti	3. 17. 49 N.	+14,70
113	α *Ceti*	3. 18. 0 N.	
114	β Virginis	2. 53. 35 N.	—19,97
115	β *Virginis*	2. 53. 38 N.	
116	γ Ophiuchi	2. 47. 42 N.	— 1,97
117	δ Aquilæ	2. 43. 36 N.	+ 6,44
118	γ Ceti	2. 23. 17 N.	+15,77
119	α Piscium	1. 47. 40 N.	+17,73
120	η Antinoi	0. 30. 12 N.	+ 8,61
121	δ Orionis	0. 27. 17 N.	— 3,38
122	ζ Virginis	0. 25. 48 N.	—18,72
[redacted]	ι Hydræ	0. 8. 18 S.	—15,86
124	γ Virginis	0. 21. 4 S.	+19,86
125	δ Ceti	0. 32. 18 S.	—15,97
126	α Aquarii	1. 17. 7 S.	—17,15
127	α *Aquarii*	1. 17. 7 S.	
128	ε Orionis	1. 24. 12 S.	— 3,02
129	θ Antinoi	1. 24. 10 S.	—10,05
130	ζ Orionis	2. 3. 33 S.	— 2,60
131	γ Aquarii	2. 23. 31 S.	—17,81
132	δ Ophiuchi	3. 10 8 S.	+ 9,77
133	ζ Serpentis	3. 39. 32 S.	— 0,93
134	ε Ophiuchi	4. 11. 37 S.	+ 9,47
135	θ Virginis	4. 28. 4 S.	+19,39
136	β Eridani	5. 21. 16 S.	— 5,41
137	ι Orionis	6. 3. 0 S.	— 3,04
138	β Aquarii	6. 26. 37 S.	—15,39
139	φ Aquarii	7. 7. 25 S.	—19,44
140	α Hydræ	7 47. 56 S.	+15,21

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800. D. M. S.	Annual Variation. S.
141	α *Hydræ*	7. 47. 53 S.	+15,21
142	*Rigel*	8. 26. 35 S.	— 4,81
143	β Libræ	8. 38. 14 S.	+13,82
144	λ Aquarii	8. 38. 29 S.	—18,89
145	α Spica	10. 6. 46 S.	+19,01
146	*Spica*	10. 6. 45 S.	
147	ζ Ophiuchi	10. 9. 1 S.	+ 8,02
148	δ Eridani	10. 27. 5 S.	—11,99
149	μ Ceti	11. 22. 48 S.	+15,71
150	λ Virginis	12 26. 26 S.	+17,01
151	1α Capricorni	13. 6. 58 S.	—10,47
152	1α *Capricorni*	13. 7. 0 S.	
153	2α Capricorni	13. 9. 15 S.	—10,50
154	2α *Capricorni*	13. 9. 17 S.	
155	γ Libræ	14. 6 42 S.	+12,63
156	γ Eridani	14. 5. 3 S.	—10,90
157	α Libræ	15. 12. 0 S.	+15,40
158	α *Libræ*	15. 12. 0 S.	
159	δ Corvi	15. 24. 5 S.	+19,98
160	β Capricorni	15. 24. 10 S.	—10,71
161	γ Canis majoris	15. 21. 6 S.	+ 4,69
162	η Ophiuchi	15. 27. 58 S.	+ 5,33
163	ι Aquarii	15. 48. 54 S.	—17,14
164	γ Corvi	16. 25. 47 S.	+ 20,04
165	Sirius	16. 27. 7 S.	+ 4,33
166	*Sirius*	16. 27. 5 S.	
167	δ Aquarii	16. 52. 59 S.	—18,85
168	δ Capricorni	17. 1. 38 S.	—16,19
169	α Crateris	17. 14. 11 S.	+19,11
170	γ Capricorni	17. 33. 22 S.	—15,82
171	ι Capricorni	17. 39. 58 S.	—14,97
172	β Canis majoris	17. 51. 55 S.	+ 1,18
173	α Leporis	17. 58. 22 S.	— 3,18
174	ϑ Capricorni	18. 1. 3 S.	—13,81
175	ν Scorpii	18. 55. 46 S.	+10,03

ZACH's CATALOGUE OF THE DECLINATIONS OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Declination for the Year 1800. D. M. S.	Annual Variation. S.
176	β Ceti	19. 5. 9 S.	— 19,84
177	β Scorpii	19. 14. 46 S.	+ 10,52
178	μ Sagittarii	21. 5. 50 S.	— 0,09
179	π Sagittarii	21. 19. 45 S.	— 4,95
180	ε Corvi	21. 30. 32 S.	+ 20,05
181	δ Scorpii	22. 2. 30 S.	+ 10,92
182	ο Sagittarii	22. 1. 21 S.	— 4,51
183	β Corvi	22. 17. 17 S.	+ 19,94
184	γ Leporis	22. 31. 15 S.	— 2,11
185	α Corvi	23. 36. 50 S.	+ 20,04
186	γ Scorpii	24. 29. 10 S.	+ 14,67
187	ϑ Ophiuchi	24. 47. 17 S.	+ 4,43
188	σ Scorpii	25. 6. 8 S.	+ 9,38
189	π Scorpii	25. 31. 36 S.	+ 11,06
190	Antares	25. 58. 38 S.	+ 8,75
191	*Antares*	25. 58. 23 S.	
192	δ Canis majoris	26. 5. 10 S.	+ 5,18
193	ε Canis majoris	28. 42. 29 S.	+ 4,38
194	ζ Canis majoris	29. 58. 50 S.	+ 1,07
195	*Fomalhaut*	30. 40. 38 S.	— 19,01

In these two Catalogues, the right ascensions and declinations of the stars denoted by *Italic* characters, were taken from Dr. Maskelyne's Catalogue of fundamental stars; by a comparison with which, M. de Zach found the right ascensions of the other stars, as given in the first Catalogue, by a transit instrument eight feet long, made by Mr. Ramsden. In the second Catalogue, the declinations of the stars which were not taken from Dr. Maskelyne's Catalogue, were computed from observations made by M. Cassini at Paris, from 1778 to 1790, by a quadrant of six feet radius. The variations in right ascension and declination were taken from Mr. Wollaston's Catalogue, except those of *Arcturus* and *Sirius*, to which are added their proper motions in declination, as determined by Dr. Maskelyne.

TOBIAS MAYER's CATALOGUE OF THE PRINCIPAL STARS,

FOR THE BEGINNING OF 1790.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Preces.	Right Ascension in Time.	Annual Preces.	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
1	γ Pegasi	2	0. 36. 35	46,07	0. 2. 26,3	3,071	75. 59. 0	—20,04
2	Piscium	7	1. 2. 41	46,07	0. 4. 10,7	3,071	82. 20. 44	—20,04
3		7	1. 27. 1	46,08	0. 5. 48 1	3,072	82 55. 32	—20,04
4		8	1. 28. 38	46,03	0. 5. 54,5	3,069	89 18. 59	-20,04
5		7	1. 45. 20	46,03	0. 7. 1,3	3,069	89. 28. 46	—20,04
6	d Piscium	6	2. 27. 10	46,12	0. 9. 48,7	3,075	82. 58 34	—20,03
7		7	3. 16. 41	45,95	0. 13. 6,7	3,063	93. 22. 47	—20,01
8	Piscium	6,7	3. 39. 48	46,04	0. 14 39,2	3,069	89. 13. 27	—20,01
9	Ceti	7	3. 58. 3	45,99	0. 15. 52,2	3,066	91. 12 47	—20,00
10		8	4. 8. 48	46,06	0. 16. 34,9	3,071	88. 21. 54	—20,00
11		6,7	4. 50 10	46,18	0. 19. 20,7	3,079	84. 30. 18	—19,97
12	Piscium	6,7	5. 23. 42	46 21	0. 21. 34,8	3,081	84 12 21	—19,96
13		9	5. 41. 19	45,96	0. 22. 45,3	3,064	91. 45. 57	—19,95
14		6,7	6. 11. 40	45,96	0. 26. 46,7	3,064	91. 39. 32	—19.93
15		7,8	6. 50. 21	45,95	0. 27. 21,4	3,063	91. 39. 30	—19,91
16		6	7. 30. 27	45,77	0 30. 1,8	3,051	95 30 37	—19,88
17		8	8. 35. 33	45.97	0. 34. 22,2	3,065	90. 53. 38	—19,82
18		6	9. 8. 17	46,33	0. 36. 33,1	3,089	84. 24. 23	—19,79
19		6,7	9. 20. 25	46,26	0. 37. 21,6	3,084	85. 47. 26	—19.78
20		7,8	9. 20. 58	46,37	0. 37. 23,9	3,091	83. 50. 57	—19,78
21	∂ Piscium	4	9. 27. 6	46.39	0. 37. 48,4	3 093	83 33. 33	—19,77
22		7,8	10. 7. 42	46,16	0. 40. 30,7	3 077	87. 45. 16	—19,73
23		6	10. 34. 25	45,87	0. 42 17,7	3,058	92. 17. 10	—19,71
24		8	10. 37. 59	46,21	0. 42. 31,9	3 081	87. 3. 16	—19,70
25		8,9	11. 6. 49	46,40	0. 44. 27,2	3,093	84. 17. 10	—19,67
26		7	11. 17. 20	46,92	0. 45. 9,3	3.128	77. 11. 18	—19,66
27		7	11. 43. 40	46,93	0. 46. 54,7	3.129	77. 26. 31	—19,63
28		7,8	11. 51. 45	46,43	0. 47. 27,0	3,095	84. 17. 28	—19,62
29		7	12 14. 27	46,42	0. 48. 57,8	3,095	84. 39. 9	—19,59
30	ι Piscium	4	13. 1. 1	46 55	0. 52. 4,1	3,103	83. 14. 34	—19,53
31		8	13 13. 35	46,47	0. 52. 54,3	3 098	84. 21. 53	—19,51
32	Piscium	7,8	13 30. 13	46,39	0. 54. 0.9	3,093	85. 28. 19	—19,49
33		7,8	13. 44. 43	46,33	0. 54. 58,8	3,089	86. 12. 41	—19,47
34		8	14. 21. 8	46,79	0. 57. 24,5	3,119	81. 13. 0	—19,42
35	e Piscium	5	14 23. 43	46,42	0. 57. 34,9	3,095	85. 27. 45	—19,42

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + s.	North polar Distance. D. M. S.	Annual Preces. s.
36		8	14. 58. 40	46,79	0. 59. 54,7	3,119	81. 34. 22	—19,37
37		7,8	15. 7. 33	46,86	1. 0. 30,2	3,124	80. 49. 38	—19,36
38	1. ζ Piscium	6	15. 41. 42	46,63	1. 2. 46,8	3,109	83. 32. 17	—19,30
39	2. ζ Piscium	7	15. 42. 4	46,63	1. 2. 48,3	3,109	83. 32. 4	—19,30
40	f Piscium	6	16. 44. 49	46,27	1. 6. 59,3	3,085	87. 29. 40	—19,20
41		8,9	17. 8. 16	46,24	1. 8. 33,1	3,083	87. 48. 57	—19,16
42		6	17. 16. 23	45,85	1. 9. 5,5	3,057	91. 36. 54	—19,14
43		7	17. 57. 22	46,09	1. 11. 49,5	3,073	89. 22. 22	—19,07
44		8	17. 58. 6	46,41	1. 11. 52,4	3,094	86. 21. 50	—19,07
45		7,8	17. 59. 52	46,71	1. 11. 59,5	3,114	83. 41. 53	—19,07
46	ρ Piscium	5	18. 44. 24	48,12	1. 14. 57,6	3,208	71. 55. 29	—18,99
47		5	18. 50. 34	48,14	1. 15. 22,2	3,209	71. 51. 6	—18,98
48		7	19. 16. 53	47,92	1. 17. 7,5	3,195	74. 0. 43	—18,93
49	μ Piscium	5	19. 47. 53	46,62	1. 19. 11,5	3,108	84. 56. 31	—18,86
50	η Piscium	4,3	20. 4. 0	47,77	1. 20. 16,0	3,185	75. 44. 27	—18,83
51		7,8	20. 9. 30	47,21	1. 20. 38,0	3,147	80. 11. 51	—18,82
52		7,8	20. 34. 46	46,90	1. 22. 19,1	3,127	82. 52. 21	—18,77
53		8	20. 58. 21	46,93	1. 23. 53,4	3,129	82. 48. 17	—18,72
54	π Piscium	5,4	21. 29. 51	47,46	1. 25. 59,4	3,164	78. 56. 15	—18,66
55		8,9	21. 37. 50	47,46	1. 26. 31,3	3,164	78. 59. 50	—18,64
56		7.8	22. 23. 9	47,05	1. 29. 32,6	3,037	82. 18. 32	—18,54
57		9	22. 35. 47	47,10	1. 30. 23,1	3,140	81. 59. 40	—18,51
58	ν Piscium	5	22. 37. 52	46,62	1. 30. 31,5	3,108	85. 34. 46	—18,51
59	ο Piscium	5	23. 34. 53	47,16	1. 34. 19,5	3,144	81. 54. 15	—18,38
60		8,9	24. 0. 25	47,43	1. 36. 1,7	3,162	80. 12. 42	—18,32
61		7,8	24. 56. 13	47,51	1. 39. 44,9	3,167	80. 0. 5	—18,18
62	1. γ Arietis	4	25. 30. 27	48,87	1. 42. 1,5	3,258	71. 44. 19	—18,09
63	2. γ Arietis		25. 30. 29	48,87	1. 42. 1,9	3,258	71. 44. 28	—18,09
64	β Arietis	3	25. 45. 54	49,15	1. 43. 3,6	3,277	70. 13. 24	—18,06
65	ι Arietis	5	26. 28. 27	48,71	1. 45. 53,8	3,247	73. 12. 49	—17,95
66		8	27. 3. 9	47,84	1. 48. 12,6	3,189	78. 43. 47	—17,85
67	γ Andromedæ	2	27. 45. 55	54,23	1. 51. 3,7	3,615	48. 41. 7	—17,74
68	α Piscium	3	27. 48. 4	46,30	1. 51. 12,3	3,087	88. 15. 21	—17,73
69	κ Arietis	6	28. 42. 36	49,84	1. 54. 50,4	3,323	68. 21. 33	—17,58
70	α Arietis	2	28. 50. 21	50,02	1. 55. 21,4	3,335	67. 32. 11	—17,56

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Precess.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
71	Piscium	8	29. 3. 56	49,00	1. 56. 15,7	3,267	72. 58. 36	—17,52
72		7	29. 45. 3	49,35	1. 59. 0,2	3,290	71. 29. 47	—17,40
73		9	29. 54. 15	49,73	1. 59. 37,0	3,315	69. 37. 12	—17,38
74	κ Arietis	6	30. 15. 49	49,74	2. 1. 3,3	3,316	69. 47. 0	—17,32
75	1. ξ Ceti	4	30. 28. 29	47,42	2. 1. 53,9	3,161	82. 8. 39	—17,28
76	Arietis	7	31. 24. 17	48,68	2. 5. 37,1	3,245	75. 42. 46	—17,11
77	1. θ Arietis	6	31. 36. 58	49,62	2. 6. 27,9	3,308	71. 4. 41	—17,08
78		7	32. 17. 3	45,03	2. 9. 8,2	3,002	95. 18. 58	—16,95
79	ξ Arietis	5	33. 23. 47	47,90	2. 13. 35,1	3,193	80. 20. 53	—16,74
80	2. ξ Ceti	4	34. 15. 18	47,51	2. 17. 1,2	3,167	82. 29. 18	—16,57
81		7,8	34. 43. 13	49,93	2. 18. 52,9	3,329	71. 5. 5	—16,48
82	ν Ceti	4,5	36. 13. 11	46,99	2. 24. 52,7	3,133	85. 19. 49	—16,18
83	ε Arietis	6	36. 43. 46	50,63	2. 26. 55,1	3,375	68. 57. 18	—16,07
84	δ Ceti		37. 11. 10	45,90	2. 28. 44,6	3,060	90. 35. 3	—15,97
85	μ Arietis	6	37. 38. 10	50,26	2. 30. 32,7	3,351	70. 58. 28	—15,88
86	ο Arietis	6	38. 14. 56	49,21	2. 32. 59,7	3,281	75. 35. 8	—15,75
87	μ Ceti	4	38. 24. 3	48,04	2. 33. 36,2	3,203	80. 46. 50	—15,71
88	π Arietis	6	39. 23. 53	49,81	2. 37. 35,5	3,321	73. 25. 4	—15,49
89	σ Arietis	6	39. 58. 45	49,28	2. 39. 55,0	3,285	75. 47. 32	—15,37
90	2. ρ Arietis	6	41. 0. 28	50,16	2. 44. 1,9	3,344	72. 31. 32	—15,13
91	3. ρ Arietis	—	41. 8. 55	50,10	2. 44. 35,7	3,340	72. 49. 16	—15,10
92	ε Arietis	5	41. 48. 25	51,01	2. 47. 13,6	3,401	69. 30. 33	—14,95
93	λ Ceti	4	42. 7. 15	47,92	2. 48. 29,0	3,195	81. 56. 19	—14,87
94	α Ceti	2	42. 49. 54	46,79	2. 51. 19,6	3,119	86. 44. 29	—14,70
95	β Persei	3	43. 38. 15	57,68	2. 54. 33,0	3,845	49. 51. 55	—14,51
96		[illegible]	44. 19. 44	51,10	2. 57. 18,9	3,407	70. 2. 55	—14,34
97		7	44. 24. 43	50,73	2. 57. 38,8	3,382	71. 25. 55	—14,32
98	δ Arietis	4	44. 54. 32	50,87	2. 59. 30,1	3,391	71. 4. 42	—14,20
99	ζ Arietis	5	45. 42. 48	51,31	3. 2. 51,1	3,421	69. 44. 35	—14,00
100	1. τ Arietis	5	47. 16. 48	51,49	3. 9. 7,2	3,433	69. 37. 13	—13,60
101	α Persei	2	47. 21. 19	63,05	3. 9. 25,3	4,203	40. 54. 3	—13,59
102	2. τ Arietis	6	47. 40. 20	51,41	3. 10. 41,3	3,427	70. 1. 11	—13,50
103		7	48. 5. 15	51,46	3. 12. 21,0	3,431	69. 57. 15	—13,39
104	ο Tauri	4	48. 23. 6	48,20	3. 13. 32,4	3,213	81. 43. 12	—13,31
105	ξ Tauri	4	48. 57. 10	48,41	3. 15. 48,6	3,227	81. 0. 37	—13,16

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Precess. S.
106	[illegible]	7	49. 17. 21	48,89	3. 17. 9,4	3,259	79. 20. 45	—13,07
107	[illegible]	7,8	49. 28. 10	50,39	3. 17. 52,7	3,359	73. 58. 21	—13,03
108	s Tauri	6	49. 44. 18	48,88	3. 18. 57,2	3,259	79. 23. 43	—12,96
109	f Tauri	5	49. 49. 30	49,38	3. 19. 18,0	3,289	77. 47. 37	—12,93
110	t Tauri	6	50. 18. 50	48,37	3. 21. 15,3	3,225	81. 20. 47	—12,80
111		7	50. 33. 25	50,79	3. 22. 13,6	3,353	72. 51. 48	—12,74
112		7	51. 30. 24	50,15	3. 26. 1,6	3,343	75. 16. 14	—12,48
113			51. 52. 38	50,58	3. 27. 34,5	3,367	74. 9. 21	—12,38
114	g Pleiadum	[illegible]	53. 5. 15	53,03	3. 32. 21,0	3,535	66. 22. 59	—12,04
115	b Pleiadum	5	53. 6. 29	52,97	3. 32. 25,9	3,531	66. 33. 33	—12,04
116		7	53. 9. 44	53,15	3. 32. 38,9	3,543	66. 2. 9	—12,02
117	e Pleiadum	5	53. 11. 1	53,10	3. 32. 44,0	3,540	66. 12. 14	—12,02
118	c Pleiadum	6	53. 20. 21	53,08	3. 33. 21,4	3,539	66. 18. 2	—11,97
119	k Pleiadum	7	53. 21. 20	53,14	3. 33. 25,3	3,543	66. 6. 45	—11,96
120	l Pleiadum	7	53. 23. 29	53,14	3. 33. 33,9	3,543	66. 8. 23	—11,96
121	d Pleiadum	5	53. 28. 23	52,95	3. 33. 53,5	3,530	66. 43. 1	—11,94
122	p Pleiadum	7	53. 43. 22	53,03	3. 34. 53,5	3,535	66. 32. 41	—11,88
123	η Pleiadum	3	53. 45. 23	53,03	3. 35. 1,5	3,535	66. 33. 23	—11,86
124	f Pleiadum	5	54. 10. 30	53,05	3. 36. 42,0	3,537	66. 36. 5	—11,74
125	h Pleiadum	6	54. 10. 43	53,08	3. 36. 42,9	3,539	66. 31. 1	—11,74
126	e Tauri	5	54. 11. 36	49,03	3. 36. 46,4	3,269	79. 30. 50	—11,73
127		7,8	54. 13. 15	53,17	3. 36. 52,3	3,545	66. 16. 22	—11,73
128		7	55. 17. 35	50,96	3. 41. 10,3	3,397	73. 18. 38	—11,41
129			56. 7. 11	52,70	3. 44. 28,7	3,513	68. 8. 18	—11,17
130	λ Tauri	4	57. 16. 1	49,56	3. 49. 4,1	3,304	78. 6. 55	—10,88
131	a Tauri	5	58. 4. 30	52,72	3. 52. 18,0	3,515	68. 30. 9	—10,60
132		7	58. 13. 47	52,71	3. 52. 55,1	3,514	68. 34. 11	—10,55
133	ψ Tauri	5	58. 30. 48	55,27	3. 54. 2,9	3,685	61. 34. 50	—10,47
134	1. ω Tauri	6	59. 14. 4	51,97	3. 56. 56,2	3,465	70. 57. 32	—10,25
135	ρ Tauri	6	59. 31. 0	54,42	3. 58. 4,0	3,628	64. 4. 51	—10,18
136	2. ω Tauri	6	61. 14. 35	52,43	4. 4. 58,3	3,495	69. 57. 4	— 9,65
137	φ Tauri	5	61. 51. 57	54,96	4. 7. 27,8	3,664	63. 9. 54	— 9,45
138	γ Tauri	3	61. 57. 47	50,80	4. 7. 57,1	3,387	74. 53. 34	— 9,42
139	Tauri	6,7	62. 2. 4	50,28	4. 8. 8,3	3,354	76. 29. 3	— 9,40
140	b Tauri		62. 10. 42	50,63	4. 8. 42,8	3,375	75. 25. 13	— 9,36

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
141			62. 16. 13	50,23	4. 9. 4,9	3,349	76. 39. 5	— 9,33
142	χ Tauri	5	62. 27. 17	54,35	4. 9. 49,1	3,623	64. 52. 48	— 9,27
143			62. 33. 31	50,31	4. 10. 14,1	3,354	76. 25. 58	— 9,23
144	1. δ Tauri	4	62. 42. 33	51,48	4. 10. 50,2	3,432	72. 57. 48	— 9,19
145			62. 50. 29	51,16	4. 11. 21,9	3,411	73. 54. 43	— 9,15
146	2. δ Tauri	5	63. 0. 3	51,46	4. 12. 0,2	3,431	73. 3. 22	— 9,10
147	3. δ Tauri	6	63. 20. 31	51,65	4. 13. 22,1	3,443	72. 33. 58	— 8,99
148	1. υ Tauri	5	63. 26. 15	53,38	4. 13. 45,0	3,559	67. 40. 31	— 8,97
149			63. 35. 46	50,88	4. 14. 23,1	3,392	74. 52. 15	— 8,91
150	2. υ Tauri	6	63. 41. 16	53,47	4. 14. 45,1	3,565	67. 29. 25	— 8,88
151	π Tauri	5	63. 41. 17	50,58	4. 14. 45,1	3,372	75. 46. 28	— 8,88
152		7	63. 41. 50	51,20	4. 14. 47,3	3,413	73. 54. 55	— 8,88
153	ε Tauri	3,4	64. 5. 29	52,12	4. 16. 21,9	3,475	71. 17. 56	— 8,76
154			64. 7. 36	50,60	4. 16. 30,4	3,373	75. 44. 23	— 8,75
155	1. ϑ Tauri	5	64. 8. 51	51,02	4. 16. 35,4	3,401	74. 30. 59	— 8,75
156	2. ϑ Tauri	5	64. 10. 16	50,99	4. 16. 41,1	3,399	74. 36. 28	— 8,74
157		8	64. 29. 56	52,38	4. 17. 59,7	3,492	70. 37. 52	— 8,63
158			64. 32. 38	50,95	4. 18. 10,5	3,397	74. 45. 4	— 8,62
159			64. 38. 18	51,12	4. 18. 33,2	3,408	74. 16. 34	— 8,61
160			64. 40. 14	50,95	4. 18. 40,9	3,397	74. 46. 38	— 8,58
161		7	64. 41. 35	51,10	4. 18. 46,3	3,407	74. 19. 16	— 8,57
162		7,8	65. 24. 32	51,20	4. 21. 38,1	3,413	74. 7. 59	— 8,34
163	ϱ Tauri	5	65. 29. 12	50,70	4. 21. 56,8	3,380	75. 36. 36	— 8,31
164		8	65. 44. 3	51,29	4. 22. 56,2	3,419	73. 55. 18	— 8,24
165			65. 51. 19	52,48	4. 23. 25,3	3,499	70. 34. 0	— 8,20
166	α Tauri	1	65. 58. 15	51,30	4. 23. 53,0	3,420	73. 55. 32	— 8,16
167			66. 32. 13	51,16	4. 26. 8,9	3,411	74. 24. 4	— 8,00
168	1. σ Tauri	7	66. 47. 43	51,09	4. 27. 10,8	3,406	74. 37. 39	— 7,90
169	2. σ Tauri	6	66. 49. 9	51,13	4. 27. 16,6	3,409	74. 30. 43	— 7,89
170	τ Tauri	5	67. 24. 53	53,72	4. 29. 39,5	3,581	67. 27. 36	— 7,70
171		7	68. 30. 25	52,20	4. 34. 1,7	3,480	71. 39. 31	— 7,35
172		8	69. 6. 9	52,23	4. 36. 24,6	3,482	71. 39. 6	— 7,16
173	i Tauri	6	69. 46. 29	52,30	4. 39. 5,9	3,487	71. 31. 54	— 6,94
174	1. φ Orionis	4,5	70. 10. 1	50,68	4. 40. 40,1	3,379	76. 6. 45	— 6,81
175	[illegible]	9	70. 28. 50	51,68	4. 41. 55,3	3,445	73. 20. 2	— 6,71

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
176		8	70. 42. 57	51,46	4. 42. 51,8	3,431	73. 58. 10	— 6,63
177		8	70. 48. 50	51,55	4. 43. 15,3	3,437	73. 43. 58	— 6,60
178		6,7	71. 16. 17	54,32	4. 45. 5,1	3,621	66. 23. 39	— 6,44
179		6	71. 18. 49	51,76	4. 49. 15,3	3,451	73. 11. 25	— 6,43
180		6,7	71. 46. 19	50,84	4. 47. 5,3	3,389	75. 47. 29	— 6,28
181		7	71. 55. 27	51,34	4. 47. 41,8	3,423	74. 24. 37	— 6,22
182	ι Tauri	4	72. 38. 19	53,47	4. 50. 33,3	3,565	68. 43. 30	— 5,99
183			72. 57. 49	53,37	4. 51. 51,3	3,558	69. 1. 53	— 5,87
184	1. y Orionis		73. 8. 42	51,19	4. 52. 34,8	3,413	74. 54. 4	— 5,82
185			73. 17. 29	52,82	4. 53. 9,9	3,531	70. 29. 49	— 5,76
186	m Tauri	6	73. 45. 22	52,40	4. 55. 1,5	3,493	71. 39. 8	— 5,61
187	1. l Tauri	6	73. 50. 54	53,08	4. 55. 23,6	3,539	69. 52. 23	— 5,58
188	2. y Orionis	6	74. 25. 21	51,31	4. 57. 41,4	3,421	74. 41. 6	— 5,39
189	α Aurigæ	1	75. 17. 56	65,94	5. 1. 11,7	4,396	44. 13. 53	— 5,09
190		7	75. 42. 42	53,88	5. 2. 50,8	3,592	67. 58. 14	— 4,94
191	β Orionis	1	76. 6. 50	43,13	5. 4. 27,3	2,875	98. 27. 24	— 4,80
192	n Tauri	6	76. 40. 1	53,84	5. 6. 40,1	3,589	68. 8. 6	— 4,62
193	2. l Tauri	7	76. 42. 21	53,08	5. 6. 49,4	3,539	70. 6. 2	— 4,61
194		8	76. 58. 36	52,48	5. 7. 54,4	3,499	71. 42. 37	— 4,52
195			77. 8. 3	52,97	5. 8. 32,2	3,531	70. 24. 50	— 4,47
196			77. 13. 41	56,30	5. 8. 54,7	3,753	62. 16. 22	— 4,43
197		7,8	77. 31. 21	56,75	5. 10. 5,4	3,783	61. 16. 45	— 4,33
198	β Tauri	2	78. 15. 23	56,64	5. 13. 1,5	3,776	61. 35. 6	— 4,08
199	γ Orionis	2	78. 28. 15	48,13	5. 13. 53,0	3,209	83. 51. 16	— 4,01
200			78. 43. 50	52,32	5. 14. 55,3	3,488	72. 14. 4	— 3,92
201	ο Tauri	5	78. 45. 25	53,86	5. 15. 1,7	3,591	68. 15. 29	— 3,91
202	χ Aurigæ	5	79. 46. 2	58,36	5. 19. 4,1	3,891	57. 58. 50	— 3,56
203	β Leporis	5	79. 48. 47	38,47	5. 19. 15,1	2,565	110. 56. 13	— 3,55
204		7,8	80. 17. 32	53,33	5. 21. 10,1	3,555	69. 41. 23	— 3,38
205	∂ Orionis	2	80. 19. 22	45,86	5. 21. 17,5	3,057	90. 28. 6	— 3,37
206	Orionis	6	80. 30. 4	51,01	5. 22. 0,3	3,401	75. 50. 5	— 3,31
207	α Leporis	3	80. 52. 7	39,59	5. 23. 32,4	2,639	107. 59. 3	— 3,18
208	ζ Tauri	3	81. 16. 29	53,63	5. 25. 5,9	3,575	69. 0. 3	— 3,05
209	ι Orionis	2	81. 23. 32	45,55	5. 25. 34,1	3,037	91. 20. 56	— 3,00
210			81. 40. 58	55,59	5. 26. 43,9	3,706	64. 14. 11	— 2,91

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
211	σ Orionis	4	82. 3. 15	45,07	5. 28. 13,0	3,005	92. 44. 2	— 2,77
212	ζ Orionis	2	82. 32. 35	45,30	5. 30. 10,3	3,020	92. 3. 59	— 2,60
213			83. 12. 57	52,72	5. 32. 51,8	3,513	71. 24. 5	— 2,37
214		7, 8	83. 37. 7	53,54	5. 34. 28,4	3,556	69. 49. 0	— 2,22
215		7,8	83. 45. 35	54,14	5. 35. 2,3	3,609	65. 24. 49	— 2,17
216			83. 47. 53	52,35	5. 35. 11,5	3,490	72. 21. 53	— 2,16
217	γ Leporis	4,3	83. 55. 55	37,75	5. 35. 43 7	2,517	112. 31. 26	— 2,11
218			83. 57. 45	53,59	5. 35. 51,0	3,573	69. 13. 5	— 2,10
219		6,7	84. 2. 0	55,10	5. 36. 8,0	3,673	65. 31. 11	— 2,08
220		8	84. 5. 31	49,42	5. 36. 22,1	3,295	80. 2[illegible]. 3	— 2,06
221	κ Orionis	3	84. 27. 1	42,59	5. 37. 48,1	2,839	99. 45. 23	— 1,93
222		7,8	85. 12. 45	53,3[illegible]	5. 40. 51,0	3.559	69. 45. 57	— 1,68
223	1. χ Orionis	5	85. 29. 19	53,38	5. 41. 57,8	3,559	69. 47. 37	— 1,58
224	2. χ Orionis	7	85. 37. 38	53,17	5. 42. 30,6	3,54[illegible]	70. 18. 26	— 1,53
225	α Orionis	1	85. 57. 9	48,60	5. 43. 48,6	3,240	82. 38. 50	— 1,41
226			86. 14. 28	55,74	5. 44. 57,5	3,716	64. 5. 20	— 1,32
227		8	86. 56. 2	54,46	5. 47. 44,1	3,631	67. 7. 40	— 1,07
228		7	87. 15. 11	54.26	5. 49. 0,7	3,617	67. 37. 16	— 0,96
229		6,7	87. 45. 3	52,45	5. 51. 0,2	3,497	72. 11. 55	— 0,79
230	4. χ Orionis	5	87. 45. 17	53,18	5. 51. 1,1	3,545	70. 19. 15	— 0,79
231	h Geminorum	4	87. 50. 23	54,63	5. 51. 21,5	3,642	66. 44. 24	— 0,75
232	3. χ Orionis	7	87. 51. 42	53 30	5. 51. 26,8	3,557	69. 52. 19	— 0,75
233			88. 46. 4	54,32	5. 55. 4 3	3 621	67. 29. 42	— 0,43
234		7,8	89. 13. 16	54,20	5. 56. 53 0	3.613	67. 47. 33	— 0,27
235			89. 14. 45	54,58	5. 56. 59,0	3.639	66. 52. 5	— 0,27
236		7,8	89. 26. 37	54,54	5. 57. 46,5	3,636	66. 58. 37	— 0,20
237		6	89. 53. 44	53,24	5. 59. 34,9	3,549	70. 10. 45	— 0,04
238	Geminorum	7,8	89. 53. 49	54,50	5. 59. 35,3	3,633	67. 3. 34	— 0,04
239	η Geminorum	4	90. 33. 1	54,3[illegible]	6. 2. 12,1	3,623	67. 26. 52	+ 0,19
240		6	90. 37. 20	53,01	6. 2. 28,3	3,534	70. 47. 11	+ 0,22
241			90. 52. 16	[illegible]5	[illegible] 3. 29,1	3,663	65. 58. 39	+ 0,30
242	Geminorum		91. 2. 25	[illegible]	[illegible] 4. 9,7	3,656	66. 14. 21	+ 0,36
243			91. 31. [illegible]	[illegible]	[illegible] 6,6	3,653	66. 19. 51	+ 0,53
244		4	[illegible]	[illegible]	[illegible]	3,649	66. 27. 50	+ 0,57
245			[illegible]	[illegible]	[illegible]	[illegible]	66. 39. 32	+ 0,58

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
246	–	8	92. 10. 11	53,79	6. 8. 40,7	3,586	68. 47. 22	+ 0,76
247		9	92. 12. 5	53,82	6. 8. 48,3	3,588	68. 43. 13	+ 0,77
248			92. 14. 52	54,87	6. 8. 59,5	3,658	66. 9. 24	+ 0,78
249	μ Geminorum	3	92. 33. 39	54,36	6. 10. 14,6	3,624	67. 23. 34	+ 0,90
250	Geminorum		93. 16. 37	54,00	6. 13. 6,5	3,600	68. 15. 16	+ 1,14
251	Geminorum		93. 48. 52	53,66	6. 15. 15,6	3,577	69. 5. 44	+ 1,33
252			93. 51. 46	53,54	6. 15. 27,1	3,569	69. 23. 36	+ 1,35
253	ν Geminorum	4	94. 7. 20	53,43	6. 16. 29,3	3,562	69. 40. 11	+ 1,[illegible]
254	Geminorum	8	95. 0. 25	52,48	6. 20. 1,7	3,499	72. 5. 16	+ 1,75
255		7,6	95. 0. 41	52,48	6. 20. 2,7	3,499	72. 5. 0	+ 1,75
256		8	95. 41. 48	51,87	6. 22. 47,2	3,458	78. 38. 47	+ 1,99
257	γ Geminorum	2,3	96. 23. 34	51,95	6. 25. 34,3	3,463	78. 26. 8	+ 2,23
258			96. 29. 27	51,95	6. 25. 57,8	3,463	78. 25. 8	+ 2,26
259	Geminorum	8	96. 54. 20	53,20	6. 27. 37,3	3,547	70. 9. 48	+ 2,41
260		6,7	97. 32. 29	52,41	6. 30. 9,9	3,494	72. 9. 48	+ 2,63
261	ι Geminorum	3	97. 45. 2	55,42	6. 31. 0,1	3,695	64. 40. 37	+ 2,70
262	1. ξ Geminorum	5,4	98. 2. 10	50,76	6. 32. 8,7	3,384	76. 34. 26	+ 2,80
263	2. ξ Geminorum		98. 22. 35	50,64	6. 33. 30,3	3,376	76. 53. 25	+ 2,92
264	α Canis	1	98. 58. 40	40,18	6. 35. 54,7	3,679	106. 25. 39	+ 3,13
265			99. 28. 48	50,82	6. 37. 55,2	3,388	76. 21. 30	+ 3,30
266	d Geminorum	6	99. 44. 17	54,00	6. 38. 57,1	3,600	68. 0. 23	+ 3,39
267	e Geminorum	6	100. 41. 51	50,72	6. 42. 47,4	3,381	76. 34. 10	+ 3,72
268		7,8	101. 0. 49	52,41	6. 44. 3,2	3,494	72. 0. 17	+ 3,83
269			101. 22. 10	52,47	6. 45. 28,7	3,498	71. 50. 12	+ 3,96
270	Geminorum	7,8	101. 27. 33	55,75	6. 45. 50,2	3,717	63. 39. 29	+ 3,98
271		7	101. 37. 15	55,67	6. 46. 29,0	3,711	63. 48. 59	+ 4,04
272		9	101. 58. 53	54,63	6. 47. 55,5	3,642	66. 16. 57	+ 4,16
273		8,9	102. 2. 45	51,77	6. 48. 11,0	3,451	79. 38. 42	+ 4,18
274	1. α Geminorum	6	102. 24. 2	54,94	6. 49. 36,1	3,663	65. 30. 1	+ 4,31
275	ζ Geminorum	3	102. 54. 36	53,47	6. 51. 38,4	3,565	69. 8. 11	+ 4,48
276	2. α Geminorum		103. 9. 46	54,28	6. 52. 39,1	3,619	67. 3. 46	+ 4,56
277		8	103. 31. 10	52,37	6. 54. 4,6	3,491	71. 57. 5	+ 4,68
278	o Geminorum	7	104. 4. 42	51,69	6. 56. 18,8	3,446	73. 44. 47	+ 4,86
279	τ Geminorum	5	104. 26. 20	57,49	6. 57. 45,3	3,833	59. 25. 33	+ 4,99
280	m Geminorum	6	104. 54. 54	54,88	6. 59. 39,6	3,655	65. 32. 7	+ 5,16

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. [illegible] Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Preces.	Right Ascension in Time.	Annual Preces.	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	[illegible]	D. M. S.	S.
281		5	105. 19. 32	51,75	7. 1. 18,1	3,450	73. 29. 55	+5,30
282	n Geminorum	[illegible]	105. 27. 33	55,12	7. 1. 50,2	3,675	64. 45. 54	+5,35
283		7,8	106. 0. 52	55,86	7. 4. 3,5	3,724	62. 56. 47	+5,58
284		7	106. 10. 16	51,72	7. 4. 41,1	3,448	73. 29. 57	+5,59
285	λ Geminorum	5	106. 30. 14	51,86	7. 6. 0,9	3,457	73. 5. 41	+5,70
286	∂ Geminorum	3	106. 53. 28	53,91	7. 7. 33,9	3,594	67. 38. 45	+5,82
287	q Geminorum	6,7	107. 23. 5	53,17	7. 9. 32,3	3,545	69. 30. 29	+5,99
288	a Geminorum	5,6	107. 39. 54	55,11	7. 10. 39,6	3,674	64. 33. 38	+6,07
289		7,8	107. 50. 40	51,47	7. 11. 22,7	3,431	71. 20. 10	+6,14
290		7	107. 52. 2	56,18	7. 11. 28,1	3,745	61. 58. 22	+6,15
291	ι Geminorum	4,5	108. 10. 2	56,24	7. 12. 40,1	3,749	61. 47. 57	+6,25
292		7,8	108. 36. 15	53,69	7. 14. 25,0	3,579	68. 1. 32	+6,40
293	p Geminorum	[illegible]	108. 48. 53	53,64	7. 15. 15,5	3,576	68. 8 20	+6,47
294	s Geminorum	5	108. 53. 33	57,96	7. 15. 34,2	3,864	57. 48. 58	+6,50
295	1.b Geminorum	[illegible]	109. 3. 35	56,32	7. 16. 14,3	3,755	61. 27. 50	+6,55
296	2.b Geminorum		109. 10. 58	56,23	7. 16. 43,9	3,749	61. 39. 56	+6,58
297	α Geminorum	1,2	110. 17. 35	57,93	7. 21. 10,3	3,862	57. 40. 0	+6,96
298	k Geminorum	6	110. 24. 8	51,50	7. 21. 36,3	3,433	73. 44. 9	+6,99
299	υ Geminorum	5	110. 44. 22	55,72	7. 22. 57,5	3,715	62. 39. 6	+7,10
300			111. 18. 53	52,59	7. 25. 15,5	3,506	70. 37. 18	+7,29
301		7	111. 22. 38	54,65	7. 25. 30,5	3,643	65. 11. 6	+7,31
302		7	111. 37. 21	54,59	7. 26. 29,4	3,639	65. 18. 58	+7,39
303	f Geminorum	6	111. 49 59	52,12	7. 27. 19,9	3,475	71. 51. 37	+7,45
304	α Canis min.	1	112. 4. 57	47,89	7. 28. 19,8	3,193	84. 14. 22	+7,53
305		7,8	112. 14. 36	47,89	7. 28. 58,4	3,193	84. 15. 16	+7,58
306	σ Geminorum	5	112. 32. 24	56,45	7. 30. 9,5	3,763	60. 37. 15	+7,66
307		7	112. 42. 36	53,83	7. 30. 50,4	3,589	67. 7. 15	+7,73
308	t Geminorum	6	112. 49. 16	55,14	7. 31. 7,1	3,676	63. 43. 47	+7,75
309	κ Geminorum	4,5	112. 56. 14	54,59	7. 31. 44,9	3,639	65. 6. 49	+7,81
[illegible]10	β Geminorum	2	113. 6. 59	56,04	7. 32. 27,9	3,736	61. 28. 48	+7,86
[illegible]	g Ge[illegible]	6	113. 29. 15	52,35	7. 33. 57,0	3,490	70. 59. 25	+7,98
[illegible]	[illegible]	[illegible]	114. 55. 49	52,58	7. 39. 43,3	3,505	70. 9. 4	+8,45
[illegible]	[illegible]	6	115. 9. 20	55,38	7. 40. 37,3	3,692	62. 42. 19	+8,52
[illegible]	[illegible]	[illegible]	115. 50. 50	52,74	7. 43. 23,3	3,516	69. 34. 29	+8,74
[illegible]	[illegible]	[illegible]	117. 2. 58	54,71	7. 48. 11,9	3,647	64. 2. 49	+9,11

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Preces. S.
316	Cancri	5	117. 38. 54	55,61	7. 50. 35,6	3,707	61. 37. 52	+ 9,30
317	Cancri		118. 20. 28	50,32	7. 53. 21,9	3,355	76. 17. 44	+ 9,51
318		7	118. 27. 7	50,44	7. 53. 48,5	3,363	75. 54. 39	+ 9,55
319	2. μ Cancri	5	118. 50. 42	53,17	7. 55. 22,8	3,545	67. 49. 17	+ 9,67
320	1. ψ Cancri	7	119. 22. 18	54,71	7. 57. 29,2	3,647	63. 33. 3	+ 9,83
321	2. ψ Cancri	4	119. 26. 37	54,58	7. 57. 46,5	3,639	63. 51. 56	+ 9,85
322			119. 29. 0	49,61	7. 57. 56,0	3,307	78. 22. 43	+ 9,86
323	ψ Cancri	6	120. 1. 30	56,15	8. 0. 5,9	3,743	59. 43. 39	+10,03
324	1. ζ Cancri	5	120. 2. 11	51,75	8. 0. 8,7	3,450	71. 43. 53	+10,03
325	2. ζ Cancri	5	120. 2. 13	51,75	8. 0. 8,9	3,450	71. 43. 50	+10,03
326		7	120. 12. 10	49,21	8. 0. 48,7	3,281	79. 34. 5	+10,08
327			120. 32. 11	51,71	8. 2. 8,7	3,447	71. 45. 44	+10,18
328	β Cancri	4,3	121. 16. 46	48,99	8. 5. 7,1	3,266	80. 10. 43	+10,40
329			121. 32. 10	48,88	8. 6. 8,7	3,259	80. 29. 44	+10,48
330	χ Cancri	6	121. 49. 8	55,03	8. 7. 16,5	3,669	62. 6. 42	+10,57
331	λ Cancri	6	122. 0. 20	53,83	8. 8. 1,3	3,589	65. 19. 43	+10,62
332	1. d Cancri	7	122. 49. 46	51,82	8. 11. 19,1	3,455	71. 0. 19	+10,87
333		6,7	123. 24. 52	55,12	8. 13. 39,5	3,675	61. 28. 39	+11,04
334	2. d Cancri	7	123. 28. 54	51,37	8. 13. 55,6	3,425	72. 16. 23	+11,06
335	2. φ Cancri		123. 30. 51	54,76	8. 14. 3,4	3,651	62. 23. 30	+11,07
336		7	123. 32. 5	53,76	8. 14. 8,3	3,584	65. 8. 17	+11,07
337	2. υ Cancri	6,7	124. 2. 1	53,70	8. 16. 8,1	3,580	65. 10. 13	+11,22
338		7	124. 17. 38	53,75	8. 17. 10,5	3,583	64. 58. 1	+11,30
339		8	124. 17. 47	54,41	8. 17. 11,1	3,627	63. 7. 12	+11,30
340	3. υ Cancri	6	124. 45. 55	53,62	8. 19. 3,7	3,575	65. 13. 22	+11,43
341	ϑ Cancri	6,5	124. 53. 57	51,61	8. 19. 35,8	3,441	71. 12. 24	+11,47
342			124. 54. 14	51,90	8. 19. 36,9	3,460	70. 18. 58	+11,47
343	η Cancri	6,7	125. 8. 8	52,36	8. 20. 32,5	3,491	68. 51. 25	+11,54
344	4. υ Cancri		125. 8. 30	53,59	8. 20. 34,0	3,573	65. 12. 41	+11,54
345			125. 31. 30	50,08	8. 22. 6,0	3,339	76. 2. 9	+11,66
346	Cancri	8	125. 48. 24	52,03	8. 23. 13,6	3,402	69. 41. 58	+11,73
347		8	125. 54. 37	53,09	8. 23. 38,5	3,539	69. 30. 59	+11,76
348		8	126. 4. 55	50,67	8. 24. 19,7	3,378	73. 58. 14	+11,81
349	1. c Cancri	6	126. 25. 24	48,97	8. 26. 41,6	3,265	79. 37. 35	+11,91
350	--		126. 25. 46	51,89	8. 25. 43,1	3,459	70. 0. 46	+11,91

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
351	2. c Cancri		126. 40. 32	48,94	8. 26. 42,1	3,263	79. 42. 8	+ 11,98
352	ο Cancri	7	126. 54. 23	52,00	8. 27. 37,5	3,467	69. 29. 39	+ 12,03
353		7	126. 56. 37	51 93	8. 27. 46,5	3,462	69. 43. 50	+ 12,05
354			126. 59. 37	52,21	8. 27. 58,4	3,481	68. 51. 35	+ 12,07
355		7	127. 0. 2	52,08	8. 28. 0,1	3,472	69. 15. 42	+ 12,07
356		7	127. 1. 21	52,07	8. 28. 5,4	3,471	69. 17. 56	+ 12,08
357		7	127. 4. 22	51,97	8. 28. 17,4	3,465	69. 35. 56	+ 12,09
358		7	127. 5. 46	51,93	8. 28. 23,1	3,462	69. 43. 27	+ 12,10
359	e Cancri	7	127. 9. 37	51,98	8. 28. 38,5	3,465	69. 32. 53	+ 12,11
360		7	127. 13. 8	51,93	8. 28. 52,5	3,462	69. 41. 12	+ 12,13
361		7	127. 26. 15	52,01	8. 29. 45,0	3,467	69. 23. 19	+ 12,19
362	γ Cancri	[illegible]	127. 46. 42	52,49	8. 31. 6,8	3,499	67. 47. 15	+ 12,28
363	1. a Cancri	6	127. 54. 17	49,80	8. 31. 37,1	3,320	76. 34. 38	+ 12,32
364	δ Cancri	4	128. 10. 55	51,42	8. 32. 43,7	3,428	71. 4. 57	+ 12,40
365	b Cancri	6	128. 20. 7	49,03	8. 33. 20,5	3,269	79. 10. 7	+ 12,44
366	ι Cancri	5	128. 29. 22	54,90	8. 33. 57,5	3,660	60. 29. 0	+ 12,48
367	2. a Cancri	6	128. 51. 7	49,59	8. 35. 24,5	3,306	77. 7. 51	+ 12,58
368		8	129. 16. 28	49,69	8. 37. 5,9	3,313	76. 41. 27	+ 12,70
369		8	129. 41. 12	51,28	8. 38. 44,8	3,419	71. 10. 1	+ 12,81
370		7,8	129. 41. 34	51,51	8. 38. 46,3	3,434	70. 23. 46	+ 12,81
371		8	129. 49. 41	50,47	8. 39. 18,7	3,365	73. 52. 54	+ 12,84
372		7,8	130. 20. 9	51,03	8. 41. 20,6	3,402	71. 50. 53	+ 12,98
373			130. 28. 11	51,80	8. 41. 52,7	3,453	69. 14. 57	+ 13,02
374		9	130. 29. 30	51,07	8. 41. 58,0	3,405	71. 40. 16	+ 13,02
375			130. 30. 59	50,16	8. 42. 3,9	3,344	74. 48. 23	+ 13,02
376		9	130. 41. 41	50,11	8. 42. 46,7	3,341	74. 56. 32	+ 13,07
377		9	130. 47. 52	48,43	8. 43. 11,5	3,229	80. 57. 45	+ 13,10
378		7	130. 52. 53	50,95	8. 43. 31,5	3,397	71. 58. 48	+ 13,12
379		8,9	130. 59. 26	50,07	8. 43. 57,7	3,338	75. 1. 27	+ 13,15
380	1. α Cancri	6,7	131. 6. 37	49,35	8. 44. 26,5	3,290	77. 34. 55	+ 13,18
381			131. 19. 36	50,90	8. 45. 18,4	3,393	72. 3. 29	+ 13,24
382	1. ο Cancri	6	131. 22. 38	50,37	8. 45. 30,5	3,358	73. 52. 56	+ 13,25
383			131. 24 54	50,98	8. 45. 39,0	3,399	71. 44. 45	+ 13,26
384	2. ο Cancri	6	131 27. 32	50,44	8. 45. 50,1	3,363	73 37. 24	+ 13,27
385	2. α Cancri	4,3	131. 44. 45	49,36	8. 46. 59,0	3,292	77. 20. 20	+ 13,34

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	[illegible]
386		7	131. 49. 25	51,14	8. 47. 17,7	3,409	71. 3. 27	+13,37
387			131. 58. 18	49,70	8. 47. 53,2	3,313	76. 7. 6	+13,40
388	ι Cancri	6	132. 36. 25	52,98	8. 50. 25,7	3,532	64. 43. 55	+13,57
389		7,8	133. 17. 52	49,03	8. 53. 11,4	3,269	78. 19. 31	+13,75
390	κ Cancri	4,5	134. 5. 22	48,95	8. 56. 21,5	3,263	78. 29. 49	+13,95
391	ξ Cancri	5,6	134. 18. 53	52,08	8. 57. 15,5	3,472	67. 6. 54	+14,01
392			134. 33. 50	52,04	8. 58. 15,3	3,469	67. 9. 42	+14,07
393			184. 57. 52	49,14	8. 59. 51,5	3,276	77. 35. 20	+14,17
394		7	135. 54. 14	49,97	9. 3. 36,9	3,331	74. 11. 49	+14,39
395		7	136. 44. 5	53,09	9. 6 56,3	3,539	62. 45. 23	+14,60
396		7	136. 48. 49	49,05	9. 5. 55,3	3,270	77. 37. 37	+14,53
397		7,8	137. 3. 1	48,59	9. 8. 12,1	3,239	79. 20. 1	+14,67
398		8,9	137. 11. 47	50,97	9. 8. 47,2	3,390	70. 1. 44	+14,70
399		7	137. 58. 13	52,80	9. 11. 52,9	3,528	63. 11. 16	+14,89
400		7,8	138. 4. 27	48,05	9. 12. 17,8	3,203	81. 23. 34	+14,92
401	κ Leonis	4	138. 5. 50	52,86	9. 12. 23,3	3,524	62. 55. 23	+14,92
402	ν Leonis	5	139. 17. 56	48,32	9. 17. 11,7	3,221	80. 2. 14	+15,20
403	α Hydræ	2	139. 19. 4	44,24	9. 17. 16,2	2,989	97. 45. 21	+15,20
404	λ Leonis	[illegible]	139. 55. 35	51,74	9. 19. 42,8	3,249	66. 6. 53	+15,34
405	ξ Leonis	4	140. 9. 11	48,80	9. 20. 36,7	3,253	77. 46. 41	+15,39
406	h Leonis	6	140. 10. 19	48,43	9. 20. 41,3	3,229	79. 21. 59	+15,39
407		7,8	140. 53. 35	49,08	9. 23. 34,8	3,272	76. 25. 1	+15,56
408	Leonis	6,7	141. 21. 28	49,94	9. 25. 25,8	3,329	72. 37. 50	+15,66
409	Leonis		141. 31. 45	47,72	9. 26. 7,0	3,181	82. 13. 49	+15,69
410			141. 57. 58	49,16	9. 27. 51,9	3,277	75. 44. 50	+15,79
411	ο Leonis	5,4	142. 29. 0	48,36	9. 29. 56,0	3,224	79. 9. 35	+15,90
412	ψ Leonis	6	143. 4. 5	49,24	9. 32. 16,3	3,283	75. 1. 30	+16,03
413	ι Leonis	3	143. 28. 26	51,51	9. 33. 53,7	3,434	65. 16. 1	+16,11
414		6,7	143 45. 48	48,71	9. 35. 3,2	3,247	77. 13. 49	+16,17
415		8	143. 58. 35	50,68	9. 35. 54,3	3,379	68. 26. 6	+16,21
416		7,8	144. 1. 50	48,64	9. 36. 7,3	3,243	77. 28. 3	+16,23
417		6	144. 4. 24	48,61	9. 36. 17,6	3,241	77. 36. 6	+16,24
418		8	144. 52. 34	48,64	9. 39. 30,3	3,243	77. 10. 57	+16,40
419		7	144. 54. 44	48,90	9. 39. 38,9	3,260	75. 57. 32	+16,40
420		7,8	145. 44. 47	47,88	9. 42. 59,1	3,192	80. 56. 29	+16,57

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Preces. S.
421	Sextantis	6,7	146. 19. 22	47,97	9. 45. 17,5	3,198	80. 4. 44	+16,68
422	ι Leonis	4,5	146. 43. 36	48,65	9. 46. 54,4	3,243	76. 33. 39	+16,76
423		7	146. 44. 48	47,82	9. 46. 59,2	3,188	80. 41. 23	+16,77
424	π Leonis	4	147. 16. 33	47,75	9. 49. 6,2	3,183	80. 57. 18	+16,87
425		8	148. 5. 32	48,06	9. 52. 22,1	3,204	79. 4. 32	+17,02
426		8	148. 13. 4	48,39	9. 52. 52,3	3,226	77. 21. 43	+17,04
427		8,9	148. 29. 31	47,68	9. 53. 58,1	3,179	80. 59. 45	+17,09
428		7,8	148. 33. 55	49,17	9. 54. 15,7	3,278	73. 13. 49	+17,10
429	η Leonis	3,4	148. 57. 52	49,33	9. 55. 51,4	3,289	72. 13. 12	+17,18
430	a Leonis	5	149. 11. 14	48,02	9. 56. 44,9	3,201	78. 58. 45	+17,22
431	α Leonis	1	149. 17. 42	48,38	9. 57. 10,8	3,225	77. 0. 45	+17,24
432		8	149. 35. 32	47,92	9. 58. 22,1	3,195	79. 23. 1	+17,29
433		7,8	150. 4. 43	48,59	10. 0. 18,8	3,239	75. 36. 48	+17,37
434		8	150. 43. 43	50,02	10. 2. 54,9	3,331	67. 47. 30	+17,48
435		8	150. 46. 45	49,05	10. 3. 7,0	3,270	72. 49. 31	+17,49
436			151. 11. 6	49,31	10. 4. 47,7	3,287	71. 13. 14	+17,56
437		6,7	151. 20. 48	48,56	10. 5. 23,2	3,237	75. 13. 47	+17,58
438		7,8	151. 47. 43	48,32	10. 7. 10,8	3,221	76. 19. 54	+17,66
439		6	152. 4. 15	49,54	10. 8. 17,0	3,303	69. 28. 2	+17,71
440	γ Leonis	2	152. 5. 16	49,60	10. 8. 21,1	3,307	69. 6. 1	+17,71
441		7	152. 37. 47	48,67	10. 10. 31,1	3,245	73. 58. 12	+17,80
442		7,8	152. 47. 18	47,64	10. 11. 9,2	3,176	79. 58. 3	+17,83
443		8	153. 17. 49	47,88	10. 13. 11,3	3,192	78. 21. 13	+17,90
444		7	153. 32. 41	47,57	10. 14. 10,7	3,171	80. 9. 9	+17,95
445		7	154. 8. 9	47,69	10. 16. 32,6	3,179	79. 10. 18	+18,04
446		6	154. 23. 23	48,41	10. 17. 33,5	3,227	74. 35. 21	+18,08
447		7,8	154. 31. 12	47,73	10. 18. 4,8	3,182	78. 46. 26	+18,10
448	i Leonis	6	155. 14. 31	48,30	10. 20. 58,1	3,220	74. 47. 23	+18,20
449	ρ Leonis	4	155. 26. 10	47,55	10. 21. 44,7	3,170	79. 37. 3	+18,23
450	Leonis	7	155. 57. 36	47,17	10. 23. 50,4	3,145	81. 58. 11	+18,31
451		7	156. 0. 8	47,42	10. 24. 0,5	3,161	80. 16. 10	+18,32
452		9	156. 37. 10	48,66	10. 26. 28,7	3,244	71. 37. 0	+18,41
453		7	156. 54. 18	48,46	10. 27. 37,2	3,231	72. 47. 1	+18,45
454		9	157. 10. 19	47,38	10. 28. 41,3	2,159	80. 4. 5	+18,48
455		6	158. 6. 43	46,79	10. 32. 26,9	3,119	84. 9. 16	+18,61

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			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
456			158. 38. 48	47,13	10. 34. 35,2	3,142	81. 23. 2	+18,67
457	Sextantis	7,8	158. 47. 53	46,97	10. 35. 11,5	3,131	82. 31. 22	+18,69
458	k Leonis	6	158. 49. 18	48,00	10. 35. 17,2	3,200	74. 42. 3	+18,70
459		8	159. 5. 55	46,96	10. 36. 23,7	3,131	82. 32. 56	+18,73
460	l Leonis	6	159. 33. 5	47,47	10. 38. 12,3	3,165	78. 20. 48	+18,79
461		8,9	160. 57. 55	46,82	10. 43. 51,7	3,121	83. 2. 17	+18,96
462	d Leonis	5	162. 25. 45	46,52	10. 49. 43,1	3,101	85. 15. 28	+19,11
463	c Leonis	5	162. 27. 56	46,79	10. 49. 51,7	3,119	82. 46. 21	+19,12
464			163. 17. 18	46,50	10. 53. 9,2	3,100	85. 13. 56	+19,20
465	χ Leonis	4	163. 32. 54	46,87	10. 54. 11,6	3,125	81. 31. 55	+19,22
466	b Leonis	7,8	165. 39 35	47,95	11. 2. 38,3	3,197	68. 43. 21	+19,43
467	δ Leonis	2,3	165. 43. 35	47,98	11. 2 54,3	3,199	68. 19. 38	+19,43
468	θ Leonis	3	165. 48. 4	47,48	11. 3. 12,3	3,165	73. 25. 26	+19,43
469	n Leonis	6	166. 12. 55	47,25	11. 4. 51,7	3,150	75. 32. 57	+19,46
470	φ Leonis	4	166. 30. 5	45,82	11. 6. 0,3	3,055	92. 30. 17	+19,49
471	σ Leonis	4,5	167. 34. 45	46,56	11. 10. 19,0	3,104	82. 49. 16	+19,57
472	ι Leonis	4	168. 14. 29	46,87	11. 12. 57,9	3,125	78. 18. 55	+19,62
473			168. 19. 8	46,20	11. 13. 16,5	3,080	87. 26. 26	+19,63
474		7	169. 2. 28	46,30	11. 16. 9,8	3,087	85. 50. 41	+19,68
475	τ Leonis	4	169. 17. 12	46,28	11. 17. 8,8	3,085	85. 59. 17	+19,70
476	e Leonis	4,5	169. 53. 58	45,91	11. 19. 35,9	3,061	91. 50. 44	+19,74
477		7,8	170. 19. 15	45,71	11. 21. 17,0	3,047	95. 18. 21	+19,76
478		7	170. 31. 59	45,63	11. 22. 7,9	3,042	96 40. 9	+19,77
479	υ Leonis	4	171. 33. 7	46,04	11. 26. 12,5	3,069	89. 39. 57	+19,83
480	ω Virginis	6	171. 54. 28	46,48	11. 27. 37,9	3,099	80. 42. 14	+19,84
481		8	172. 51. 2	46,28	11. 31. 24,1	3,085	84. 5. 26	+19,89
482	1. ξ Virginis	5	173. 36. 50	46,39	11 34. 27,3	3,093	80. 34. 31	+19,93
483	ν Virginis	5	173. 46. 5	46,31	11. 35. 4,3	3,087	82. 17. 34	+19,93
484	2. ξ Virginis		174. 16. 57	46,35	11. 37. 7,8	3,090	80. 35. 19	+19,95
485	β Leonis	1,2	174. 35. 22	46,55	11. 38. 21,4	3,103	74. 15. 12	+19,96
486	β Virginis	3	174. 56. 7	46,11	11. 39. 44,5	3,074	87. 3. 3	+19,97
487	a Virginis	6	176. 4. 3	46,25	11. 44. 16,2	3,083	80. 23. 20	+20,00
488		8	176. 9. 57	46,07	11. 44. 39,8	3,071	87. 43. 59	+20,00
489		7	176. 52. 10	46,11	11. 47. 28,7	3,074	85. 21. 10	+20,02
490			177. 4. 57	46,05	11. 48. 19,8	3,070	88. 18. 8	+20,02

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			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
491	b Virginis	5,6	177. 18. 2	46,10	11. 49. 12,1	3,073	85. 10. 28	+20,02
492	π Virginis	5	177. 31. 41	46,14	11. 50. 6,7	3,076	82. 12. 50	+20,03
493		7,8	177. 50. 14	46,01	11. 51. 20,9	3,067	90. 35. 50	+20,03
494		7	178. 15. 28	46,09	11. 53. 1,9	3,073	83. 15 58	+20,03
495	ρ Virginis	5	178. 37. 51	46,10	11. 54. 31,4	3,073	80. 6. 1	+20,04
496		7,8	178. 56. 26	46,01	11. 55. 45,7	3,067	91. 57. 34	+20,04
497			179. 19. 1	46,03	11. 57. 16,1	3,069	88. 12. 21	+20,04
498	r Virginis	6	179. 44. 11	46,02	11. 58. 56,7	3,068	86. 44. 0	+20,05
499	s Virginis	6	179. 50. 22	46,03	11. 59. 21,5	3,069	83. 1. 31	+20,05
500		7,8	180. 52. 51	46,04	12. 3. 31,5	3,069	94. 33. 19	+20,04
501	n Virginis	8	181. 58. 49	46,02	12. 7. 55,3	3,068	89. 37. 4	+20,03
502	ν Virginis	4,3	182. 17. 42	46,01	12. 9 10,8	3,067	89. 29. 53	+20,03
503	o Virginis	5	182. 25. 35	45,95	12. 9. 42,3	3,063	85. 30. 55	+20,03
504		6,7	182. 58. 2	45,90	12. 11. 52,1	3,060	83. 31. 27	+20,02
505		7	183. 7. 3	46,09	12. 12. 28,2	3,073	93. 44. 48	+20,02
506		8,9	183. 23. 14	46,21	12. 13. 32,9	3,081	99. 18. 52	+20,01
507		7,8	184. 16. 36	46,11	12. 17. 6,4	3,074	93. 27. 0	+19,99
508			184. 24. 9	45,87	12. 17. 36,6	3,058	84. 26. 21	+19,99
509		7	184. 49. 17	46,39	12. 19. 17,1	3,093	102. 13. 32	+19,98
510			185. 1. 15	46,11	12. 20. 5,0	3,074	92. 53. 59	+19,97
511		7	185. 13. 1	46,14	12. 20. 52,1	3,076	93. 53. 30	+19,96
512	q Virginis	6	185. 44. 28	46,31	12. 22. 57,9	3,087	98. 17. 26	+19,95
513			185. 54. 40	46,03	12. 23. 38,7	3,069	90. 14. 51	+19,94
514	f Virginis	6	186. 29. 54	46,20	12. 25. 59,6	3,080	94. 40. 18	+19,92
515	χ Virginis	5	187. 6. 28	46,30	12. 28. 25,9	3,087	96. 50. 11	+19,90
516			187. 10. 18	46,24	12. 48. 41,1	3,083	94. 56. 30	+19,89
517	1. γ Virginis	3	187. 45. 50	46,03	12. 31. 3,3	3,069	90. 17. 41	+19,86
518	2. γ Virginis		187. 45. 54	46,03	12. 31. 3,5	3,069	90. 17. 46	+19,86
519			189. 10. 47	46,31	12. 36. 43,1	3,087	95. 9. 0	+19,79
520			189. 49. 20	46,41	12. 39. 17,3	3,094	96. 29. 12	+19,75
521		7	190. 7. 8	46,59	12. 40. 28,5	3,106	99. 11. 25	+19,73
522	Virginis	7	190. 36. 58	46,18	12. 42. 27,9	3,079	92. 24. 27	+19,70
523	ψ Virginis	5	190. 51. 50	46,58	12. 43. 27,3	3,105	98. 23. 38	+19,69
524	δ Virginis	3	191. 15. 51	45,71	12. 45. 3,4	3,047	85. 27. 29	+19,66
525	k Virginis	6	192. 13. 1	46,22	12. 48. 52,1	3,081	92. 40. 30	+19,60

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			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
526	η Virginis	3	192. 56. 1	45,06	12. 51. 44,1	3,004	77. 54. 30	+19,53
527		7	193. 16. 36	46,22	12. 53. 6,4	3,081	92. 31. 37	+19,51
528	g Virginis	5	194. 13. 47	46,85	12. 56. 55,1	3,123	99. 36. 46	+19,43
529	δ Virginis	4	194. 46. 34	46,41	12. 59. 6,3	3,094	94. 24. 46	+19,39
530	Virginis		195. 13. 35	47,44	13. 0. 54,5	3,163	105. 3. 24	+19,35
531		7	195. 49. 12	45,77	13. 3. 16,8	3,051	87. 25. 23	+19,29
532			196. 52. 23	47,81	13. 7. 29,5	3,187	107. 7. 42	+19,18
533	γ Hydræ cont.	4	196. 53. 7	48,38	13. 7. 32,5	3,225	112. 3. 26	+19,18
534		9	197. 45. 56	47,27	13. 11. 3,7	3,151	101. 28. 23	+19,09
535	Virginis		197. 57. 2	47,87	13. 11. 48,1	3,191	106. 37. 43	+19,08
536	α Virginis	1	198. 32. 21	47,15	13. 14. 9,4	3,143	100. 3. 33	+19,01
537	i Virginis	6	198. 54. 50	47,35	13. 15. 39,3	3,157	101. 36. 32	+18,97
538	Virginis		199. 4. 13	47,76	13. 16. 16,9	3,184	104. 52. 38	+18,95
539		7	199. 37. 27	46,05	13. 18. 29,8	3,070	90. 16. 4	+18,89
540	1. l Virginis	7	199. 52. 27	46,66	13. 19. 29,8	3,111	95. 22. 47	+18,86
541	2. l Virginis	6	200. 16. 9	46,65	13. 21. 4,6	3,110	95. 9. 50	+18,81
542	h Virginis	6	200. 29. 2	47,15	13. 21. 56,1	3,143	99. 4. 32	+18,78
543	ζ Virginis	3	201. 0. 25	45,96	13. 24. 1,7	3,064	89. 30. 59	+18,72
544	3. l Virginis	6	201. 9. 19	46,57	13. 24. 37,3	3,105	94. 19. 12	+18,70
545	m Virginis	6	202. 39. 18	47,05	13. 30. 37,2	3,137	97. 38. 11	+18,51
546		7,8	203. 15. 0	46,63	13. 33. 0,0	3,109	94. 26. 1	+18,43
547		7	203. 22. 43	47,88	13. 33. 30,9	3,192	103. 9. 4	+18,41
548			203. 41. 40	47,64	13. 34. 46,7	3,176	101. 22. 1	+18,36
549			204. 0. 27	48,48	13. 36. 1,8	3,232	106. 47. 58	+18,32
550			204. 37. 28	48,59	13. 38. 29,8	3,239	107. 4. 46	+18,23
551	η Ursæ	2	204. 48. 58	35,88	13. 39. 15,9	2,392	39. 37. 59	+18,20
552	p Virginis	6	205. 59. 15	46,09	13. 43. 57,0	3,073	90. 27. 43	+18,02
553		7	207. 15. 34	47,17	13. 49. 2,2	3,145	97. 7. 52	+17,82
554		7	208. 18. 59	47,40	13. 53. 15,9	3,160	98. 14. 28	+17,65
555	θ Centauri	2	208. 36. 13	52,82	13. 54. 24,9	3,521	125. 19. 27	+17,60
556		7,8	208. 48. 7	47,36	13. 55. 12,5	3,157	97. 52. 52	+17,57
557		6,7	208. 54. 34	47,43	13. 55. 38,2	3,162	98. 18. 11	+17,55
558		8	208. 57. 22	48,65	13. 55. 49,4	3,243	105. 10. 38	+17,54
559		6,7	209. 51. 3	48,75	13. 59. 24,2	3,250	105. 18. 2	+17,38
560	κ Virginis	4	210. 25. 51	47,68	14. 1. 43,4	3,179	99. 17. 19	+17,29

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Precess.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
561			210. 51. 53	46,91	14. 3. 27,5	3,127	94. 57. 48	+ 17,11
562	ι Virginis	4	211. 15. 30	46,93	14. 5. 2,0	3,129	94. 59. 8	+ 17,15
563	α Bootis	1	211. 31. 54	42,14	14. 6. 7,6	2,809	69. 41. 55	+ 17,09
564	λ Virginis	4	211. 56. 40	48,35	14. 7. 46,7	3,233	102. 23. 41	+ 17,01
565		8	212. 13. 14	47,10	14. 8. 52,9	3,140	95. 46. 8	+ 16,96
566		8	212. 53. 26	47,32	14. 11. 33,7	3,155	96. 47. 35	+ 16,84
567	Libræ	8	213. 2. 25	48,09	14. 12. 9,7	3,206	100. 44. 40	+ 16,81
568		8	213. 29. 17	48,45	14. 13. 57,1	3,230	102. 23. 34	+ 16,72
569		8,9	214. 6. 0	48,49	14. 16. 24,0	3,233	102. 24. 19	+ 16,60
570		8	214. 20. 8	47,82	14. 17. 20,5	3,188	99. 3. 5	+ 16,55
571		7,8	215. 45. 53	50,17	14. 23. 3,5	3,345	109. 30. 25	+ 16,27
572		7,8	216. 28. 15	48,42	14. 25. 53,0	3,228	101. 24. 21	+ 16,12
573		7,8	216. 56. 13	48,07	14. 27. 44,9	3,205	99. 38. 12	+ 16,02
574		8	217. 40. 7	48,48	14. 30. 40,5	3,232	101. 19. 33	+ 15,87
575	μ Virginis	4	218. 0. 19	47,04	14. 32. 1,3	3,136	94. 43. 53	+ 15,80
576		7,8	218. 34. 38	50,64	14. 34. 18,5	3,376	110. 16. 18	+ 15,67
577		7,8	218. 49. 55	50,71	14. 35. 19,7	3,381	110. 25. 50	+ 15,62
578	μ Libræ	5	219. 27. 36	49,02	14. 37. 50,4	3,268	103. 15. 37	+ 15,48
579	Libræ	2,3	219. 46. 36	49,48	14. 39. 6,4	3,299	105. 6. 40	+ 15,41
580	α Libræ		219. 49. 28	49,50	14. 39. 17,9	3,300	105. 9. 24	+ 15,40
581	1. ξ Libræ	6	220. 45. 10	48,57	14. 43. 0,7	3,238	101. 1. 46	+ 15,19
582	2. ξ Libræ	6	221. 21. 3	48,49	14. 45. 24,2	3,233	100. 33. 1	+ 15,05
583			221. 43. 7	48,44	14. 46. 52,5	3,229	100. 17. 59	+ 14,97
584		6,7	221. 53. 33	48,45	14. 47. 34,2	3,230	100. 17. 13	+ 14,93
585	δ Libræ	4,5	222. 26. 47	47,84	14. 49. 47,1	3,189	97. 40. 26	+ 14,79
586		3	222. 57. 18	52,23	14. 51. 49,2	3,482	114. 26. 33	+ 14,67
587	1. ν Libræ	5	223. 44. 12	49,84	14. 54. 56,8	3,323	105. 25. 45	+ 14,48
588	2. ν Libræ	7,8	223. 46. 57	49,91	14. 55. 7,8	3,327	105. 39. 29	+ 14,47
589	1. ι Libræ	4	225. 4. 18	50,90	15. 0. 17,2	3,393	108. 58. 58	+ 14,17
590	2. ι Libræ	7,8	225. 20. 52	50,89	15. 1. 23,5	3,393	108. 50. 36	+ 14,09
591		7	226. 3. 43	51,74	15. 4. 14,9	3,449	111. 36. 28	+ 13,92
592	β Libræ	2	226. 26. 7	48,21	15. 5. 44,4	3,214	98. 35. 44	+ 13,82
593		9	227. 1. 4	48,18	15. 8. 4,3	3,212	98. 21. 57	+ 13,67
594	1. ο Libræ	7	227. 19. 40	49,91	15. 9. 18,7	3,327	104. 46. 41	+ 13,59
595		6,7	228. 4. 3	49,08	15. 12. 16,2	3,272	101. 36. 23	+ 13,39

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
596	ε Libræ	4	228. 12. 42	48,53	15. 12. 50,8	3,235	99. 33. 9	+ 13,36
[illegible]	1. ζ Libræ	4	229. 6. 38	50,85	15. 16. 26,5	3,357	105. 58. 13	+ 13,12
598			229. 25. 59	50,59	15. 17. 43,9	3,373	106. 42. 3	+ 13,04
599	3. ζ Libræ		229. 42. 51	50,37	15. 18. 51,4	3,358	105. 52. 29	+ 12,97
600		7,8	229. 55. 2	51,43	15. 19. 40,1	3,429	109. 25. 57	+ 12,91
601		7	230. 8. 44	51,30	15. 20. 34,9	3,420	108. 56. 25	+ 12,85
602	4. ζ Libræ		230. 16. 18	50,48	15. 21. 5,2	3,365	106. 7. 36	+ 12,82
603			230. 28. 58	54,01	15. 21. 56,9	3,601	117. 19. 24	+ 12,76
604	γ Libræ	3,4	230. 57. 4	49,92	15. 23. 48,3	3,328	104. 4. 31	+ 12,63
[illegible]			231. 4. 45	54,11	15. 24. 19,0	3,607	117. [illegible] 25	+ 12,60
606		8	231. 13. 40	53,50	15. 24. 54,7	3,567	115. 34. 5	+ 12,56
607		9	231. 34. 29	49,87	15. 26. 17,9	3,325	103. 47. 6	+ 12,47
608		8	231. 42. 8	49,89	15. 26. 48,5	[illegible]	103. 48. 31	+ 12,43
609			231. 42. 41	51,31	15. 26. 50,7	3,421	108. 35. [illegible]	+ 12,43
610	Libræ	7	231. 58. 33	52,76	15. 27. 54,2	3,517	113. 7. 12	+ 12,36
611	κ Libræ	4	232. 28. 14	51,49	15. 29. 52,9	3,433	108. [illegible] 56	+ 12,22
612		8	232. 44. 54	50,39	15. 30. [illegible]	3,359	105. 19. 29	+ 12,14
613	η Libræ	4	233. 4. 21	50,31	15. 32. 17,4	3,354	104. 59. 19	+ 12,05
614	α Serpentis		233. 29. 5	44,01	15. 33. 56,3	2,934	82. 54. 7	+ 11,93
615	b Scorpii	6	234. 35. 44	53,67	15. 38. 22,9	3,578	115. 5. 48	+ 11,62
616	1. a Scorpii	5	235. 15. 32	53,59	15. 41. 2,1	3,573	114. 41. 0	+ 11,42
617	λ Libræ	4	235. 17. 34	51,86	14. 41. 10,3	3,457	[illegible] 31. 24	+ 11,41
618	θ Libræ	4	235. 28. 21	50,79	15. 41. 53,4	3,386	106. 5. 56	+ 11,36
619	2. a Scorpii	7,8	235. 31. 9	53,58	15. 42. 4,6	3,572	114 36. 16	+ 11,35
620		8	235. 51. 6	51,65	15. 43. 24,4	3,443	108. 44. 47	+ 11,25
621	ρ Scorpii	4,3	235. 59. 15	55,05	15. 43. 57,0	3,670	118. 35. 1	+ 11,21
622	π Scorpii	3	236. 32. 38	54,00	15. 46. 10,5	3,600	115 29. 33	+ 11,06
623	ψ Libræ		236. 36. 51	50,09	15. 46. 27,4	3,339	103. 39. 31	+ 11,04
[illegible]	δ Scorpii	3,2	236. 59. 11	52,82	15. 47. 56,7	3,521	112 0. 27	+ 10,92
625		7	237. 40. 17	54,01	15. 50. 41,1	3,601	115. 15 45	+ 10,72
626		4,3	238. 12. 44	49,26	15. 52. 50,8	3,284	100. 46 [illegible]	+ 10,56
627	1. β Scorpii	4	238. 18. 46	51,97	15. 53. 15,1	3,465	109 12. 52	+ 10,53
628	2. β Scorpii	8	238. 18. 55	51,97	15. 53. 15,7	3,465	109. 12 40	+ 10,53
629	1. ω Scorpii	5	238. 33. 17	52,28	15. 54. 33,1	3,485	110. 5. 3	+ 10,4[illegible]
630	2. ω Scorpii	5	238. 46. 47	52,86	15. 55. 7,1	3,494	110. 17. 4	+ 10,39

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

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631	1. c Scorpii	6	239. 49. 47	55,17	15. 59. 19,1	3,678	117. 51. 10	+ 10,07
632	2. c Scorpii	6,5	239 50. 57	54,99	15. 59. 23,8	3,666	117. [illegible] 48	+ 10,07
633		8	239. 56. 58	51,96	15. 59. 47,9	3,464	108. 53. 16	+ 10,04
634	υ Scorpii	4	239. 57. 17	51,96	15. 59 49,1	3,464	108. 53. 57	+ 10,03
635		8	240. 32. 48	54,13	16. 2. 11,2	3,609	114. 58. 38	+ 9,86
636	d Scorpii	6,5	241. 19. 34	55,40	16. 5. 18,3	3,693	118. 4. 21	+ 9,62
637	Scorpii		242. 0. 29	53,77	16. 8. 1,9	3,585	113. 38. 47	+ 9,41
638	σ Scorpii	4	242 6. 47	54,31	16. 8. 27,1	3,621	115. 4. 16	+ 9,38
639		8	242. 52. 25	55,99	16. 11. 29,7	3,733	119. 11. 37	+ 9,14
640	ψ Ophiuchi	5	242. 57. 37	52,35	16. 11. 50,5	3,490	109. 31. 43	+ 9,11
641	g Ophiuchi		243. 15. 19	53,60	16. 13. 1,3	3,573	112. 56. 45	+ 9,02
642	χ Ophiuchi	5	243. 43. 10	51,85	16. 14. 52,7	3,457	107. 57. 49	+ 8,88
643	α Scorpii	1	244. 8. 21	54,80	16. 16. 33,4	3,653	115. 56. 55	+ 8,75
644		7,8	244. 37. 47	54,88	16. 18. 31,1	3,659	116. 3. 53	+ 8,59
645	φ Ophiuchi	5	244. 47. 11	51,27	16. 19. 8,7	3,418	106. 8. 17	+ 8,54
646	ω Ophiuchi	5	244. 55. 42	52,99	16. 19. 42,8	3,533	111. 0. 0	+ 8,49
647	τ Scorpii	4	245. 42. 31	55,64	16 22. 50,1	3,709	117. 45. 42	+ 8,25
648		8	246. 34. 48	51,91	16. 26. 19,1	3,461	107. 44. 42	+ 7,97
649		8	247. 3. 16	52,73	16. 28. 13,1	3,515	109. 58. 54	+ 7,81
650		7,8	247. 17. 15	51,90	16. 29. 9,0	3,460	107. 38. 4	+ 7,74
651	m Scorpii	6	247. 21. 47	51,79	16. 29. 27,1	3,453	107. 19. 8	+ 7,71
652		7	247. 23. 38	52,58	16. 29. 34,5	3,505	109. 30. 23	+ 7,71
653		7	247. 58. 27	55,94	16. 31. 53,8	3,729	118. 6. 6	+ 7,52
654	ι Scorpii		249. 9. 17	58,61	16. 36. 37,1	3,907	123. 53. 36	+ 7,14
655	Ophiuchi	8	249. 14. 35	54,47	16. 36. 58,3	3,631	114. 15. 12	+ 7,11
656		8	249. 42. 58	51,47	16. 38. 51,9	3,431	106. 10. 6	+ 6,9[illegible]
657		7	250. 15. 40	52,91	16. 41. 2,7	3,527	110. 2. 48	+ 6,7[illegible]
658	Scorpii		250. 53. 1	58,28	16. 43. 32,1	3,885	122. 54. 28	+ 6,57
659		6,7	251. 2. 23	53,99	16. 44. 9,5	3,599	112. 47. 55	+ 6,52
660		7,8	251. 11. 11	52,63	16. 44. 44,7	3,509	109. 11. 28	+ 6,47
661		7	251. 46. 54	54,80	16. 47. 7,6	3,653	114. 45. 17	+ 6,28
662		6,7	251. 49. 42	54,77	16. 47. 18,8	3,651	114. 39. 18	+ 6,26
663	Ophiuchi	7,8	252. 23. 54	52,44	16. 49. 35,6	3,496	108. 33. 36	+ 6,07
664		8	252. 46. 33	55,11	16. 51. 6,2	3,674	115. 22. 50	+ 5,95
665		8	252. 57. 20	55,09	16. 51. 49,3	3,673	115. 19. 37	+ 5,88

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			D M. S.	S.	H. M. S.	S.	D. M. S.	S.
666		7,8	253. 5. 7	53,07	16. 52. 20,5	3,538	110. 10 58	+ 5,8[illegible]
667	Scorpii	7,8	253. 25 12	53,49	16. 53. 40,8	3,566	111. 15. 18	+ 5,72
668		7,8	253. 28. 21	54,48	16. 53. 53,4	3,699	116. 12. 37	+ 5,71
669		6,7	254. 1. 0	52,03	16. 56. 4,0	3,469	107. 18. 48	+ 5,52
670	Ophiuchi	3	254. 35. 18	51,36	16. 58. 21,2	3,424	105. 26. 55	+ 5,33
671		5	255. 35. 31	52,80	17. 2. 22,1	3,520	109. 14. 22	+ 4,99
672	a Ophiuchi	5	255. 37. 6	55,60	17. 2. 28,4	3,707	116. 14. 18	+ 4,97
673	Scorpii		255. 50. 19	55,59	17. 3. 21,2	3,706	116. 12. 51	+ 4,90
674			256. 18. 19	54,71	17. 5. 13,3	3,647	114. 2. 23	+ 4,74
675		8	256. 19. 34	54,62	17. 5. 18,2	3,641	113. 49. 17	+ 4,73
676	ϱ Ophiuchi	4	257. 6. 22	53,47	17. 8. 25,4	3,565	110. 52. 6	+ 4,48
677	Scorpii	8	257. 12. 27	55,02	17. 8. 49,8	3,668	114. 43. 59	+ 4,44
678	ϑ Ophiuchi	3	257. 16. 55	55,04	17. 9. 7,7	3,669	114. 46. 15	+ 4,42
679	Ophiuchi	7	257. 32. 22	56,39	17. 10. 9,4	3,759	117. 55. 17	+ 4,33
680			258. 2. 25	53,64	17. 12. 9,7	3,576	111. 13. 37	+ 4,16
681	ι Scorpii	7	258. 4. 12	54,76	17. 12. 16,8	3,651	114. 2. 0	+ 4,15
682	b Ophiuchi	5	258. 23. 23	54,75	17. 13. 33,5	3,650	113. 57. 46	+ 4,04
683	c Ophiuchi	5	259. 39. 13	54,71	17. 18. 36,8	3,647	113. 46. 52	+ 3,60
684	λ Scorpii	2,3	259. 50. 31	60,85	17. 19. 22,3	4,057	126. 55. 54	+ 3,54
685			260. 11. 41	52,18	17. 20. 46,7	3,479	107. 19. 36	+ 3,41
686	Ophiuchi	8	260. 40. 20	53,96	17. 22. 41,3	3,597	111. 53. 1	+ 3,25
687	α Ophiuchi	2	261. 17. 59	41,54	17. 25. 11,9	2,76[illegible]	77. 16. 18	+ 3,04
688		7	261. 23. 31	51,49	17. 25. 34,0	3,433	105. 25. 32	+ 3,00
689	ξ Serpentis		261. 23. 40	51,42	17. 25. 34,6	3,428	105. 14. 55	+ 3,00
690	β Draconis	3	261. 26. 5	20,22	17. 25. 44,3	1,348	37. 32. 12	+ 2,99
691		7	262. 39. 24	51,51	17. 30. 37,6	3,434	105. 26. 15	+ 2,56
692	d Ophiuchi	6	262. 43. 14	53,88	17. 30. 52,9	3,592	111. 33. 42	+ 2,54
693	p Sagittarii	6	263. 35. 16	56,49	17. 34. 21,1	3,766	117. 43. 55	+ 2,23
694			263. 50. 1	56,12	17. 35. 20,0	3,741	116. 52. 45	+ 2,15
695		7,8	264. 35. 52	54,42	17. 38. 23,5	3,628	112. 50. 11	+ 1,88
696		7,8	264. 59. 15	52,91	17. 39. 57,0	3,527	109. 2. 52	+ 1,75
697		6	265. 21. 0	49,84	17. 41. 24,0	3,323	100. 49. 47	+ 1,63
698		7,6	265. 33. 37	52,81	17. 43. 34,5	3,521	108. 44. 51	+ 1,44
699		6	265. 36. 34	50,01	17. 42. 26,3	3,334	101. 16. 31	+ 1,54
700		7	265. 55. 40	53,65	17. 43. 42,7	3,577	110. 53. 49	+ 1,43

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			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
701		6,7	266. 3. 41	51,66	17. 44. 14,7	3,444	105. 45. 22	+ 1,38
702	Sagittarii	6	266. 44. 41	54,84	17. 46. 58,7	3,656	113. 46. 40	+ 1,14
703	i Sagittarii		266. 49. 48	55,03	17. 47. 19,8	3,669	114. 14. 48	+ 1,11
704		7,8	266. 52. 48	58,43	17. 47. 30,8	3,562	110. 18. 16	+ 1,09
705		6,7	267. 17. 48	54,42	17. 49. 11,2	3,628	112. 45. 26	+ 0,95
706	a Sagittarii		267. 29. 51	55,04	17. 49. 59,4	3,669	114. 15. 44	+ 0,88
707		8	267. 34. 48	54,40	17. 50. 18,8	3,627	112. 41. 57	+ 0,85
708	Sagittarii		267. 45. 2	55,08	17. 51. 0,1	3,672	114. 20. 45	+ 0,79
709	γ Draconis	2	267. 56. 28	20,83	17. 51. 45,5	1,389	38. 28. 44	+ 0,72
710	γ Sagittarii	3	268. 4. 52	57,77	17. 52. 19,5	3,851	120. 24. 23	+ 0,68
711		7,8	268. 39. 3	53,89	17. 54. 36,2	3,593	111. 26. 50	+ 0,47
712		6	268. 41. 45	56,88	17. 54. 47,0	3,792	118. 27. 35	+ 0,46
713		6,7	269. 8. 18	57,94	17. 56. 33,2	3,863	120. 44. 31	+ 0,30
714		6	269. 43. 33	54,83	17. 58. 54,2	3,655	113. 46. 28	+ 0,10
715	1. μ Sagittarii	4	270. 18. 11	53,75	18. 1. 12,7	3,583	111. 5. 44	— 0,10
716		7	270. 24. 46	54,02	18. 1. 38,9	3,601	111. 44. 59	— 0,14
717	2. μ Sagittarii	4	270. 40. 23	53,62	18. 2. 41,5	3,575	110. 46. 20	— 0,23
718		7	270. 40. 50	53,51	18. 2. 43,3	3,567	110. 25. 54	— 0,23
719	δ Sagittarii	3	271. 53. 8	57,54	18. 7. 32,5	3,836	119. 53. 51	— 0,66
720	ι Sagittarii	2,3	272. 33. 30	59,76	18. 10. 14,0	3,984	124. 27. 47	— 0,90
721		6	273. 12. 41	53,56	18. 12. 50,7	3,571	110. 36. 7	— 1,12
722	λ Sagittarii	4	273. 45. 8	55,57	18. 15. 0,5	3,705	115. 31. 2	— 1,31
723		7,8	274. 6. 10	55,50	18. 16. 24,7	3,700	115. 22. 14	— 1,43
724		7	274. 27. 49	52,84	18. 17. 51,8	3,523	108. 50. 37	— 1,56
725		6	274. 45. 7	52,91	18. 19. 0,5	3,527	109. 1. 16	— 1,66
726		7	274. 47. 2	52,71	18. 19. 8,1	3,514	108. 31. 42	— 1,68
727		8	275. 2. 2	52,94	18. 20. 8,1	3,529	109. 6. 18	— 1,76
728		6,7	275. 10. 58	51,37	18. 20. 43,8	3,425	105. 0. 6	— 1,81
729			275. 12. 40	52,70	18. 20. 50,7	3,518	108. 30. 7	— 1,82
730		7	275. 15. 49	54,98	18. 21. 3,3	3,665	114. 10. 20	— 1,84
731		6,7	275. 24. 47	51,86	18. 21. 39,1	3,424	104. 59. 36	— 1,89
732		8	275. 25. 25	55,06	18. 21. 41,6	3,651	114. 21. 57	— 1,90
733		7	275. 45. 7	53,05	18. 23. 0,5	3,537	109. 24. 51	— 2,01
734		7	276. 11. 13	53,02	18. 24. 44,9	3,535	109. 21. 56	— 2,16
735			276. 19. 56	53,89	18. 25. 19,7	3,593	111. 33. 10	— 2,20

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MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Preces. S.
736		7,8	276. 25. 1	52,26	18. 25. 40,1	3,484	107. 23. 30	— 2,24
737		7	276. 26. 5	54,75	18. 25. 44,3	3,650	118 39. 57	— 2,24
738		7	276. 35. 33	53,74	18. 26. 22,2	3,583	111. 11. 11	— 2,30
739	α Lyræ	1	277. 27. 8	30,16	18 29. 48,5	2,011	51. 24. 17	— 2,59
740	φ Sagittarii	[illegible]	278. 7. 55	56,21	18. 32. 31,7	3,747	117. 11. 18	— 2,84
741		7,8	278. 24. 17	53,16	18 33. 37,1	3,544	109. 48. 29	— 2,93
742		8	278. 50. 58	53,42	18. 35. 23,9	3,561	110. 26. 58	— 3,09
743			279. 18. 6	53,43	18. 37. 12,4	3,562	110. 32. 45	— 3,24
744		8	279. 33. 18	54,16	18. 38. 13,2	3,611	112. 29. 6	— 3,33
745		8	279. 52. 53	54,06	18. 39. 31,5	3,604	112. 8. 54	— 3,44
746			280. 21. 43	53,83	18. 41. 25,9	3,589	111. 35. 59	— 3,60
747	1 ν Sagittarii	5	280 22. 20	54,38	18. 41 29,3	3,625	112. 50. 7	— 3,61
748	σ Sagittarii	3	280. 32. 58	55,86	18. 42. 11,8	3,724	116. 32. 18	— 3,67
749	2. ν Sagittarii	5	280. 36. 15	54,35	18. 42. 25,0	3,623	112 54. 55	— 3,69
750	1. ξ Sagittarii	7	281. 12. 59	53,53	18. 44. 51,5	3,569	110. 54. 46	— 3,90
751	2. ξ Sagittarii	[illegible]	281. 17. 58	53,71	18. 45. 11,9	3,581	111 21. 50	— 3,93
752		[illegible]	281. 25. 55	53,44	18. 45. 43,7	3,563	112. 41. 9	— 3,98
753		6,7	282. 14. 33	54,32	18. 48. 58,2	3,621	112. 58 25	— 4,25
754	ζ Sagittarii	3	282. 18. 36	57,40	18. 49. 14,4	3,827	120. 9. 42	— 4,27
755		7	282. 40. 4	54,19	18. 50 40,2	3,613	115. 6. 55	— 4,40
756	ο Sagittarii	[illegible]	283. 1. 23	53,92	18. 52. 5,5	3,595	112. 1. 52	— 4,52
757	τ Sagittarii	4	283. 27. 20	56,36	18. 53. 49,3	3,757	177. 57. 24	— 4,66
758		7	283. 34. 9	56,79	18. 54. 16,6	3,786	118. 56. 22	— 4,70
759		8	283. 35. 2	54,21	18. 54. 20,1	3,614	112. 48. 8	— 4,71
760		7,8	283. 51. 4	55,08	18. 55. 24,3	3,672	114. 57. 57	— 4,79
761	π Sagittarii	3,4	284. 19. 4	53,61	18. 57. 16,3	3,574	111. 20. 23	— 4,95
762		7,8	284. 21. 11	53,13	18. 57. 24,7	3,542	110. 7. 40	— 4,96
763		8	284. 58. 43	53,84	18. 59. 54,9	3,589	111 59. 20	— 5,18
764		8	284. 59. 26	51,18	18. 59. 57,7	3,412	104. 54. 59	— 5,19
765		6,7	285. 4. 20	55,56	19. 0. 17,3	3,704	116. 14. 20	— 5,21
766	ψ Sagittarii	6	285. 38. 49	55,27	19. 2. 33,3	3,685	115. 36. 1	— 5,41
767		7	285. 41. 12	54,82	19. 2. 44,8	3,655	114. 31. 4	— 5,42
768		9	285. 55. 4	52,17	19. 3. 40,3	3,478	107. 41. 27	— 5,50
769	d Sagittarii	6	286. 20. 11	52,76	19. 5. 20,7	3,517	109. 18. 34	— 5,64
770		8,9	286. 28. 30	52,72	19. 5. 54,0	3,515	109 13. 22	— 5,68

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Precess. S.
771		6	286. 45. 25	51,48	19. 7. 1,7	3,432	106. 52. 52	— 5,78
772		7	287. 0. 40	54,07	19. 8. 2,7	3,605	112. 48. 33	— 5,86
773	1. ς Sagittarii	5	287. 22. 20	52,32	19. 9. 29,3	[illegible]	108. 13. 32	— 5,96
774	2. ς Sagittarii	6	287. 23. 49	52,49	19. 9. 35,3	3,499	108. 40. 49	— 5,99
775	ν Sagittarii	[illegible]	287. 25. 27	51,63	19. 9. 41,8	3,442	106. 19. 56	— 6,00
776	1. χ Sagittarii	5	288. 7. 16	54,86	19. 12. 29,1	3,657	114. 53. 54	— 6,24
777	2. χ Sagittarii	[illegible]	288. 11. 39	54,64	19. 12. 46,6	[illegible]	114. 21. 18	— 6,27
778		7	288. 27. 2	53,77	19. 13. 48,1	3,585	112. 10. 30	— 6,35
779		7	288. 33. 23	51,29	19. 14. 13,5	3,419	105. 30. 41	— 6,38
780		7,8	288. 42. 58	53,76	19. 14. 51,9	[illegible]	112. 10. 27	— 6,43
781		7,8	289. 9. 36	51,30	19. 16. 38,4	3,420	105. 30. 47	— 6,56
782	η Sagittarii		289. 12. 57	55,83	19. 16. 51,8	3,722	117. 23. 45	— 6,59
783		6	289. 23. 55	51,36	19. 17. 35,7	3,424	105. 46. 23	— 6,66
784		8	289. 36. 27	53,55	19. 18. 25,8	3,570	111. 43. 59	— 6,73
785		7,8	289. 42. 22	53,54	19. 18. 48,5	[illegible]	111. 43. 51	— 6,76
786		8	290. 28. 12	54,50	19. 21. 52,8	3,633	114. 17. 44	— 7,01
787	1. h Sagittarii	5	290. 48. 59	54,82	19. 23. 15,9	3,655	115. 9. 41	— 7,12
788	2. h Sagittarii	5	290. 58. 44	54,88	19. 23. 54,9	3,659	115. 19. 49	— 7,18
789		8	291. 2. 33	52,57	19. 24. 10,2	3,505	109. 17. 55	— 7,20
790		8	291. 12. 57	52,34	19. 24. 51,8	3,489	108. 40. 53	— 7,26
791		8	291. 47. 50	54,27	19. 27. 11,3	3,618	113. 53. 18	— 7,44
792		7,8	291. 52. 15	54,26	19. 27. 29,0	3,617	113. 53. 28	— 7,47
793	1. e Sagittarii	6	292. 10. 14	51,61	19. 28. 40,9	3,441	106. 45. 30	— 7,56
794	2. e Sagittarii	6,5	292. 37. 25	51,54	19. 30. 29,7	3,436	106. 36. 1	— 7,71
795		[illegible]	292. 53. 30	51,30	19. 31. 14,0	3,420	105. 56. 21	— 7,79
796	f Sagittarii	6	293. 31. 39	52,80	19. 34. 6,6	3,520	110. 14. 56	— 8,00
797		8	293. 54. 13	53,22	19. 35. 36,9	3,548	111. 27. 26	— 8,12
798		7,8	294. 3. 41	50,65	19. 36. 14,7	3,377	104. 12. 15	— 8,17
799			294. 17. 14	50,18	19. 37. 8,9	3,345	102. 49. 31	— 8,24
800	α Aquilæ	1	295. 7. 46	43,37	19. 40. 31,1	2,891	81. 40. 36	— 8,51
801	ω Sagittarii	5	295. 44. 14	55,16	19. 42. 56,9	3,677	116. 50. 24	— 8,70
802	b Sagittarii	5	296. 0. 38	55,48	19. 44. 2,5	[illegible]	117. 42. 34	— 8,79
803	g Sagittarii	5	296. 30. 26	51,17	19. 46. 1,7	3,411	106. 1. 55	— 8,94
804	a Sagittarii	5	296. 32. 8	55,06	19. 46. 8,5	3,671	116. 44. 51	— 8,96
805		6	297. 25. 51	55,59	19. 49. 43,4	3,706	118. 16. 40	— 9,23

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			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
806	Sagittarii	7	297. 32. 57	50,52	19. 50. 11,8	3,368	104. 12. 19	— 9,27
807		8	297. 49. 13	53,61	19. 51. 16,9	3,574	113. 10. 9	— 9,36
808		8,9	297. 54. 47	51,09	19. 51. 39,1	3,406	105. 59. 7	— 9,38
809			298. 8. 44	53,12	19. 52. 34,9	3,541	111. 53. 29	— 9,45
810	Sagittarii	7,8	298. 22. 16	49,83	19. 53. 29,1	3,322	102. 10. 45	— 9,52
811	Sagittarii	7	298. 26. 44	50,17	19. 53. 44,9	3,345	103. 14. 37	— 9,54
812		8	299. 1. 1	52,19	19. 56. 4,1	3,479	109. 23. 47	— 9,73
813		8	299. 9. 19	50,91	19. 56. 37,3	3,394	105. 37. 0	— 9,77
814		7,8	299. 15. 32	49,31	19. 57. 2,1	3,287	100. 39. 21	— 9,80
815			299. 19. 0	52,79	19. 57. 16,0	3,519	111. 11. 1	— 9,82
816		9	299. 33. 38	52,36	19. 58. 14,5	3,491	109. 58. 41	— 9,89
817	Capricorni	6	300. 4. 45	50,03	20. 0. 19,0	3,335	103. 0. 6	—10,04
818	ξ Capricorni	6	300. 10. 41	50,09	20. 0. 42,7	3,339	103. 13. 1	—10,07
819	1. α Capricorni	4	301. 11. 8	49,97	20. 4. 44,5	3,331	102. 57. 55	—10,38
820	2. α Capricorni		301. 29. 58	50,01	20. 5. 55,9	3,334	103. 8. 34	—10,47
821	3. α Capricorni	3	301. 35. 50	50,02	20. 6. 23,3	3,335	103. 10. 54	—10,50
822	σ Capricorni	6	301. 48. 57	52,14	20. 7. 15,8	3,476	109. 45. 31	—10,56
823		6,7	302. 14. 26	50,69	20. 8. 57,7	3,380	105. 25. 59	—10,69
824	ν Capricorni	6	302. 15. 3	50,06	20. 9. 0,2	3,337	103. 24. 21	—10,69
825	β Capricorni	3	302. 17. 59	50,70	20. 9. 11,9	3,380	105. 25. 47	—10,71
826		7,8	303. 9. 53	50,49	20. 12. 39,5	3,366	104. 54. 52	—10,97
827		7,8	303. 22. 24	50,44	20. 13. 29,6	3,363	104. 46. 29	—11,03
828		8	303. 23. 27	49,69	20. 13. 33,8	3,313	102. 22. 21	—11,04
829			303. 30. 1	52,14	20. 14. 0,1	3,476	110. 5. 51	—11,07
830	π Capricorni	6	303. 49. 18	51,72	20. 15. 17,2	3,448	108. 53. 8	—11,16
831	ρ Capricorni	6	304. 13. 1	51,57	20. 16. 52,1	3,438	108. 29. 37	—11,27
832		8	304. 15. 1	51,58	20. 17. 0,1	3,439	108. 32. 59	—11,28
833		8	304. 15. 6	51,44	20. 17. 0,4	3,429	108. 6. 57	—11,28
834	1. ο Capricorni*	8	304. 27. 16	51,85	20. 17. 49,1	3,457	109. 25. 56	—11,34
835	2. ο Capricorni	6	304. 27. 40	51,85	20. 17. 50,6	3,457	109. 25. 49	—11,34
836		8,9	305. 5. 15	53,89	20. 20. 21,0	3,593	115. 38. 17	—11,53
837		8	305. 9. 13	51,13	20. 20. 36,9	3,409	107. 18. 22	—11,54
838		[illegible]	305. 13. 31	49,05	20. 20. 54,1	3,070	100. 29. 55	—11,57
839		7	305. 37. 16	50,21	20. 22. 29,1	3,347	104. 24. 47	—11,68
840		7,8	305. 54. 23	51,06	20. 23. 37,5	3,404	107. 13. 56	—11,76

* There appears to be a mistake of 10′ in the polar distance of this star; it should probably be 109°. 15′. 56″.

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841		8	[illegible] [illegible] 59	52,34	[illegible] 24. 15,9	3,489	111. 17. 48	—11,81
842		8	[illegible] [illegible] 22	50,60	20. [illegible] 33,5	3,373	105. 51. 35	—11,90
843		8	306. 28 31	51,21	20. 25. 54,1	3,414	107. 50. 20	—11,98
844	τ Capricorni	6	306. 59. 48	50,55	20. 27. [illegible]	3,370	105. 40. 39	—12,04
845	υ Capricorni	6	307. 1. 12	51,49	20. 28. 4,8	3,433	108. 51. 30	—12,07
846		7	307. 10. 51	50,86	20. [illegible] 43,4	3,391	106. 51. 28	—12,12
847	ψ Capricorni	5	308. 24. 35	53,68	20. [illegible] 38,3	3,579	116. 0. 36	—12,46
848	α Cygni	1	[illegible] 33. 57	[illegible]	[illegible] 34. 15,8	2,039	45. 27. 44	—12,51
849	ε Aquarii	5	309. 4. 25	48,83	20. 36. 17,7	3,255	100. 15. 6	—12,64
850		7	309. 37. 1	51,28	20. 38. 28,1	3,419	108. 47. 45	—12,79
851	ω Capricorni	6	[illegible] 48. [illegible]	54,10	20. 39. 15,6	3,607	117. 41. 26	—12,84
852		8	310. 13. 39	49,83	20. 40. 54,6	3,322	103. 58. 51	—12,95
853	μ Aquarii	4,5	310. 19. 41	48,65	[illegible] 41. 18,7	3,243	99. 45. 30	—12,98
854		7	310. [illegible] 51	49,36	20. 41. 35,4	3,291	102. 21. 19	—13,00
855		7	310. 43. 44	51,16	20. 42. 54,9	3,411	108. 42. 24	—13,08
856		7,8	311. 4. 14	53,75	20. 44. 16,9	3,583	117. 4. 55	—13,17
857		7	311. 28. 28	50,56	20 45. 53,9	3,371	106. 49. 39	—13,28
858		7,8	311. 54. 46	51,39	20. 47. 39,1	3,426	109. 50. 12	—13,39
859		7	312 15. 25	50,94	20. 49. 1,7	3,396	108. 20. 4	—13,48
860	Aquarii	7,8	312. 23. 20	49,80	20. 49. 33,3	3,320	104. 20. 19	—13,52
861		7,8	312. 37. 50	49,29	20. 50. 31,3	3,286	102. 30. 22	—13,58
862	η Capricorni	5	313. 6. 33	51,54	20. 52. 26,2	3,436	110. 40. 19	—13,70
863	ϑ Capricorni	5	313. 31. 50	50,75	20. 54. 7,3	3,383	108. 3. 17	—13,81
864	a Capricorni	6	313. 42. 20	53,03	20. 54. 49,3	3,535	115. 49. 55	—13,85
865	1. χ Capricorni	6	314. 7. 41	51,84	20. 56. 30,7	3,456	112 1. 27	—13,97
866	3. χ Capricorni	6	314. 22 46	51,63	20. 57. 31,1	3,442	111. 23. 8	—14,03
867	ν Aquarii	5	314 32. 4	49,11	20. 58. 8,3	3,274	102 12. 37	—14,06
868		7,8	315. 1 7	49,90	21. 0. 4,5	3,327	105. 19. 7	—14,18
869		7,8	315. 29. 6	52,01	21. 1 56,4	3,467	113. 3. 54	—14,30
870	φ Capricorni	6	315 54. 54	51,52	21. 3. 39,6	3,435	111. 30. 40	—14,40
871		6	316. 32 23	50,73	21. 6. 9,5	3,382	108. 51. 12	—14,55
872		7,8	316. 37 40	51,36	21. 6 30,7	3,424	111. 12. 10	—14,57
873		7,8	316. 53 27	50,22	21. 7. 33,8	3,348	107. 3. 1	—14,63
874		8,9	317. 16. 2	51,43	21. 9 4,1	3,429	111. 41. 40	—14,72
875	ι Capricorni	5	317. 38. 5	50,33	21. 10. 32,3	3,355	107. 43. 1	—14,81

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[illegible] [illegible] Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Precess.
			[illegible] M. S.	S.	H. M. S.	S.	D. M. S.	S.
876		8	318. 1. 23	51,89	21. 12. 5,5	3,459	113. 38. 4	—14,90
877		7	318. 3. 31	51,36	21. 12. 14,1	3,424	111. 43. 53	—14,91
878		6	318. 10. 28	49,29	21. 12. 41,9	3,286	103. 46. [illegible]	—14,94
879	ζ Capricorni	5	318. 39. 52	51,72	21. 14. 39,5	3,448	113. 18. 21	—15,05
880	b Capricorni	6	319. 10. 51	51,50	21. 16. 43,4	3,433	112. 42. 36	—15,17
881		7	319. 12. 39	48,91	21. 16. 50,6	3,261	102. 28. 2	—15,18
882		7,8	319. 32. 46	50,77	21. 18. 11,1	3,385	110. 3. 8	—15,25
883		8	319. 47. 16	49,54	21. 19. 9,2	3,303	105. 11. 57	—15,31
884		9	319. 54. 11	50,76	21. 19. 36,7	3,384	110. 8. 50	—15,34
885	β Aquarii	3	320. 7. 28	47,48	21. 20. 29,9	3,165	96. 29. 5	—15,39
886		9	320. 30. 44	49,94	21. 22. 2,9	3,329	107. 6. 37	—15,47
887		6,7	320. 45. 35	50,89	21. 23. 2,3	3,393	111. 0. 34	—15,53
888			320. 46. 41	50,93	21. 23. 6,7	3,395	111. 10. 14	—15,53
889	ι Capricorni	4	321. 19. 37	50,68	21. 25. 18,4	3,379	110. 23. 44	—15,65
890	ξ Aquarii	5	321. 38. 26	47,94	21. 26. 33,7	3,196	98. 47. 8	—15,72
891	γ Capricorni	4,3	322. 6. 30	49,93	21. 28. 26,0	3,329	107. 36. 0	—15,82
892	1. d Capricorni		322. 31. 47	49,28	21. 30. 7,8	3,285	104. 58. 15	—15,91
893	κ Capricorni	5	322. 43. 38	50,39	21. 30. 54,5	3,359	109. 48. 47	—15,95
894		7,8	322. 51. 52	50,56	21. 31. 27,5	3,371	110. 34. 6	—15,98
895	1. c Capricorni	6	323. 27. 0	48,13	21. 33. 48,0	3,209	100. 2. 13	—16,10
896	λ Capricorni	5	323. 48. 18	48,61	21. 35. 13,2	3,241	102. 19. 28	—16,18
897	δ Capricorni	3	223. 51. 25	49,65	21. 35. 25,7	3,310	107. 4. 2	—16,19
898		7,8	324. 34. 54	48,85	21. 38. 19,6	3,257	103. 41. 33	—16,34
899		8	324. 39. 45	49,75	21. 38. 38,9	3,317	107. 48. 49	—16,35
900		8	325. 0. 12	50,11	21. 40. 0,8	3,341	109. 35. 26	—16,42
901	μ Capricorni	5	325. 27. 20	48,96	21. 41. 49,3	3,264	104. 31. 51	—16,51
902		8	325. 53. 14	49,29	21. 43. 32,9	3,286	106. 14. 21	—16,60
903		6,9	326. 17. 45	49,82	21. 45. 11,0	3,321	108. 53. 7	—16,68
904		9	326. 35. 12	49,21	21. 46. 20,8	3,281	106. 6. 49	—16,74
905		7	326. 44. 58	50,50	21. 46. 59,9	3,367	112. 10. 34	—16,77
906		8	326. 46. 2	48,69	21. 47. 4,1	3,246	103. 39. 43	—16,77
907		8	327. 39. 19	49,69	21. 50. 37,3	3,313	108. 54. 4	—16,94
908	ο Aquarii	5	328. 6. 47	46,60	21. 52. 21,1	3,107	93. 9. 39	—17,02
909	α Aquarii	3	328. 45. 3	46,26	21. 55. 0,2	3,084	91. 19. 55	—17,14
910	ι Aquarii	4	328. 46. 13	48,78	21. 55. 4,9	3,252	104. 52. 44	—17,14

MAYER's CATALOGUE OF THE PRINCIPAL STARS.

Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees. D. M. S.	Annual Variat. + S.	Right Ascension in Time. H. M. S.	Annual Variat. + S.	North polar Distance. D. M. S.	Annual Preces. [illegible]
911	e Aquarii	6	329. 50. 49	48,27	21. 59. 23,3	3,218	102. 35. 20	—17,33
912		7,8	330. 32. 58	48,29	22. 2. 11,9	3,219	102. 57. 19	—17,45
913	ϑ Aquarii	4	331. 26. 8	47,51	22. 5. 44,5	3,167	98. 49. 15	—17,60
914		7	331. 26. 43	47,72	22. 5. 46,8	3,181	100. 4. 42	—17,60
915	ρ Aquarii	5,6	332. 17. 10	47,17	22. 9. 8,7	3,165	98. 58. 4	—17,74
916	γ Aquarii	3	332. 42. 8	46,41	22. 10. 48,5	3,094	92. 26. 21	—17,81
917		6	333. 16. 37	46,95	22. 13. 6,5	3,130	95. 53. 29	—17,90
918		7	333. 26. 23	46,37	22. 13. 45,5	3,091	92. 14. 46	—17,93
919		7,8	333. 52. 56	47,94	22. 15. 31,7	3,196	102. 17. 16	—17,99
920	ζ Aquarii	[illegible]	334. 30. 17	46,18	22. 18. 1,1	3,079	91. 5. 13	—18,09
921	σ Aquarii	5	334. 52. 52	47,79	22. 19. 31,5	3,186	101. 44. 42	—18,15
922		7,8	335. 8. 8	47,81	22. 20. 32,5	3,187	101. 58. 30	—18,19
923	η Aquarii	4	336. 8. 32	46,19	22. 24. 34,1	3,079	91. 11. 33	—18,34
924	κ Aquarii	5	336. 43. 12	46,75	22. 26. 52,8	3,117	95. 17. 12	—18,42
925		8	337. 15. 35	47,45	22. 29. 2,3	3,163	100. 26. 55	—18,49
926		8	337. 18. 47	47,05	22. 29. 15,1	3,137	97. 37. 13	—18,50
927		8	337. 59. 32	47,51	22. 31. 58,1	3,167	101. 11. 46	—18,59
928		7	338. 0. 45	47,26	22. 32. 3,0	3,151	99. 24. 12	—18,59
929		8,9	338. 3. 18	47,11	22. 32. 13,1	3,141	98. 18. 32	—18,60
930		7	338. 4. 2	47,08	22. 32. 16,1	3,139	98. 3. 22	—18,60
931		9	338. 34. 31	47,41	22. 34. 18,1	3,161	100. 44. 39	—18,66
932	1. τ Aquarii	6	339. 8. 11	47,96	22. 36. 32,7	3,197	105. 10. 23	—18,74
933	2. τ Aquarii	6	339. 36. 56	47,85	22. 38. 27,7	3,190	104. 41. 40	—18,79
934			339. 57. 20	47,04	22. 39. 49,3	3,136	95. 25. 4	—18,83
935			339. 57. 20	47,04	22. 39. 49,3	3,136	98. 25. 4	—18,83
936	λ Aquarii	4	340. 24. 50	47,05	22. 41. 39,3	3,137	98. 41. 30	—18,89
937	δ Aquarii	[illegible]	340. 52. 26	48,02	22. 43. 29,7	3,201	106. 55. 58	—18,95
938	α Piscis Aust.	1	341. 30. 1	49,81	22. 46. 0,1	3,321	120. 43. 39	—19,01
939		7	342. 20. 22	47,09	22. 49. 21,5	3,139	100. 0. 1	—19,10
940		6,7	342. 37. 7	46,88	22. 50. 28,5	3,125	98. 10. 0	—19,13
941	β Piscium	4	343. 13. 0	45,75	22. 53. 12,0	3,050	87. 18. 22	—19,20
942	β Pegasi	2	343. 24. 2	43,11	22. 53. 36,1	2,874	63. 3. 11	—19,21
943	1. h Aquarii		343. 33. 4	46,90	22. 54. 12,3	3,127	98. 49. 20	—19,22
944	α Pegasi	2	343. 34. 38	44,60	22. 54. 18,5	2,973	75. 55. 12	—19,23
945	2. h Aquarii	7	343. 35. 37	46,91	22. 54. 22,6	3,127	98. 54. 7	—19,23

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946	3. h Aquarii	6,7	343. 44. 10	46,92	22. 51. 56,7	3,128	99. 3. 57	—19,24
947	4. h Aquarii	9	344. 4. 9	46,88	22. 56. 16,6	3,125	98. 50. 12	—19,28
948	a Piscium.	6	344. 28. 56	45,93	22. 57. 55,7	3,062	89. 0. 44	—19,32
949		7	344. 39. 52	45,93	22. 58. 39,5	3,062	88. 59. 28	—19,33
950		8	344. 57. 8	46,67	22. 59. 48,5	3,111	97. 5. 46	—19,36
951	φ Aquarii	5	345. 51. 41	46,64	23. 3. 26,7	3,109	97. 10. 29	—19,44
952	1. ψ Aquarii	5	346. 13. 6	46,88	23. 4. 52,4	3,125	100. 13. 37	—19,47
953	χ Aquarii	6	346. 29. 29	46,75	23. 5. 57,9	3,117	98. 52. 3	—19,49
954	γ Piscium	4	346. 33. 59	45,85	23. 6. 15,9	3,057	87. 51. 40	—19,50
955	2. ψ Aquarii	5	346. 44. 48	46,86	23. 6. 59,2	3,124	100. 19. 28	—19,51
956	3. ψ Aquarii	5	347. 0. 30	46,88	23. 8. 2,0	3,125	100. 45. 13	—19,53
957	b Piscium	5,6	347. 24. 47	45,70	23. 9. 39,1	3,047	85. 45. 36	—19,56
958		6,7	348. 11. 30	46,08	23. 12. 46,0	3,072	90. 51. 29	—19,62
959		7,8	348. 13. 7	46,71	23. 12. 52,5	3,114	99. 36. 29	—19,62
960	1. κ Piscium	5	349. 2. 39	46,01	23. 16. 10,6	3,067	89. 53. 24	—19,68
961	2. κ Piscium	6,7	349. 7. 31	46,02	23. 16. 30,1	3,068	90. 1. 28	—19,69
962	9 Piscium	5	349. 20. 2	45,68	23. 17. 20,1	3,045	84. 46. 16	—19,70
963		8	349. 42. 14	46,16	23. 18. 48,9	3,077	92. 11. 18	—19,72
964		7	349. 50. 10	46,20	23. 19. 20,7	3,080	92. 56. 33	—19,73
965		7,8	350. 10. 8	46,33	23. 20. 40,5	3,089	95. 13. 46	—19,75
966		7	350. 17. 52	46,15	23. 21. 11,5	3,077	92. 14. 32	—19,76
967		6,7	350. 50. 23	46,15	23. 23. 21,5	3,077	92. 24. 14	—19,79
968		7,8	351. 10. 34	46,48	23. 24. 42,3	3,099	98. 37. 25	—19,81
969		7	351. 11. 22	46,01	23. 24. 45,4	3,067	89. 50. 35	—19,81
970		6,7	351. 25. 19	45,97	23. 25. 41,3	3,065	89. 3. 39	—19,82
971	ι Piscium	5	352. 17. 15	45,81	23. 29. 9,0	3,054	85. 30. 16	—19,86
972	λ Piscium	5	352. 50. 15	45,99	23. 31. 21,0	3,066	89. 22. 18	—19,89
973		8	353. 31. 43	45,78	23. 34. 6,9	3,052	83. 58. 14	—19,92
974		6,7	353. 55. 11	45,93	23. 35. 40,7	3,062	87. 40. 33	—19,94
975		6,7	354. 6. 50	46,49	23. 36. 27,3	3,099	103. 4. 11	—19,94
976		6,7	354. 17. 14	45,98	23. 37. 8,9	3,065	88. 48. 21	—19,95
977		7	354. 26. 18	46,28	23. 37. 45,2	3,085	97. 31. 37	—19,95
978		7	354. 31. 20	45,99	23. 38. 5,3	3,066	88. 56. 58	—19,96
979		6,7	354. 40. 49	46,02	23. 38. 43,2	3,068	90. 5. 16	—19,96
980		6	354. 51. 3	46,37	23. 39. 24,2	3,091	101. 8. 31	—19,97

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Numb. of Stars.	Names and Places of the Stars.	Magnitude.	Right Ascension in Degrees.	Annual Variat. +	Right Ascension in Time.	Annual Variat. +	North polar Distance.	Annual Preces.
			D. M. S.	S.	H. M. S.	S.	D. M. S.	S.
981	Piscium	7	355. 32. 12	46,14	23. 42. 8,8	3,076	94. 19. 9	—19,99
982		6,7	356. 0. 40	46,05	23. 44. 2,7	3,070	91. 3. 27	—20,00
983		6,7	356. 5. 55	46,88	23. 44. 23,7	3,059	84. 5. 40	—20,00
984		6,7	356. 59. 1	46,12	23. 47. 56,1	3,075	94. 43. 8	—20,02
985	ω Piscium	5	357. 8. 5	45,92	23. 48. 32,3	3,061	84. 17. 50	—20,02
986		5	357. 46. 5	46,12	23. 51. 4,3	3,075	94. 11. 40	—20,03
987	Piscium	5	357. 47. 55	46,12	23. 51. 11,7	3,075	97. 10. 44	—20,03
988			357. 49. 16	46,07	23. 51. 17,1	3,071	93. 56. 1	—20,03
989	2. c Piscium	5,6	357. 56. 44	45,93	23. 51. 46,9	3,062	82. 40. 47	—20,03
990	Piscium		358. 38. 56	46,08	23. 54. 35,7	3,072	96. 52. 55	—20,04
991	α Andromedæ	2	359. 23. 20	45,91	23. 57. 33,3	3,061	62. 4. 1	—20,04
992		8	359. 47. 39	46,02	23. 59. 10,6	3,068	93. 43. 39	—20,05

MAYER had no transit instrument to take the right ascension of the stars, but took them with the telescope of his quadrant. He settled the places of several of the bright stars which were visible in the day time, by comparison with the sun in the same parallel. But this could only be applied to those stars whose declinations were less than 18°, as the sun's motion in declination would otherwise be too small. He composed his solar Tables by the help of these bright stars; and by comparing the transits of the said stars with those of the sun when its declination was between 18° and 23½°, taking the sun's right ascension answering to the longitude computed by his solar Tables, he found the errors of the plane of his quadrant at those altitudes; and then by observing the zodiacal stars within those limits, he filled up his catalogue.

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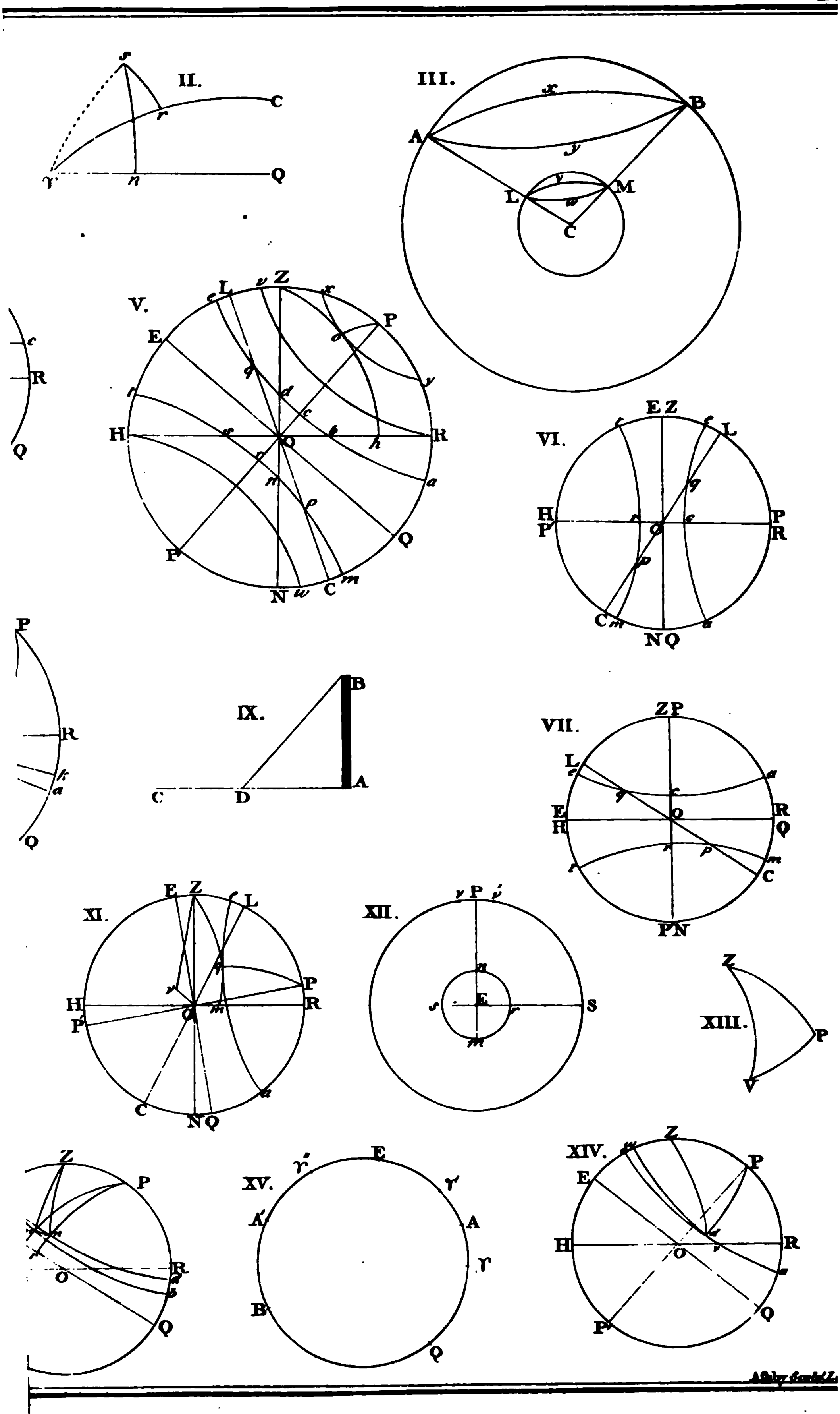

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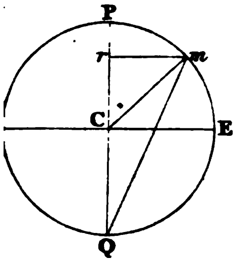

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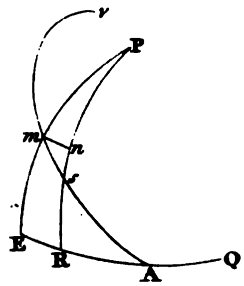

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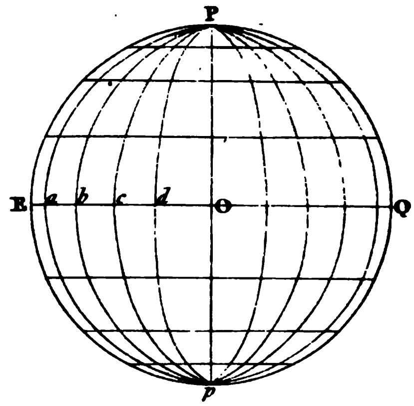

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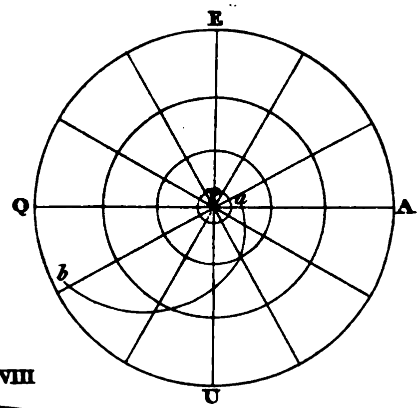

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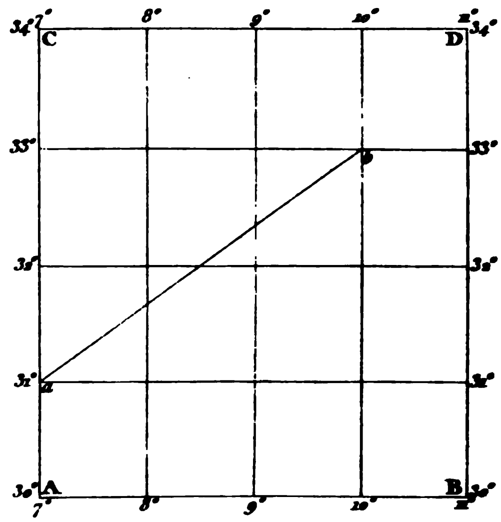

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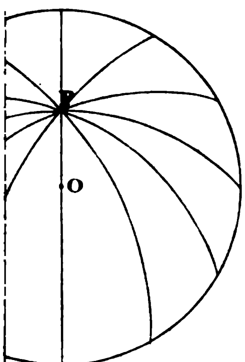

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